PHY401
Electromagnetic Theory I

Energy and Momentum in Electromagnetic Waves

Assoc. Prof. Dr. Fulya Bagci
Department of Physics
Engineering/Ankara University
fbagci@eng.ankara.edu.tr

Contents

9.2.3 Energy and momentum in electromagnetic waves

The energy per unit volume stored in the electromagnetic field is

$$u = \frac{1}{2}(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2)$$

In the case of a monochromatic plane wave $B^2 = \frac{1}{c^2} E^2 = \mu_0 \varepsilon_0 E^2$

$$u = \frac{1}{2}(\varepsilon_0 E^2 + \varepsilon_0 E^2) = \varepsilon_0 E^2$$

The contributions of E and B are equal.

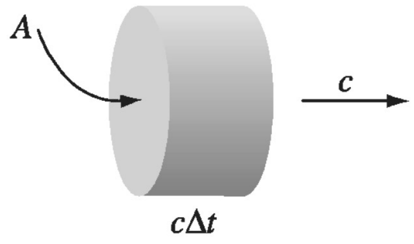
$$u = \varepsilon_0 E^2 = \varepsilon_0 \, E_0^2 cos^2(kz - \omega t + \delta)$$

As the wave travels, it carries this energy along with it.

Energy Transport and the Poynting vector

The energy flux density transported by the fields is given by the Poynting vector.

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$$



For in a time ΔT, a length $c\Delta T$ passes through area A, carrying it with an energy $uAc\Delta T$. S = Energy/(area.time)=uc

$$\vec{S} = c\varepsilon_0 E^2 \hat{z} = c\varepsilon_0 E_0^2 \cos^2(kz - \omega t + \delta)\hat{z} = uc\hat{z}$$

Electromagnetic fields not only carry energy, they also carry momentum. The momentum density stored in the fields is:

$$\vec{g} = \frac{1}{c^2}\vec{S}$$

$$\vec{g} = \frac{1}{c}\varepsilon_0 E_0^2 \cos^2(kz - \omega t + \delta)\hat{z} = \frac{1}{c}u\hat{z}$$

for monochromatic waves

Doç.Dr. Fulya Bağcı

In the case of *light,* the wavelength is so short (500 nm) and the period so brief ($\sim 10^{-15}$ s), that any macroscopic measurement will encompass many cycles. Typically, therefore, we're not interested in the fluctuating cosine-squared term in the energy and momentum densities; all we want is the ***average*** value.

The average of cosine-squared over a complete cycle is ½.

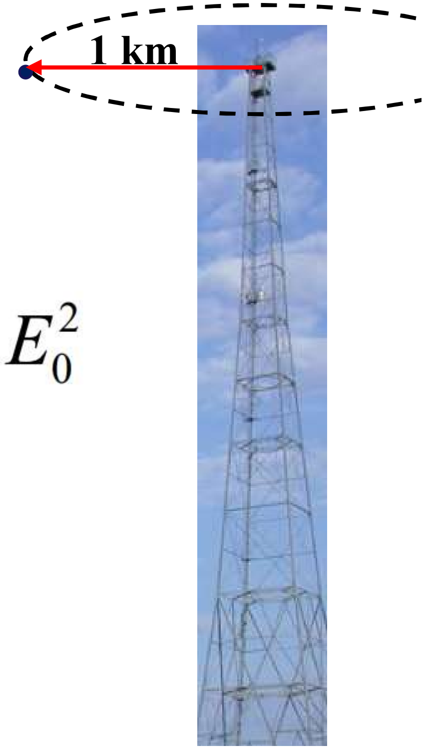
$$< u > = \frac{1}{2}\varepsilon_0 E_0^2 \qquad < S > = \frac{1}{2}c\varepsilon_0 E_0^2 \hat{z} \qquad < g > = \frac{1}{2c}\varepsilon_0 E_0^2 \hat{z}$$

The average power per unit area transported by an electromagnetic wave is called the **intensity:**

$$I \equiv < S > = \frac{1}{2}c\varepsilon_0 E_0^2 = \frac{E_0^2}{2\mu_0 c}$$

Example

A radio station transmits a 10-kW signal at a frequency of 100 MHz. For simplicity, assume that it radiates as a point source. At a distance of 1 km from the antenna, find: (a) the amplitude of the electric and magnetic field strengths, and (b) the energy incident normally on a square plate of side 10 cm in 5 min.



Solution:

$$(a)\ \ S_{av} = \frac{\text{Average power}}{4\pi r^2} = \frac{E_0^2}{2\mu_0 c}$$

$$\Rightarrow \frac{10000}{4\pi 1000^2} \times 2 \times 4\pi \times 10^{-7} \times 3 \times 10^{8} = E_0^2$$

$$\begin{cases} E_0 = 0.775 \text{ V/m} \\ B_0 = 2.58 \times 10^{-9} \text{ T} \end{cases}$$

$$(b)\ \ \Delta U = S_{av} \Delta t = 2.4 \times 10^{-3} \ \text{J}$$

Doç.Dr. Fulya Bağcı

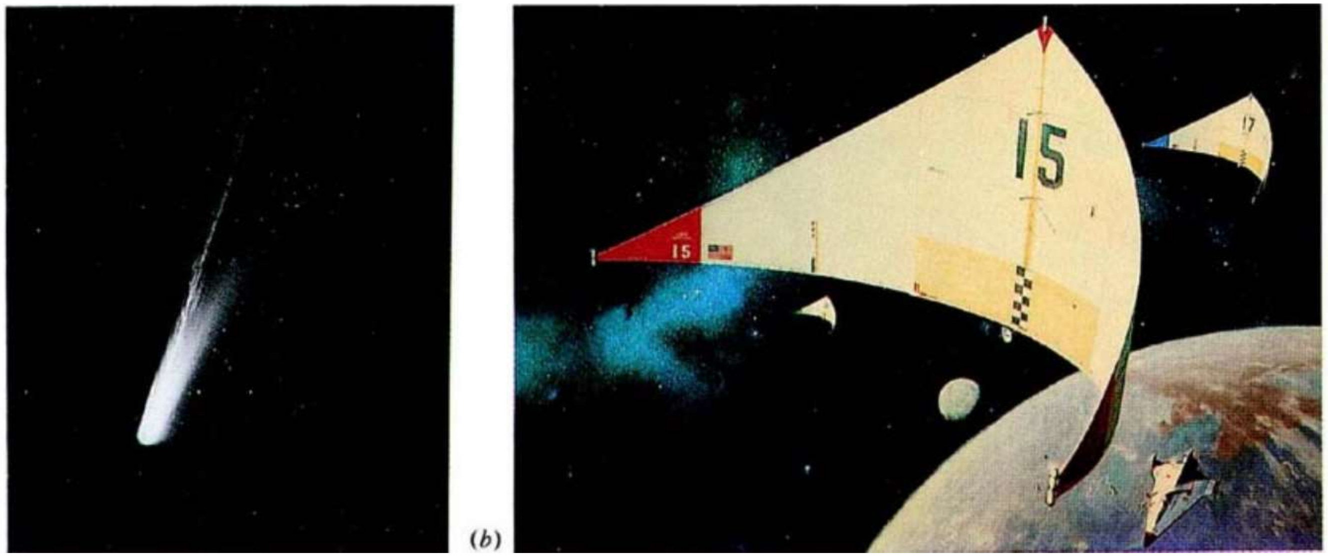
Momentum and Pressure

An electromagnetic wave transports linear momentum. The linear momentum carried by an electromagnetic wave is related to the energy it transports according to

$$p = \frac{U}{c}$$

The force exerted by an electromagnetic wave on a surface can be related to the Poynting vector. The radiation pressure at normal incidence is:

$$\frac{F}{A} = \frac{\Delta p}{A\Delta t} = \frac{\Delta U}{Ac\Delta t} = \frac{SA}{Ac} = \frac{S}{c} = u$$



(a) (b)

Examples of radiation pressure. (a) the tail of comet, (b) a ‘solar sail’

http://www.phys.nthu.edu.tw/~thschang/notes/GP34.pdf

Doç.Dr. Fulya Bağcı