PHY401
Electromagnetic Theory I

Propagation in Linear Media

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9.3.1 Propagation in linear media

Inside matter, but in regions where there is no *free* charge or *free* current, Maxwell's equations become,

$$(i)\vec{\nabla}.\vec{D} = 0 \qquad (iii)\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$(ii)\vec{\nabla}.\vec{B} = 0 \qquad (iv)\vec{\nabla}\times\vec{H} = \frac{\partial\vec{D}}{\partial t}$$

If the medium is linear,

$$\vec{D} = \varepsilon\vec{E} \qquad \vec{H} = \frac{1}{\mu}\vec{B}$$

D (electric displacement field) accounts for the effects of free and bound charge within materials.

If the medium is *linear* and *homogeneous* (ε and μ do not vary from point to point),

$$(i)\vec{\nabla}.\vec{E} = 0 \qquad (iii)\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$(ii)\vec{\nabla}.\vec{B} = 0 \qquad (iv)\vec{\nabla}\times\vec{B} = \varepsilon\mu\frac{\partial\vec{E}}{\partial t}$$

<u>Vacuum</u> $\varepsilon_0\mu_0$ ⇒ <u>Matter</u> $\varepsilon\mu$

Electromagnetic waves propagate through a linear homogeneous medium at a speed,

$$\vec{\vartheta} = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n}$$

where *n* is the index of refraction of a substance:

$$n \equiv \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}$$

μ is very close to μ_0 in most materials so,

$$n \cong \sqrt{\varepsilon_r}$$

ε_r is the dielectric constant and it is always greater than 1 (since light travels more slowly through matter)

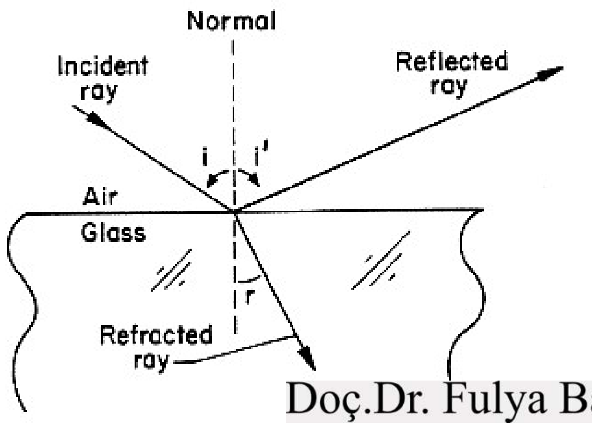
Energy Density, Poynting Vector, and Intensity in Linear Media

All of our previous results carry over, with the simple transcription $\varepsilon_0 \rightarrow \varepsilon$, $\mu_0 \rightarrow \mu$, and hence c $\rightarrow \vartheta$

$$u = \frac{1}{2}\left(\varepsilon E^2 + \frac{1}{\mu} B^2\right) \qquad \mathbf{g} = \frac{1}{v} u \hat{\mathbf{z}}$$

$$\mathbf{S} = \frac{\mathbf{E} \times \mathbf{B}}{\mu} \qquad I \equiv \langle \mathbf{S} \rangle = \frac{1}{2} v \varepsilon E_0^2$$

What happens when a wave passes from one transparent medium into another? Exp. air to water, or glass to plastic?



In Chapter 7, you derived the electrodynamic boundary conditions as:

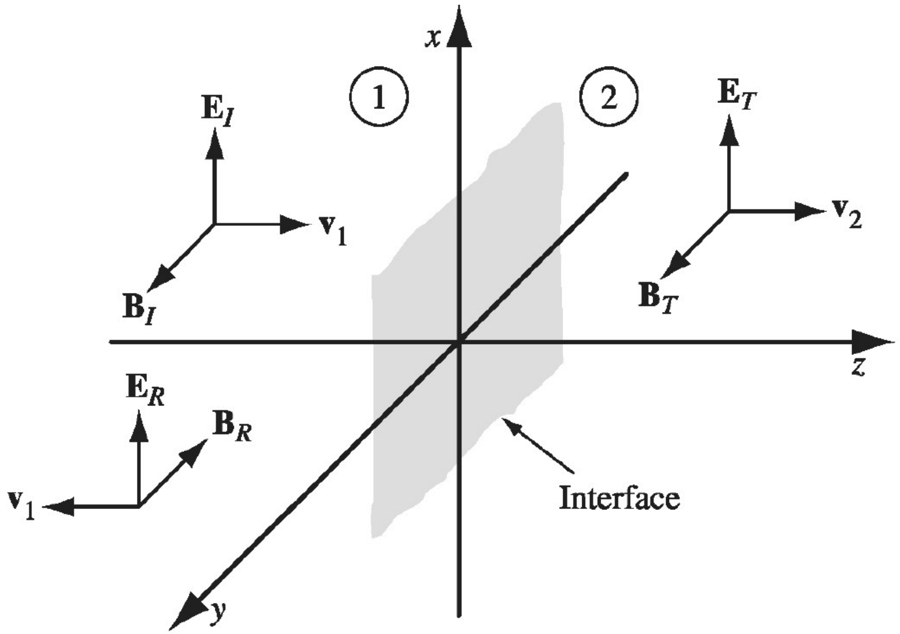
$$(i)\,\varepsilon_1 \overrightarrow{E_1^{\perp}} = \varepsilon_2 \overrightarrow{E_2^{\perp}} \qquad (iii)\,\overrightarrow{E_1^{\parallel}} = \overrightarrow{E_2^{\parallel}}$$

$$(ii)\,\overrightarrow{B_1^{\perp}} = \overrightarrow{B_2^{\perp}} \qquad (iv)\,\frac{1}{\mu_1}\overrightarrow{B_1^{\parallel}} = \frac{1}{\mu_2}\overrightarrow{B_2^{\parallel}}$$

These equations relate the electric and magnetic fields just to the left and just to the right of the interface between two linear media. In the following sections, we use them to deduce the laws governing reflection and refraction of electromagnetic waves.

9.3.2 Reflection and Transmission at Normal Incidence

Suppose a plane wave of frequency *w,* traveling in the *z* direction and polarized in the *x* direction, approaches the interface from the left.



Transmitted wave:

$$\tilde{\mathbf{E}}_T(z,t) = \tilde{E}_{0T} e^{i(k_2 z - \omega t)} \hat{\mathbf{x}}$$

$$\tilde{\mathbf{B}}_T(z,t) = \frac{1}{v_2} \tilde{E}_{0T} e^{i(k_2 z - \omega t)} \hat{\mathbf{y}}$$

Incident wave:

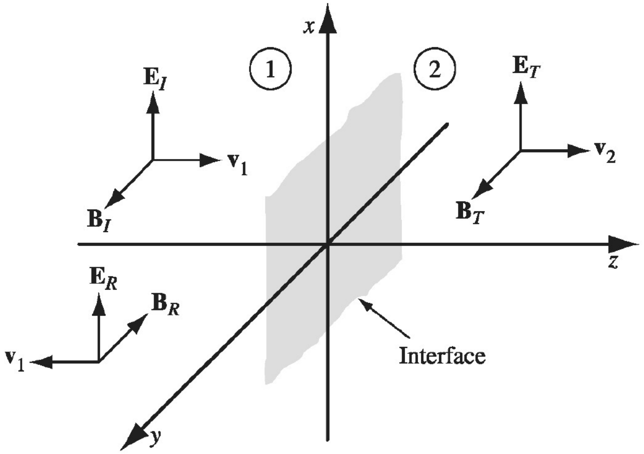
$$\tilde{\mathbf{E}}_I(z,t) = \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{x}}$$

$$\tilde{\mathbf{B}}_I(z,t) = \frac{1}{v_1} \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{y}}$$

Reflected wave:

$$\tilde{\mathbf{E}}_R(z,t) = \tilde{E}_{0R} e^{i(-k_1 z - \omega t)} \hat{\mathbf{x}}$$

$$\tilde{\mathbf{B}}_R(z,t) = -\frac{1}{v_1} \tilde{E}_{0R} e^{i(-k_1 z - \omega t)} \hat{\mathbf{y}}$$



At z=0, the combined fields on the left must join the fields on the right in accordance with the boundary conditions. There are no components perpendicular to the surface, so (i) and (ii) are trivial.

$$(iii)\overrightarrow{E_1^{\|}} = \overrightarrow{E_2^{\|}} \qquad (iv)\frac{1}{\mu_1}\overrightarrow{B_1^{\|}} = \frac{1}{\mu_2}\overrightarrow{B_2^{\|}}$$

$$\widetilde{E}_{0i} + \widetilde{E}_{0r} = \widetilde{E}_{0t}$$

$$\frac{1}{\mu_1}\left(\frac{1}{\vartheta_1}\widetilde{E}_{0i} - \frac{1}{\vartheta_1}\widetilde{E}_{0r}\right) = \frac{1}{\mu_2}\frac{1}{\vartheta_2}\widetilde{E}_{0t}$$

$$\widetilde{E}_{0i} - \widetilde{E}_{0r} = \beta\widetilde{E}_{0t}$$

where $\beta = \dfrac{\mu_1\vartheta_1}{\mu_2\vartheta_2} = \dfrac{\mu_1 n_2}{\mu_2 n_1}$

$$(1)\widetilde{E}^{\parallel}_{0i} + \widetilde{E}^{\parallel}_{0r} = \widetilde{E}^{\parallel}_{0t}$$

$$(2)\widetilde{E}_{0i} - \widetilde{E}_{0r} = \beta\widetilde{E}_{0t}$$

(1)+(2)→$\tilde{E}_{0t}$

$$\widetilde{E}_{0t} = \frac{2}{1+\beta}\widetilde{E}_{0i}$$

(1)-(2)→$\tilde{E}_{0r}$

$$\widetilde{E}_{0r} = \frac{1-\beta}{2}\widetilde{E}_{0t} \qquad \widetilde{E}_{0r} = \frac{1-\beta}{2}\frac{2}{1+\beta}\widetilde{E}_{0i}$$

$$\widetilde{E}_{0r} = \frac{1-\beta}{1+\beta}\widetilde{E}_{0i}$$

$\beta = \frac{\vartheta_1}{\vartheta_2}$ (Permeabilities are close to the value in vacuum for most media.)

$$\widetilde{E}_{0t} = \left(\frac{2\vartheta_2}{\vartheta_1 + \vartheta_2}\right)\widetilde{E}_{0i} \qquad \widetilde{E}_{0r} = \left|\frac{\vartheta_2 - \vartheta_1}{\vartheta_2 + \vartheta_1}\right|\widetilde{E}_{0i}$$

$$\widetilde{E}_{0t} = \left(\frac{2n_1}{n_1 + n_2}\right)\widetilde{E}_{0i} \qquad \widetilde{E}_{0r} = \left|\frac{n_2 - n_1}{n_2 + n_1}\right|\widetilde{E}_{0i}$$

What fraction of the incident energy is reflected, and what fraction is transmitted?

The intensity (average power per unit area) is (Eq. 9.73)

$$I = \frac{1}{2}\varepsilon \vartheta E_0^2$$

The ratio of the reflected intensity to the incident intensity is the reflection coefficient, *R*.

$$R \equiv \frac{I_r}{I_i} = \left(\frac{E_{0r}}{E_{0i}}\right)^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

The ratio of the transmitted intensity to the incident intensity is the transmission coefficient, *T*.

$$T \equiv \frac{I_t}{I_i} = \frac{\varepsilon_2 \vartheta_2}{\varepsilon_1 \vartheta_1}\left(\frac{E_{0t}}{E_{0i}}\right)^2 = \frac{4n_1 n_2}{\left(n_1 + n_2\right)^2}$$

Notice that $R+T=1$ as conservation of energy, of course, requires. For instance, when light passes from air ($n_1 = 1$) into glass ($n_2 = 1.5$), $R = 0.04$ and $T = 0.96$. No surprise, most of the light is transmitted.

One-way mirrors (used by police to interrogate bad guys) are just partial reflectors (actually, with a very thin aluminum coating).

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