An alternating series is a series whose terms are alternately positive and negative. Here are two examples:

$$\sum_{n=1}^{\infty}(-1)^n = -1+1-1+\cdots$$

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{2^n} = -\frac{1}{3}+\frac{1}{3^2}-\frac{1}{3^3}+\cdots$$

Theorem (Leibniz's Test)

If the alternating series

$$\sum_{n=1}^{\infty}(-1)^{n-1}b_n = b_1-b_2+b_3-b_4+\cdots$$

satisfies

$$(i) 0<b_{n+1}\leq b_n \; for \; all \; n\geq 1,$$

$$(ii) \lim_{n\to\infty} b_n = 0,$$

then the series is convergent.

Absolute and Conditional Convergence

Definition: A series $\sum a_n$ is absolutely convergent if the corresponding series of absolute values, $\sum |a_n|$, is convergent.

A series $\sum a_n$ is called conditionally convergent if it is convergent but not absolutely convergent.

Theorem If $\sum |a_n|$ converges, then $\sum a_n$ converges.

Power Series

Definition: A series of the form

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots$$

is called a power series in $(x-a)$ or power series centered at a or a power series about a, where x is a variable and the c_n ‘s are constants called the coefficients of the series.

In this section our question is for what values of x is the power series convergent?

Definition: Let the power series $\sum c_n(x-a)^n$ be convergent for $|x-a|<R$. The number R is called the radius of convergence of the power series. If the series converges only when $x=a$, then $R=0$. If the series converges for all x, then $R=\infty$. The interval of convergence of a power series is the interval that consits of all values of x for which the series converges.

	Series	Radius of convergence	Interval of convergence
Geometric series	$\sum_{n=0}^{\infty} x^n$	$R=1$	$(-1, 1)$
Example 1	$\sum_{n=0}^{\infty} n!\, x^n$	$R=0$	$\{0\}$
Example 2	$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$	$R=1$	$[2, 4)$
Example 3	$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$	$R=\infty$	$(-\infty, \infty)$