Computer Simulations

A practical approach to simulation

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Epidemics

Introduction to Epidemics

Computational simulation of epicemics provide a valuable tool for the study the dynamics of epidemics.

- The spread of infectious deseases such as influenza, smallpox, HIV/AIDS are studied.
- It is assumed that for contamination direct contacts between the contaminated and the susceptible individuals are necessary.
- The validity of assumptions can be tested by using available epidemics data.

Epidemics

Introduction to Epidemics

Computational tools of studying epidemics varies.

- Epidemics can be modelled by using different techniques such as
  - ordinary diferential equations (mean-field),
  - partial differential equations (spatial distribution is taken into consideration),
  - Cellularautomatom (simple local rules),
  - stochastic dynamics (Agent based simulation, random walk, Monte-Carlo techniques)

Epidemics

Introduction to Epidemics

Computational tools of studying epidemics can be used to study many apparently different pnenomena.

- Modeling of social systems
  - naming, opinion spread, population dynamics, rumour propagation.
  - . . .
- Finance - Modeling economics
- Population dynamics
- . . .

Epidemics

Introduction to Epidemics

Elements of the Epidemic Simultion:

- Susceptible
- Infected
- removed

The dynamics of the epidemics spread:

- Susceptible individuals become infected at rate proportional to the number of infected neighbours.
- Infected individuals become healty at a fixed rate δ
- Removed individuals become susceptible at a fixed rate α

Epidemics

An ordinary differential equation approach

$$\begin{aligned} u &: \text{Fraction of infected} \\ v &: \text{Fraction of removed} \\ 1-u-v &: \text{Fraction of susceptible} \end{aligned}$$

Assumption: Large, mixed, homogeneous population
(Mean-Field Approximation)

$$\begin{aligned} \frac{du}{dt} &= u \cdot (1 - u - v)) - \delta\, u \\ \frac{dv}{dt} &= \delta\, u - \alpha\, v \end{aligned}$$

Epidemics

An ordinary differential equation approach

- If $\alpha = 0$, recovered individuals do not get contaminated again.
  - If δ is big, epidemics dies out
  - for $\delta < \delta_c$: Epidemic spreads linearly with time. where δ_c is a critical value of δ. Approach to steady state distribution.
- If $\alpha > 0$, problem is more complicated.

Referance : *Dolgoarshinnykh, R. G. and Lalley, S. P. (2006), "Critical scaling for the SIS stochastic epidemic," J. Appl. Probab., 43, 892–898.*

Epidemics

An ordinary differential equation approach

```
t_Final = 30; // Time to end calculation
dt = 0.01;    // Time steps
t = 0;        // Start clock
while(t < t_Final){
  u1 = u0 +  (u0 * (1.0 -  u0 - v0 ) - delta * u0
  v1 = v0 + (delta * u0 -  alpha * v0 ) * dt;
  u0 = u1;
  v0 = v1;
  t = t + dt;
  fprintf(out1," %f %f %f %f \n",t,1-u0-v0,u0,v0)
}
```

Epidemics

An ordinary differential equation approach

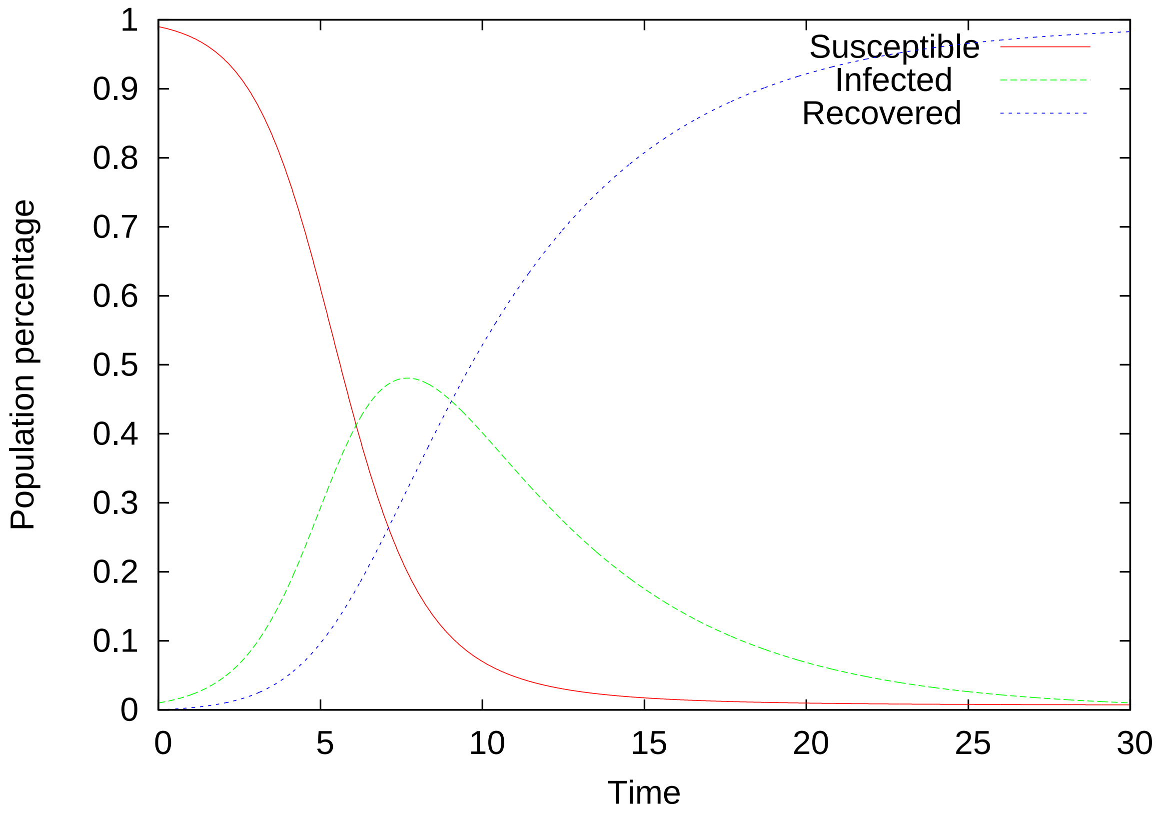

Figure 1: Epidemic spread $\alpha = 0.0, \quad \delta = 0.2$.

Epidemics

An ordinary differential equation approach

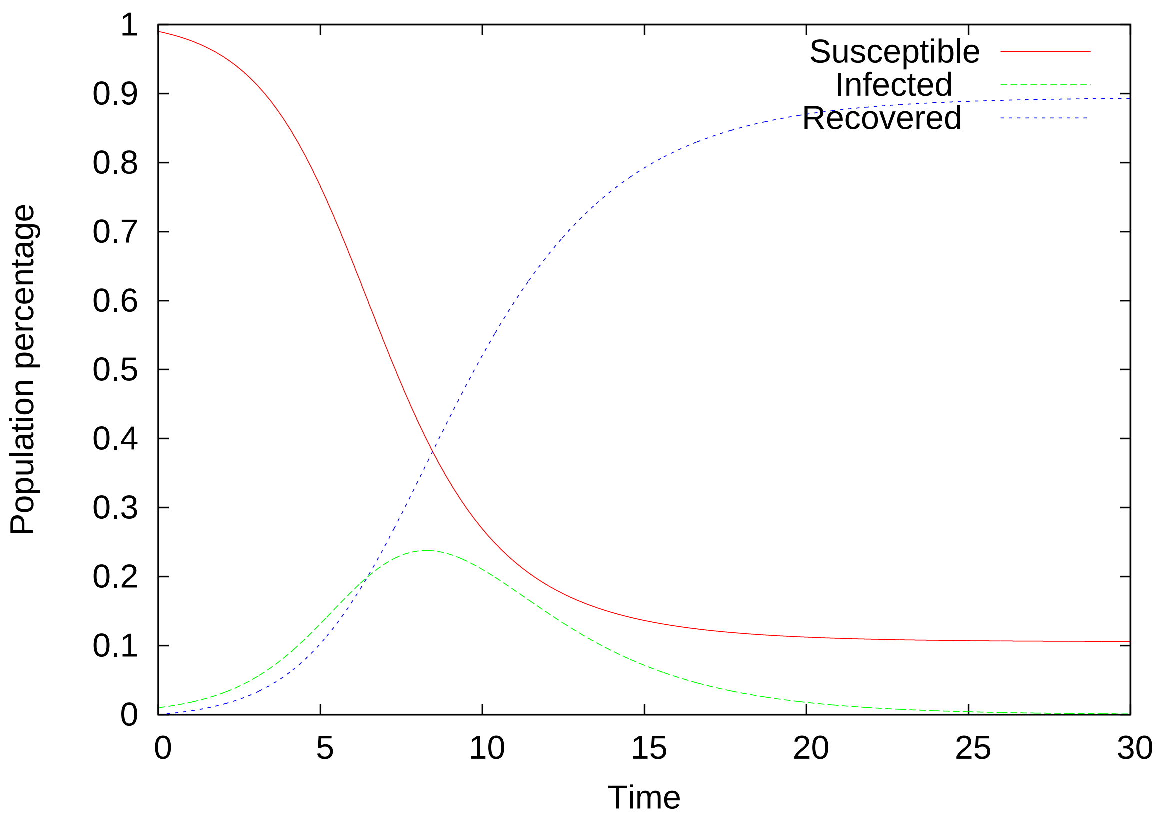

Figure 2: Epidemic spread $\alpha = 0.0, \quad \delta = 0.4$.

Epidemics

An ordinary differential equation approach

- Measle spread in Glasgow 1929
  - Measles took 24 weeks to spread over Glasgow. approximtely 150 meters per week
  - This is approximately linear spred rate of rakoon.
- Speread of muskrats over central Europa during a period of 27 years.

Spatial distribution is important. Mean-Field approach do not count the spatial distribution of population.

Epidemics

Effects of spatial distribution of population

Simple spatial models of epidemics are known to exhibit critical thresholds in one dimension: Roughly, when the density of the initially infected set exceeds a certain level, the epidemic evolves in a markedly different fashion than its branching envelope for contact between the contaminated and susseptible person.

Epidemics

Effects of spatial distribution of population

Referances:

1. Lalley, S. , “Spatial Epidemics: Critical Behavior in One Dimension,” arXiv:math/0701698v2 (2007).
2. Aldous, D., “Brownian excursions, critical random graphs and the multiplicative coalescent,” Ann. Probab., 25, 812–854 (1997).
3. Martin-Löf, A., “The final size of a nearly critical epidemic, and the first passage time of a Wiener process to a parabolic barrier,” J. Appl. Probab., 35, 671–682 (1998).

Epidemics

Partial differential equation approach

- Partial differential approach models similar phenomena. Only difference is that spatial distribution is taken into consideration.
- $P(x,t)$: Population density.

$$\frac{\partial P(x,t)}{\partial t} = \delta \Delta^2 P(x,t) + \alpha P(x,t)$$

The solution leads to linear spread reate. Partial differential equation solution approximates random walk.