# COM/BLM 453
# Data Mining

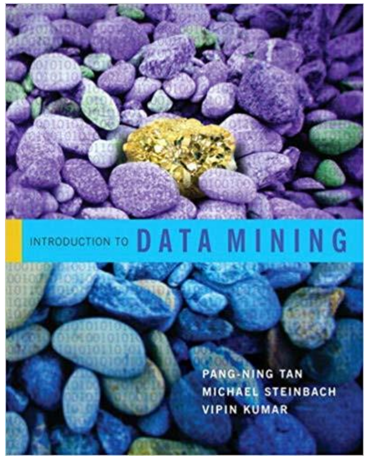
Asst. Prof. Dr. Bulent TUGRUL
btugrul@eng.ankara.edu.tr

Slides are mainly based on:

Introduction to Data Mining

by Pang-Ning Tan, Michael Steinbach, Vipin Kumar

Pearson, 1st Edition, 2005



# Data Mining
# Association Analysis: Basic Concepts and Algorithms

Lecture Notes for Chapter 6

Introduction to Data Mining

by

Tan, Steinbach, Kumar

# Association Rule Mining

- Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

**Market-Basket transactions**

| *TID* | *Items* |
|---|---|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
| 4 | Bread, Milk, Diaper, Beer |
| 5 | Bread, Milk, Diaper, Coke |

**Example of Association Rules**

{Diaper} $\rightarrow$ {Beer},
{Milk, Bread} $\rightarrow$ {Eggs,Coke},
{Beer, Bread} $\rightarrow$ {Milk},

Implication means co-occurrence, not causality!

# Definition: Frequent Itemset

- **Itemset**
  - A collection of one or more items
    - Example: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items
- **Support count ($\sigma$)**
  - Frequency of occurrence of an itemset
  - E.g. $\sigma$({Milk, Bread,Diaper}) = 2
- **Support**
  - Fraction of transactions that contain an itemset
  - E.g. s({Milk, Bread, Diaper}) = 2/5
- **Frequent Itemset**
  - An itemset whose support is greater than or equal to a *minsup* threshold

| *TID* | *Items* |
|---|---|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
| 4 | Bread, Milk, Diaper, Beer |
| 5 | Bread, Milk, Diaper, Coke |

# Definition: Association Rule

- **Association Rule**
  - An implication expression of the form $X \rightarrow Y$, where X and Y are itemsets
  - Example:
    $\{\text{Milk, Diaper}\} \rightarrow \{\text{Beer}\}$

- **Rule Evaluation Metrics**
  - Support (s)
    - Fraction of transactions that contain both X and Y
  - Confidence (c)
    - Measures how often items in Y appear in transactions that contain X

| *TID* | *Items* |
|---|---|
| **1** | **Bread, Milk** |
| **2** | **Bread, Diaper, Beer, Eggs** |
| **3** | **Milk, Diaper, Beer, Coke** |
| **4** | **Bread, Milk, Diaper, Beer** |
| **5** | **Bread, Milk, Diaper, Coke** |

Example:

$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \text{Beer}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|\text{T}|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

# Association Rule Mining Task

- Given a set of transactions T, the goal of association rule mining is to find all rules having
  - support ≥ *minsup* threshold
  - confidence ≥ *minconf* threshold

- Brute-force approach:
  - List all possible association rules
  - Compute the support and confidence for each rule
  - Prune rules that fail the *minsup* and *minconf* thresholds

  ⇒ Computationally prohibitive!

# Mining Association Rules

| *TID* | *Items* |
|---|---|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
| 4 | Bread, Milk, Diaper, Beer |
| 5 | Bread, Milk, Diaper, Coke |

## Example of Rules:

{Milk,Diaper} $\rightarrow$ {Beer} (s=0.4, c=0.67)
{Milk,Beer} $\rightarrow$ {Diaper} (s=0.4, c=1.0)
{Diaper,Beer} $\rightarrow$ {Milk} (s=0.4, c=0.67)
{Beer} $\rightarrow$ {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} $\rightarrow$ {Milk,Beer} (s=0.4, c=0.5)
{Milk} $\rightarrow$ {Diaper,Beer} (s=0.4, c=0.5)

## Observations:

- All the above rules are binary partitions of the same itemset:
  {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

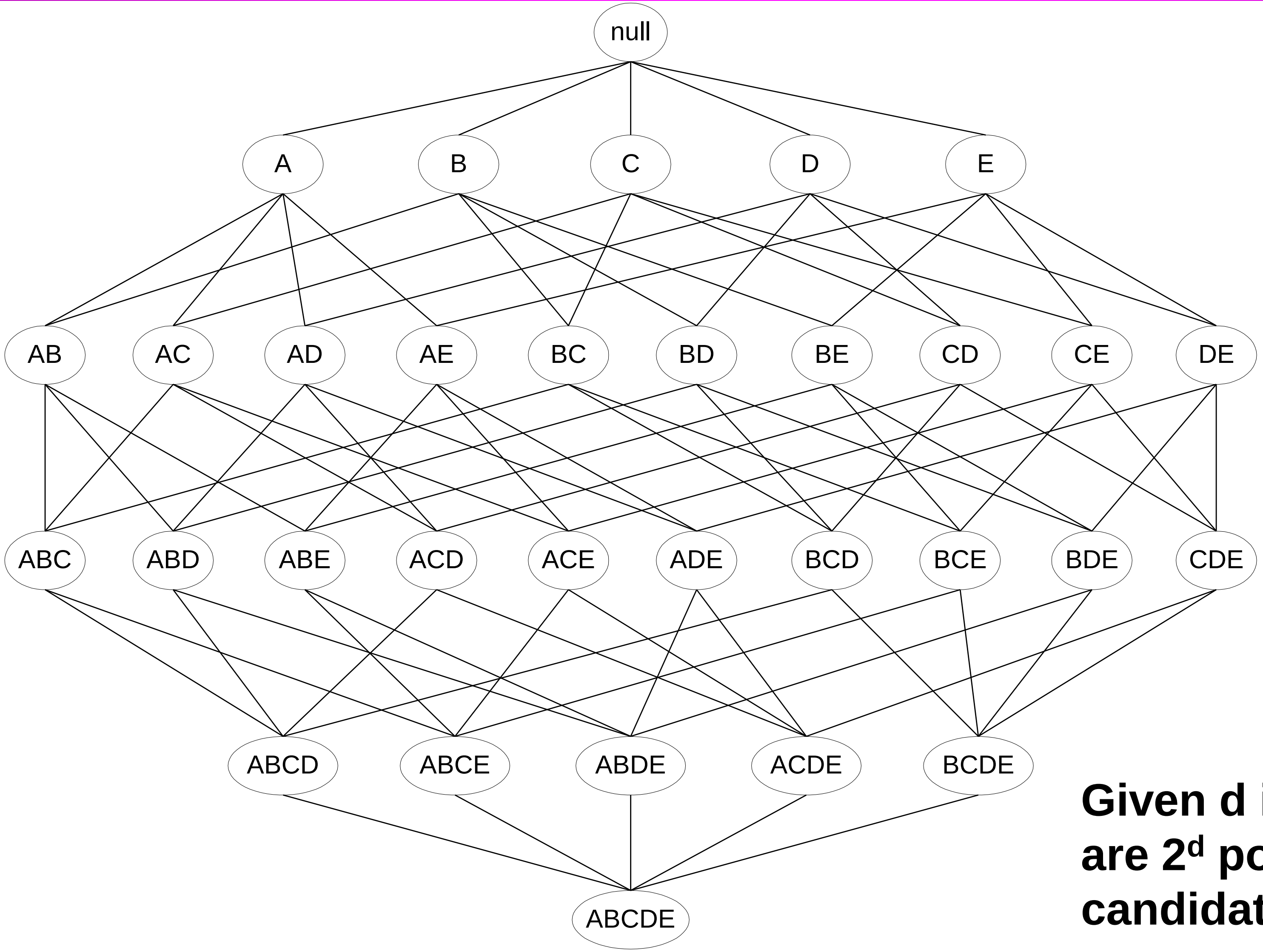
# Mining Association Rules

- Two-step approach:
  1. Frequent Itemset Generation
     – Generate all itemsets whose support $\geq$ minsup

  2. Rule Generation
     – Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset

- Frequent itemset generation is still computationally expensive

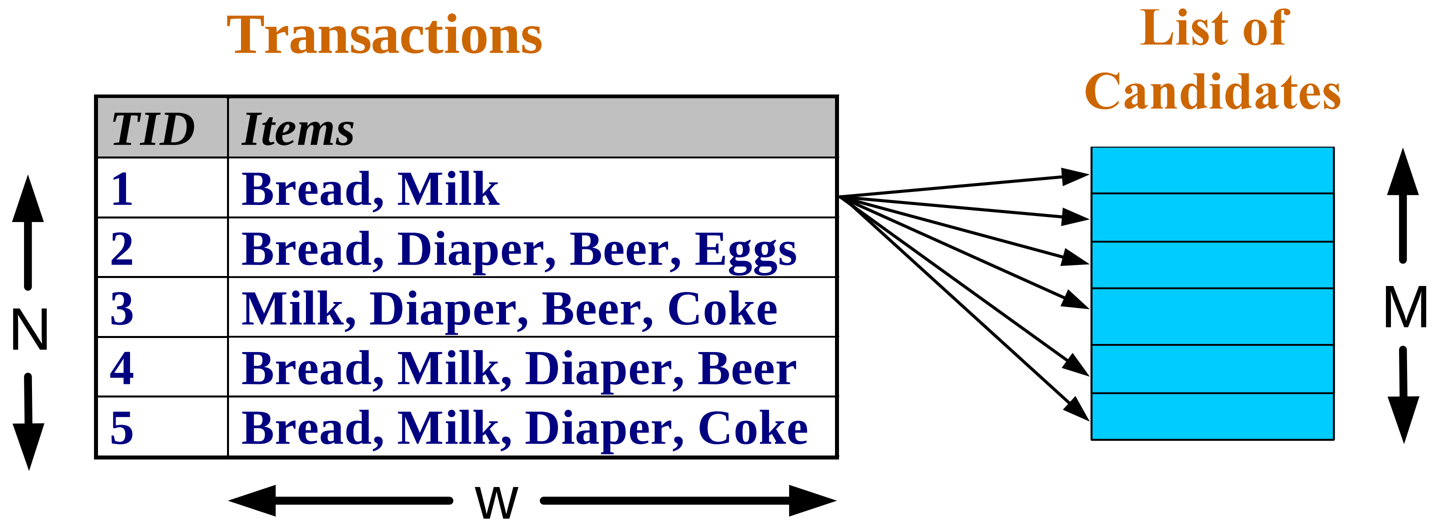
# Frequent Itemset Generation




**Given d items, there are $2^d$ possible candidate itemsets**

# Frequent Itemset Generation

- Brute-force approach:
  - Each itemset in the lattice is a candidate frequent itemset
  - Count the support of each candidate by scanning the database




| *TID* | *Items* |
|---|---|
| 1 | Bread, Milk |
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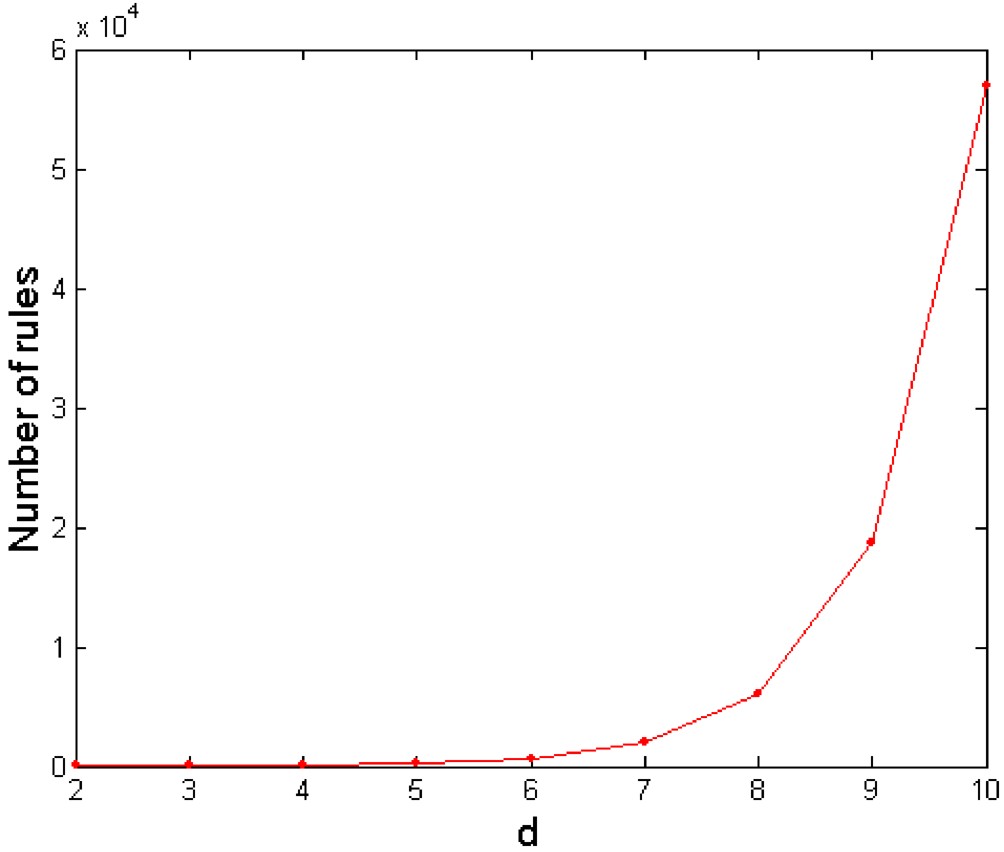
  - Match each transaction against every candidate
  - Complexity ~ O(NMw) => Expensive since $M = 2^d$ !!!

# Computational Complexity

- Given d unique items:
  - Total number of itemsets = $2^d$
  - Total number of possible association rules:




$$R = \sum_{k=1}^{d-1}\left[\binom{d}{k} \times \sum_{j=1}^{d-k}\binom{d-k}{j}\right]$$
$$= 3^d - 2^{d+1} + 1$$

**If d=6, R = 602 rules**

# Frequent Itemset Generation Strategies

- Reduce the number of candidates (M)
  - Complete search: $M=2^d$
  - Use pruning techniques to reduce M
- Reduce the number of transactions (N)
  - Reduce size of N as the size of itemset increases
  - Used by DHP and vertical-based mining algorithms
- Reduce the number of comparisons (NM)
  - Use efficient data structures to store the candidates or transactions
  - No need to match every candidate against every transaction

# Reducing Number of Candidates

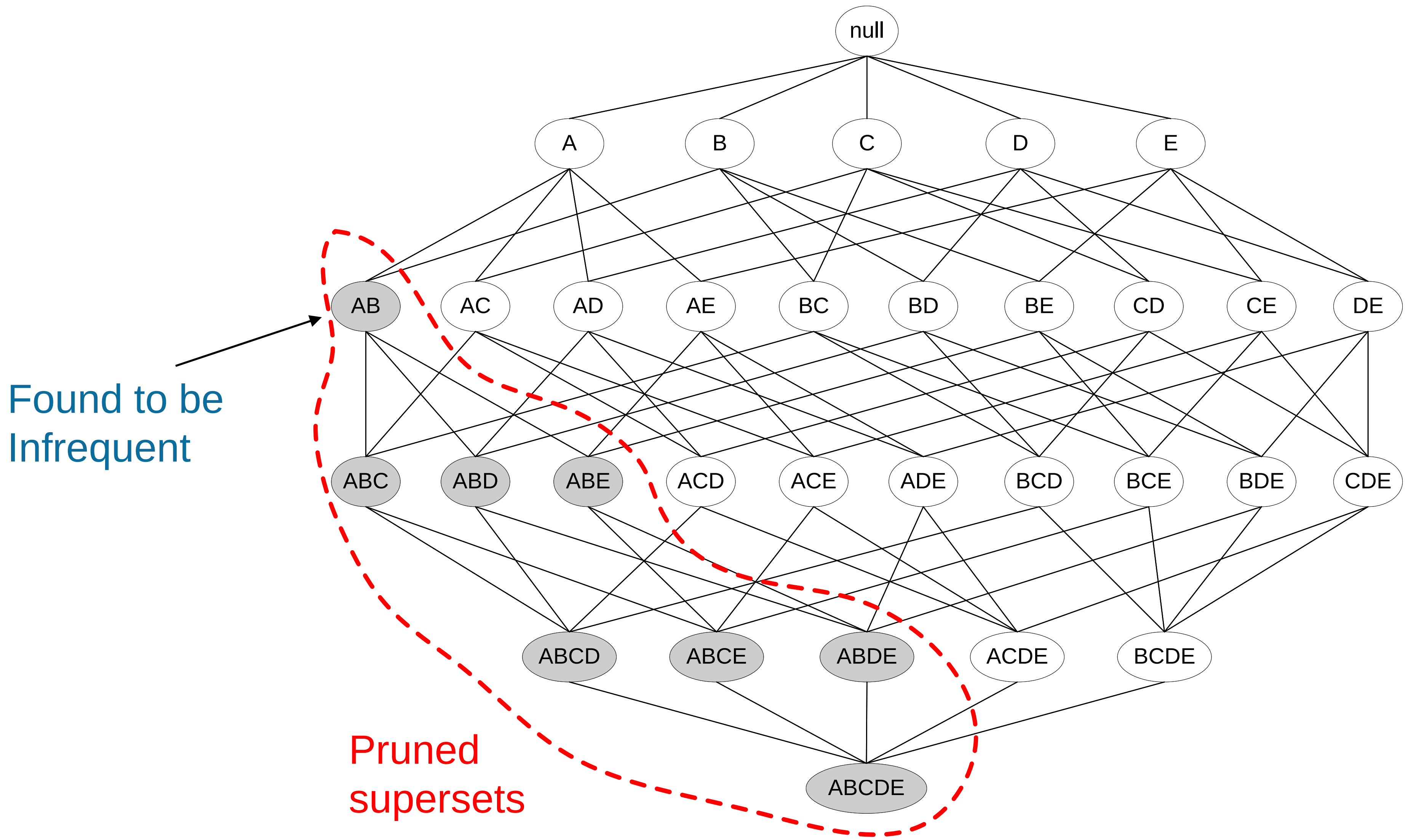
- Apriori principle:
  - If an itemset is frequent, then all of its subsets must also be frequent

- Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

  - Support of an itemset never exceeds the support of its subsets
  - This is known as the anti-monotone property of support

# Illustrating Apriori Principle



# Illustrating Apriori Principle

| Item | Count |
|---|---|
| **Bread** | **4** |
| **Coke** | **2** |
| **Milk** | **4** |
| **Beer** | **3** |
| **Diaper** | **4** |
| **Eggs** | **1** |

Items (1-itemsets)

| Itemset | Count |
|---|---|
| **{Bread,Milk}** | **3** |
| **{Bread,Beer}** | **2** |
| **{Bread,Diaper}** | **3** |
| **{Milk,Beer}** | **2** |
| **{Milk,Diaper}** | **3** |
| **{Beer,Diaper}** | **3** |

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

Triplets (3-itemsets)

| Itemset | Count |
|---|---|
| **{Bread,Milk,Diaper}** | **3** |

If every subset is considered,

$^6C_1 + {}^6C_2 + {}^6C_3 = 41$

With support-based pruning,

$6 + 6 + 1 = 13$

# Apriori Algorithm

- Method:
  - Let k=1
  - Generate frequent itemsets of length 1
  - Repeat until no new frequent itemsets are identified
    - Generate length (k+1) candidate itemsets from length k frequent itemsets
    - Prune candidate itemsets containing subsets of length k that are infrequent
    - Count the support of each candidate by scanning the DB
    - Eliminate candidates that are infrequent, leaving only those that are frequent

# Reducing Number of Comparisons

- Candidate counting:
  - Scan the database of transactions to determine the support of each candidate itemset
  - To reduce the number of comparisons, store the candidates in a hash structure
    - Instead of matching each transaction against every candidate, match it against candidates contained in the hashed buckets

**Transactions**

N

| *TID* | *Items* |
|---|---|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
| 4 | Bread, Milk, Diaper, Beer |
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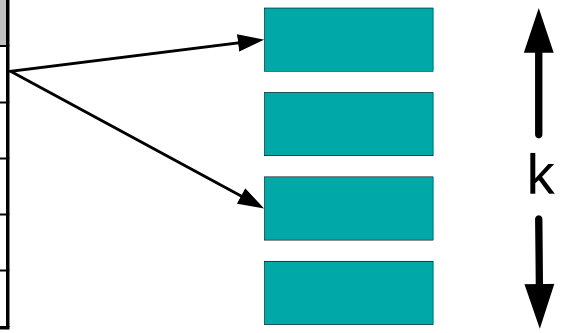
**Hash Structure**




Buckets

# Generate Hash Tree

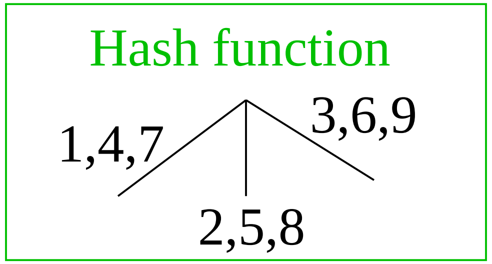
Suppose you have 15 candidate itemsets of length 3:

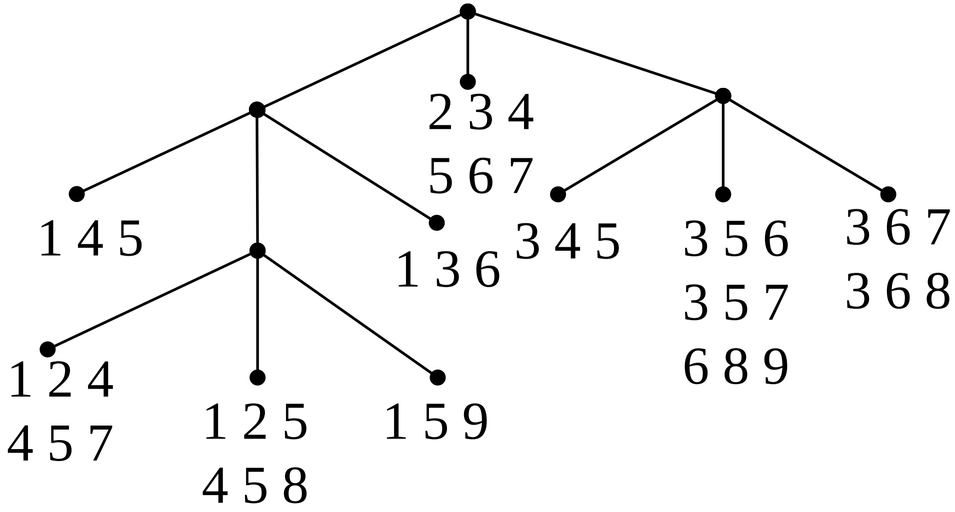
{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}

You need:

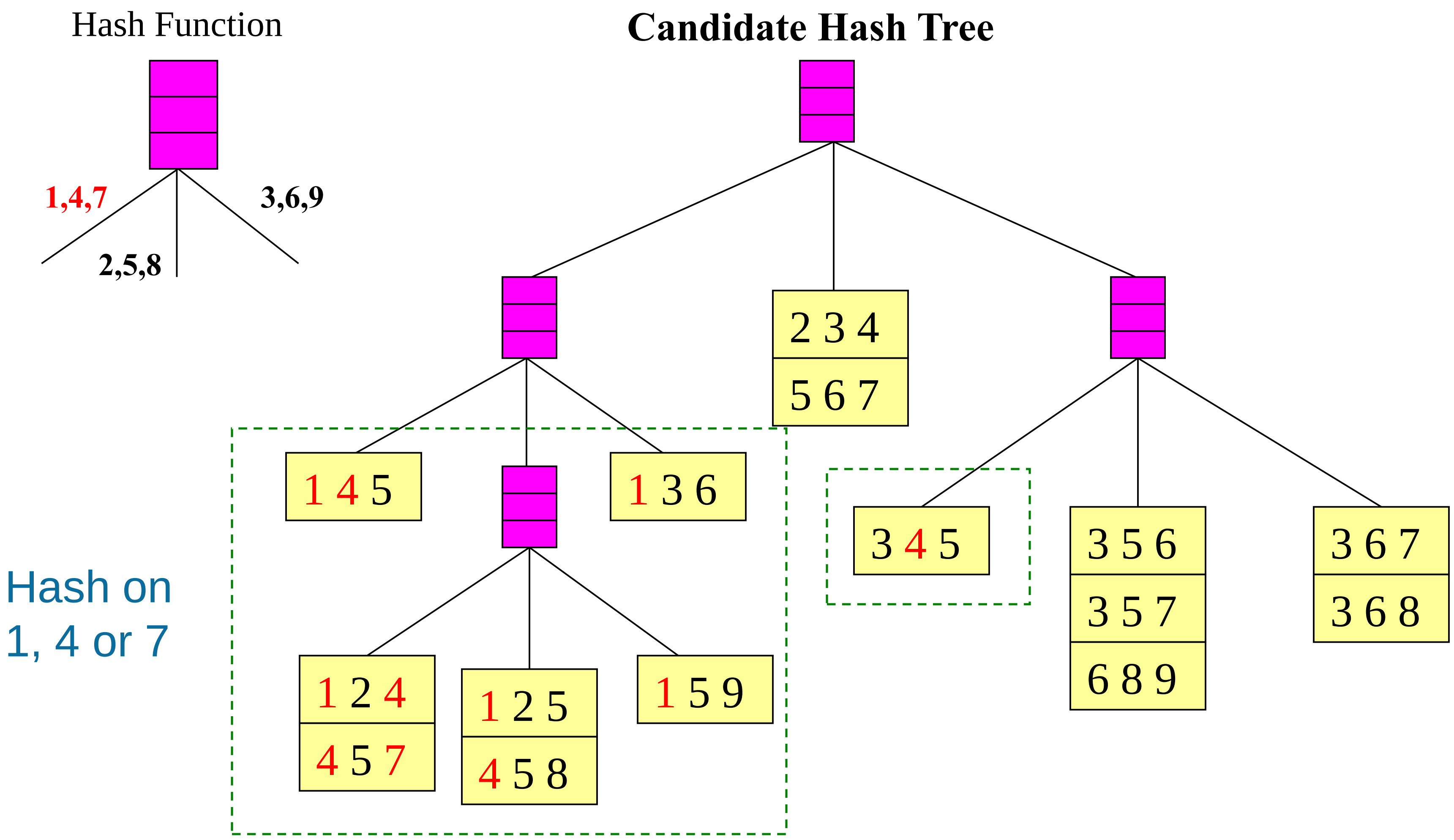
• Hash function

• Max leaf size: max number of itemsets stored in a leaf node (if number of candidate itemsets exceeds max leaf size, split the node)

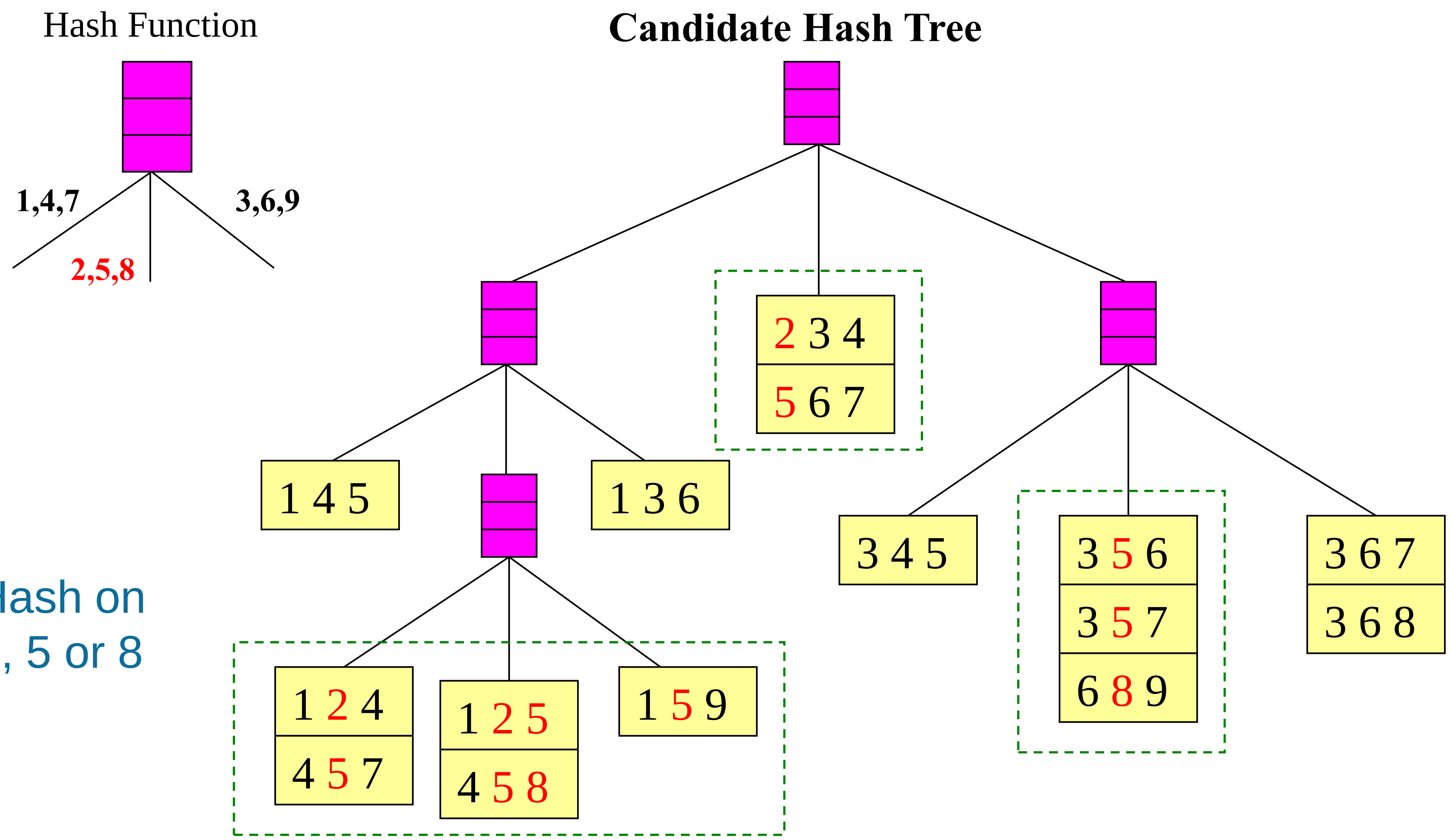


# Association Rule Discovery: Hash tree

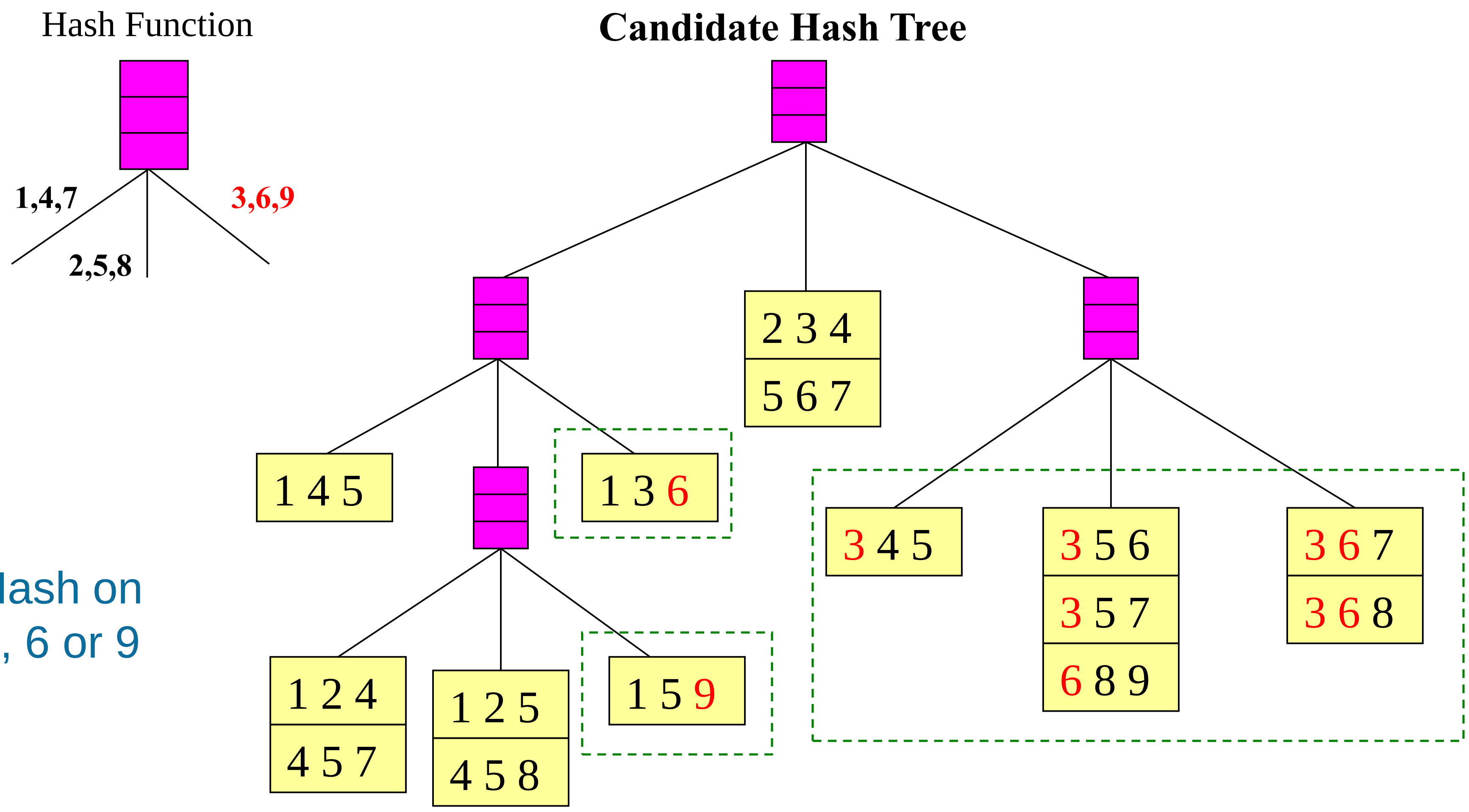


# Association Rule Discovery: Hash tree




Hash on
2, 5 or 8

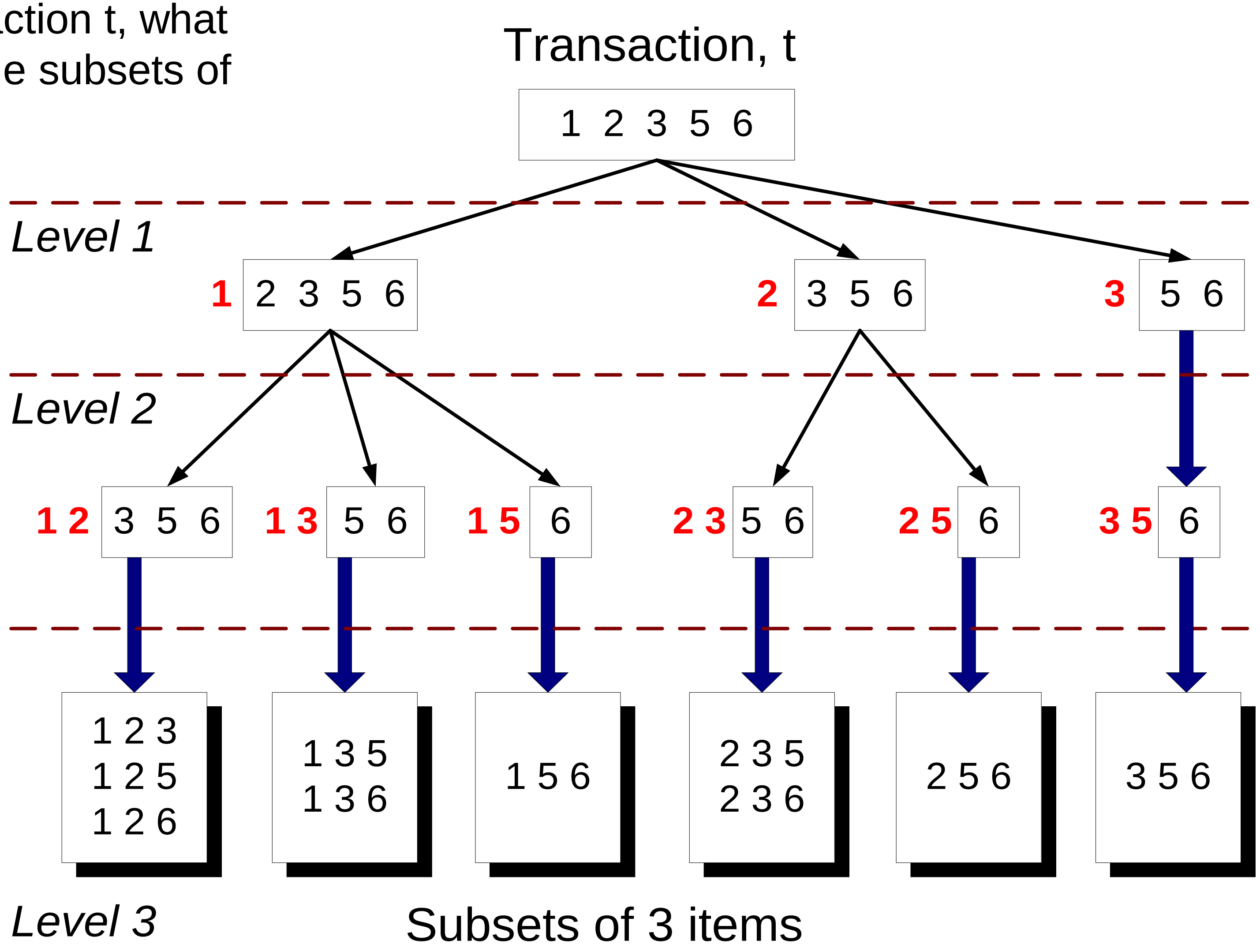
# Association Rule Discovery: Hash tree




Hash on
3, 6 or 9

# Subset Operation

Given a transaction t, what are the possible subsets of size 3?

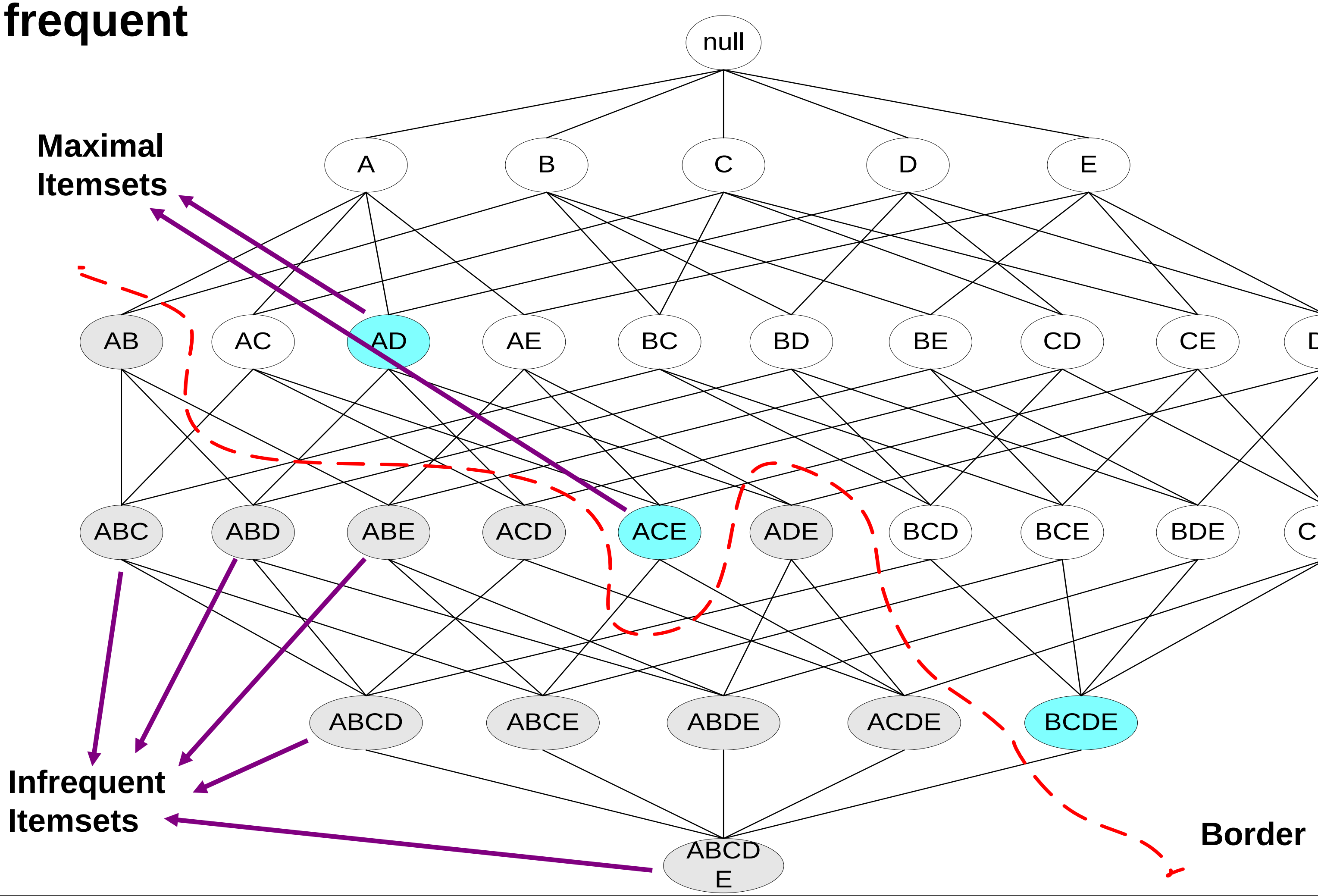


# Factors Affecting Complexity

- Choice of minimum support threshold
  - lowering support threshold results in more frequent itemsets
  - this may increase number of candidates and max length of frequent itemsets
- Dimensionality (number of items) of the data set
  - more space is needed to store support count of each item
  - if number of frequent items also increases, both computation and I/O costs may also increase
- Size of database
  - since Apriori makes multiple passes, run time of algorithm may increase with number of transactions
- Average transaction width
  - transaction width increases with denser data sets
  - This may increase max length of frequent itemsets and traversals of hash tree (number of subsets in a transaction increases with its width)

# Maximal Frequent Itemset

**An itemset is maximal frequent if none of its immediate supersets is frequent**



# Closed Itemset

- An itemset is closed if none of its immediate supersets has the same support as the itemset

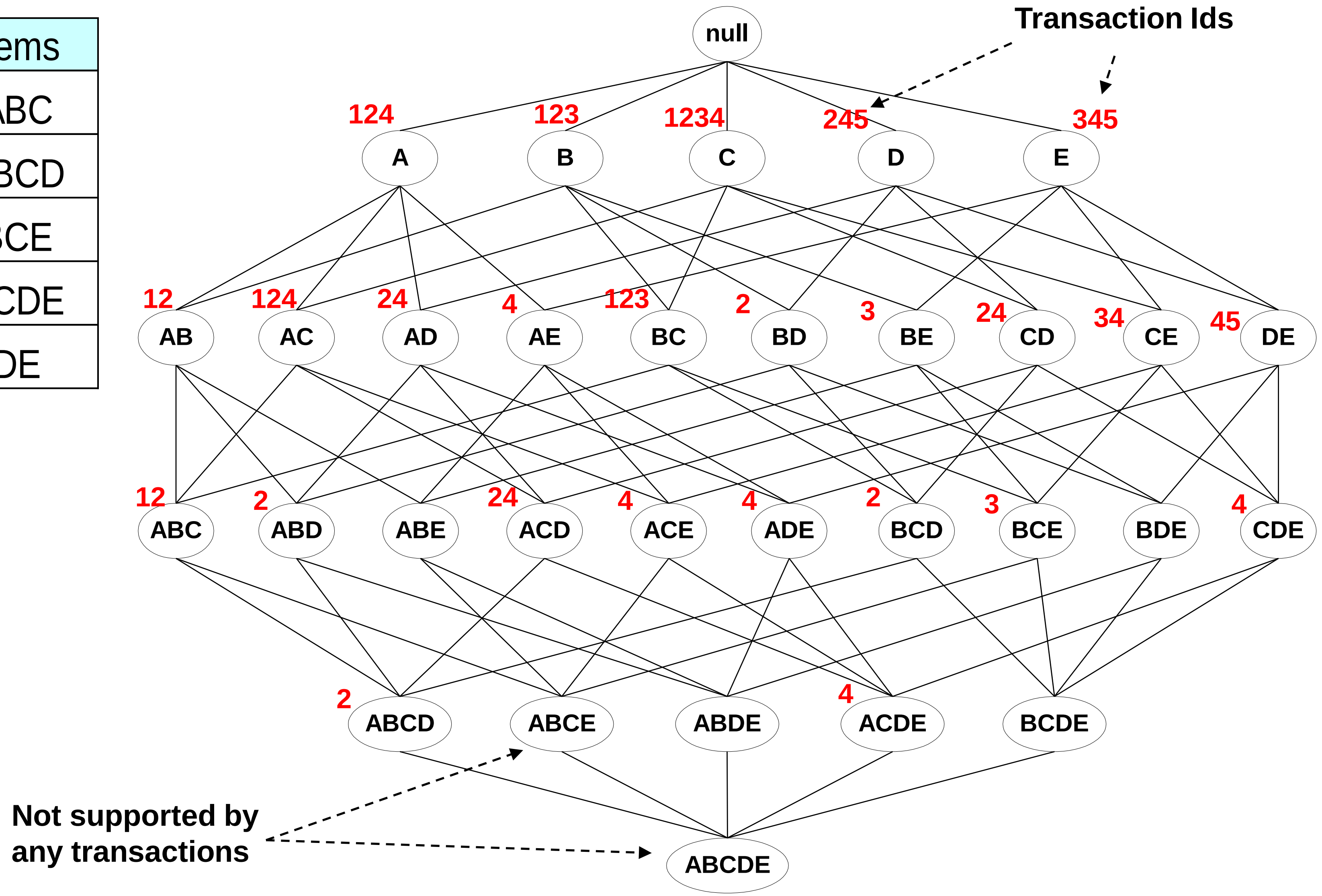
| TID | Items |
|---|---|
| 1 | {A,B} |
| 2 | {B,C,D} |
| 3 | {A,B,C,D} |
| 4 | {A,B,D} |
| 5 | {A,B,C,D} |

| Itemset | Support |
|---|---|
| {A} | 4 |
| {B} | 5 |
| {C} | 3 |
| {D} | 4 |
| {A,B} | 4 |
| {A,C} | 2 |
| {A,D} | 3 |
| {B,C} | 3 |
| {B,D} | 4 |
| {C,D} | 3 |

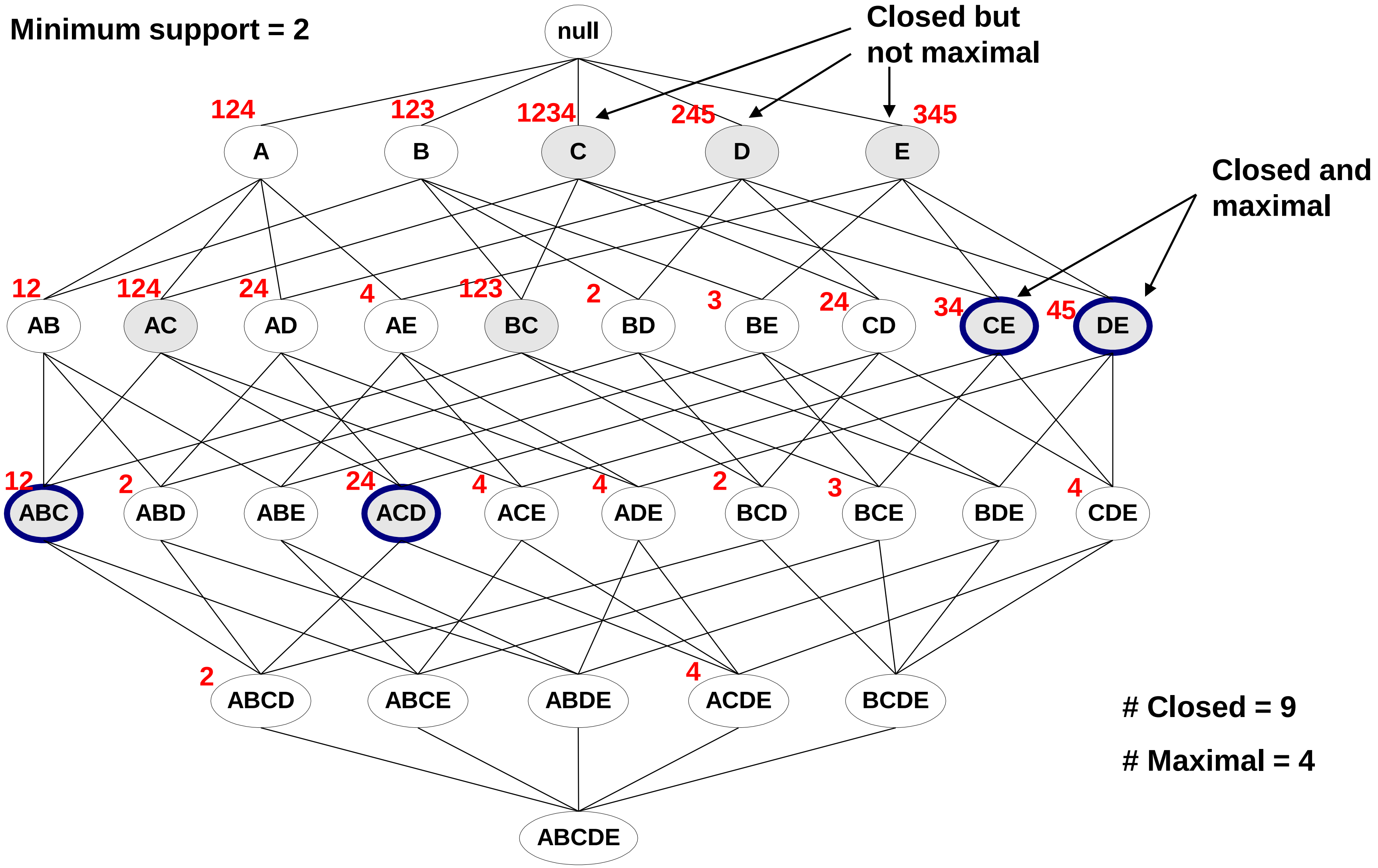
| Itemset | Support |
|---|---|
| {A,B,C} | 2 |
| {A,B,D} | 3 |
| {A,C,D} | 2 |
| {B,C,D} | 3 |
| {A,B,C,D} | 2 |

# Maximal vs Closed Itemsets

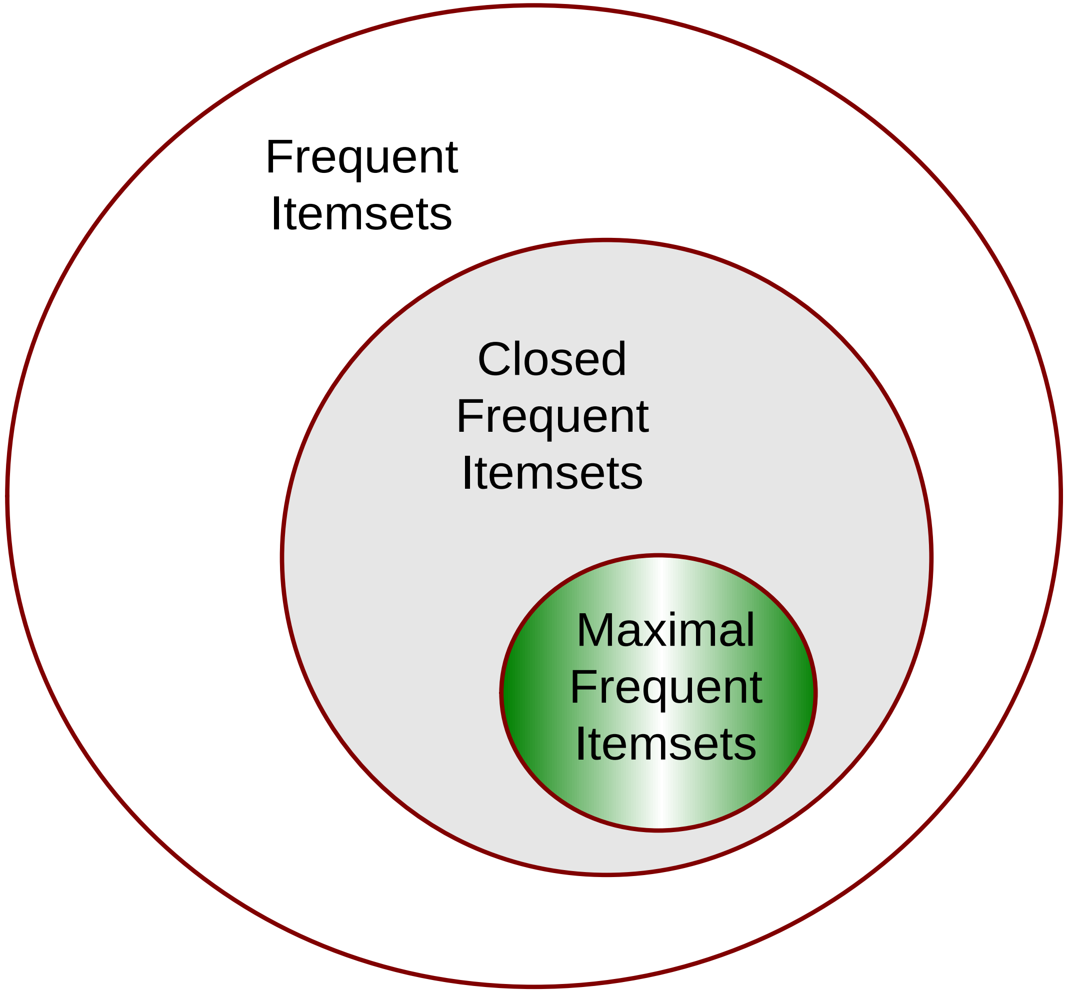
| TID | Items |
|---|---|
| 1 | ABC |
| 2 | ABCD |
| 3 | BCE |
| 4 | ACDE |
| 5 | DE |



# Maximal vs Closed Frequent Itemsets



# Maximal vs Closed Itemsets



# Rule Generation

- Given a frequent itemset L, find all non-empty subsets $f \subset L$ such that $f \rightarrow L - f$ satisfies the minimum confidence requirement
  - If {A,B,C,D} is a frequent itemset, candidate rules:

| | | | |
|---|---|---|---|
| ABC $\rightarrow$D, | ABD $\rightarrow$C, | ACD $\rightarrow$B, | BCD $\rightarrow$A, |
| A $\rightarrow$BCD, | B $\rightarrow$ACD, | C $\rightarrow$ABD, | D $\rightarrow$ABC |
| AB $\rightarrow$CD, | AC $\rightarrow$ BD, | AD $\rightarrow$ BC, | BC $\rightarrow$AD, |
| BD $\rightarrow$AC, | CD $\rightarrow$AB, | | |

- If $|L| = k$, then there are $2^k - 2$ candidate association rules (ignoring $L \rightarrow \emptyset$ and $\emptyset \rightarrow L$)

# Rule Generation

- How to efficiently generate rules from frequent itemsets?
  - In general, confidence does not have an anti-monotone property

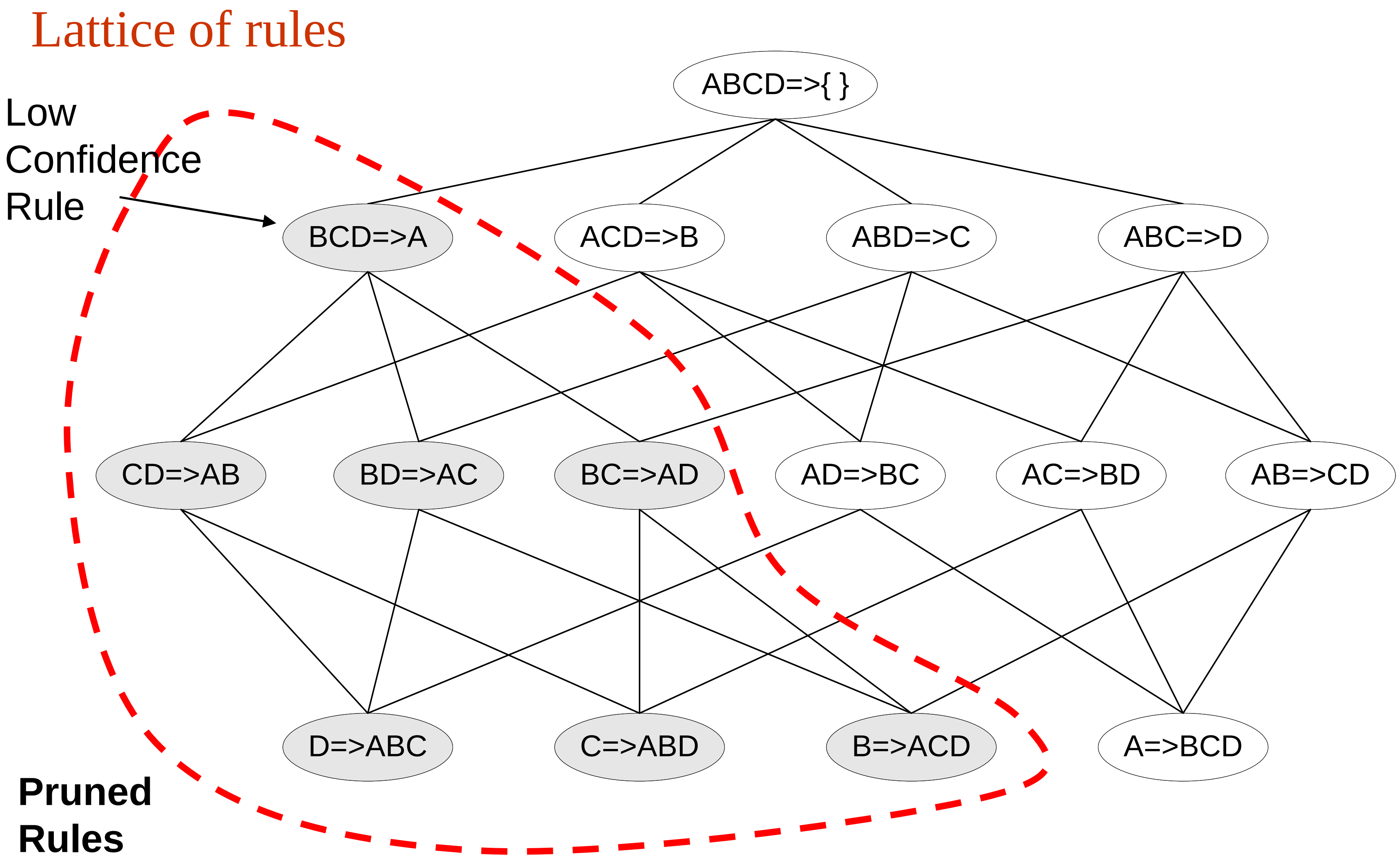
    $c(ABC \rightarrow D)$ can be larger or smaller than $c(AB \rightarrow D)$

  - But confidence of rules generated from the same itemset has an anti-monotone property
  - e.g., L = {A,B,C,D}:

$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

    - Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

# Rule Generation for Apriori Algorithm



# Statistical-based Measures

$$\text{lift}(X \rightarrow Y) = \text{lift}(Y \rightarrow X) = \frac{conf(X \rightarrow Y)}{supp(Y)} = \frac{cons(Y \rightarrow X)}{supp(X)} = \frac{P(Y \wedge X)}{P(X)P(Y)}$$

Leverage (Manivela, Kaldıraç)

Manivela yöntemi, X ve Y'nin birlikte bulunmaları arasındaki farkı ölçmektedir ve X ve Y değerlerinn istatistiksel olarak bağı olması halini hesaplamaktadır.

Manivela(X→Y) = P(X ve Y ) – (P(X)P(Y))

**Conviction (Kanaat)**

Aşağıdaki şekilde hesaplanabilir:

$$\text{Kanaat}(X \rightarrow Y) = \frac{1 - supp(Y)}{1 - conf(X \rightarrow Y)} = \frac{P(X)P(\neg Y)}{P(X \wedge \neg Y)}$$

**There are lots of measures proposed in the literature**

**Some measures are good for certain applications, but not for others**

**What criteria should we use to determine whether a measure is good or bad?**

**What about Apriori-style support based pruning? How does it affect these measures?**

| # | Measure | Formula |
|---|---|---|
| 1 | $\phi$-coefficient | $\frac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$ |
| 2 | Goodman-Kruskal's ($\lambda$) | $\frac{\sum_j \max_k P(A_j,B_k)+\sum_k \max_j P(A_j,B_k)-\max_j P(A_j)-\max_k P(B_k)}{2-\max_j P(A_j)-\max_k P(B_k)}$ |
| 3 | Odds ratio ($\alpha$) | $\frac{P(A,B)P(\overline{A},\overline{B})}{P(A,\overline{B})P(\overline{A},B)}$ |
| 4 | Yule's $Q$ | $\frac{P(A,B)P(\overline{AB})-P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{AB})+P(A,\overline{B})P(\overline{A},B)} = \frac{\alpha-1}{\alpha+1}$ |
| 5 | Yule's $Y$ | $\frac{\sqrt{P(A,B)P(\overline{AB})}-\sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{AB})}+\sqrt{P(A,\overline{B})P(\overline{A},B)}} = \frac{\sqrt{\alpha}-1}{\sqrt{\alpha}+1}$ |
| 6 | Kappa ($\kappa$) | $\frac{P(A,B)+P(\overline{A},\overline{B})-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A)P(B)-P(\overline{A})P(\overline{B})}$ |
| 7 | Mutual Information ($M$) | $\frac{\sum_i \sum_j P(A_i,B_j) \log \frac{P(A_i,B_j)}{P(A_i)P(B_j)}}{\min(-\sum_i P(A_i)\log P(A_i), -\sum_j P(B_j)\log P(B_j))}$ |
| 8 | J-Measure ($J$) | $\max\Big(P(A,B)\log(\frac{P(B\|A)}{P(B)}) + P(A\overline{B})\log(\frac{P(\overline{B}\|A)}{P(\overline{B})}),$ $P(A,B)\log(\frac{P(A\|B)}{P(A)}) + P(\overline{A}B)\log(\frac{P(\overline{A}\|B)}{P(\overline{A})})\Big)$ |
| 9 | Gini index ($G$) | $\max\Big(P(A)[P(B\|A)^2+P(\overline{B}\|A)^2]+P(\overline{A})[P(B\|\overline{A})^2+P(\overline{B}\|\overline{A})^2]$ $-P(B)^2-P(\overline{B})^2,$ $P(B)[P(A\|B)^2+P(\overline{A}\|B)^2]+P(\overline{B})[P(A\|\overline{B})^2+P(\overline{A}\|\overline{B})^2]$ $-P(A)^2-P(\overline{A})^2\Big)$ |
| 10 | Support ($s$) | $P(A,B)$ |
| 11 | Confidence ($c$) | $\max(P(B\|A), P(A\|B))$ |
| 12 | Laplace ($L$) | $\max\Big(\frac{NP(A,B)+1}{NP(A)+2}, \frac{NP(A,B)+1}{NP(B)+2}\Big)$ |
| 13 | Conviction ($V$) | $\max\Big(\frac{P(A)P(\overline{B})}{P(A\overline{B})}, \frac{P(B)P(\overline{A})}{P(B\overline{A})}\Big)$ |
| 14 | Interest ($I$) | $\frac{P(A,B)}{P(A)P(B)}$ |
| 15 | cosine ($IS$) | $\frac{P(A,B)}{\sqrt{P(A)P(B)}}$ |
| 16 | Piatetsky-Shapiro's ($PS$) | $P(A,B)-P(A)P(B)$ |
| 17 | Certainty factor ($F$) | $\max\Big(\frac{P(B\|A)-P(B)}{1-P(B)}, \frac{P(A\|B)-P(A)}{1-P(A)}\Big)$ |
| 18 | Added Value ($AV$) | $\max(P(B\|A)-P(B), P(A\|B)-P(A))$ |
| 19 | Collective strength ($S$) | $\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})} \times \frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$ |
| 20 | Jaccard ($\zeta$) | $\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$ |
| 21 | Klosgen ($K$) | $\sqrt{P(A,B)}\max(P(B\|A)-P(B), P(A\|B)-P(A))$ |