The volume V of a circular cylinder depends on its radius r and its height h. In fact, we know that $V = \pi r^2 h$. We say that V is a function of r and h, and we write $V(r,h) = \pi r^2 h$.

Definition

A function f of two variables is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the domain of f and its range is the set of values that f takes on, that is, $\{f(x,y): (x,y) \in D\}$.

We often write $z = f(x, y)$. So, x and y are independent variables; z is a dependent variable.

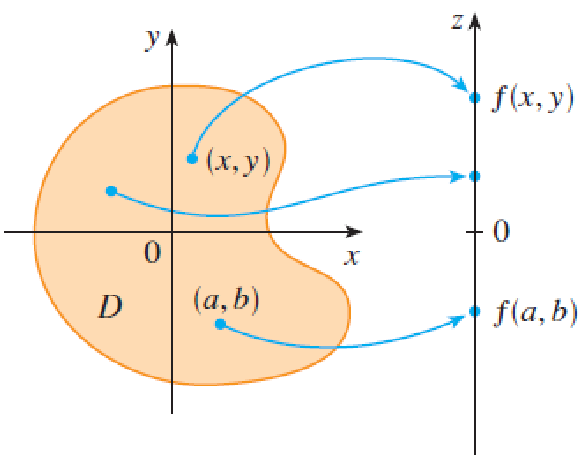
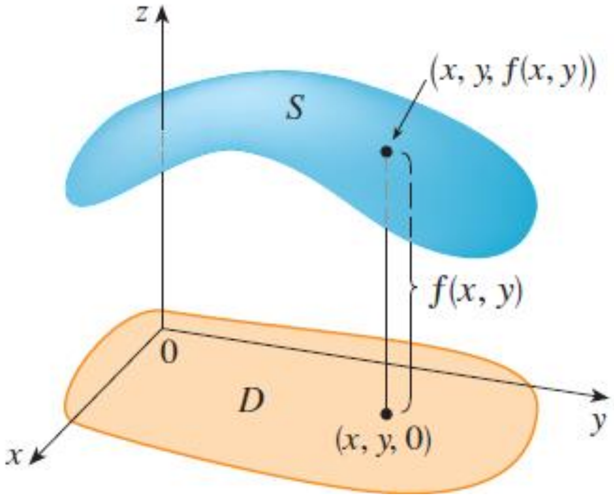


FIGURE 1

Figure 1 shows the domain and range of two variable functions.

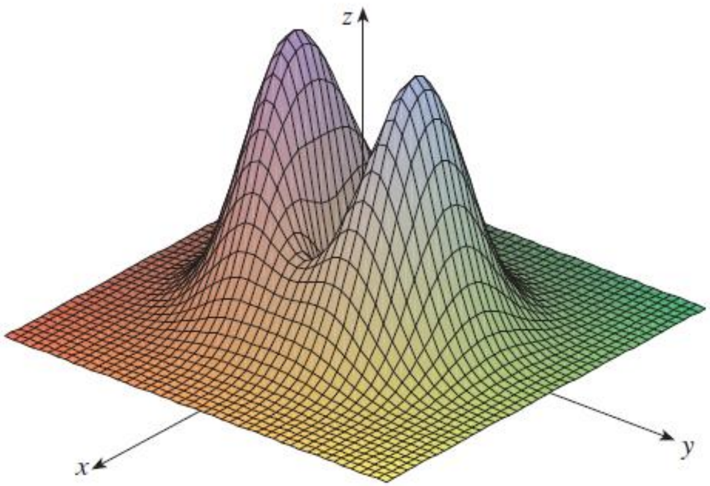
- The set of points in the plane where a function $f(x, y)$ has a constant value $f(x, y) = c$ is called a **level curve** of f.

- The set of all points $(x, y, f(x, y))$ in space is called the **graph** of f. The graph of f is also called the surface $z = f(x, y)$.

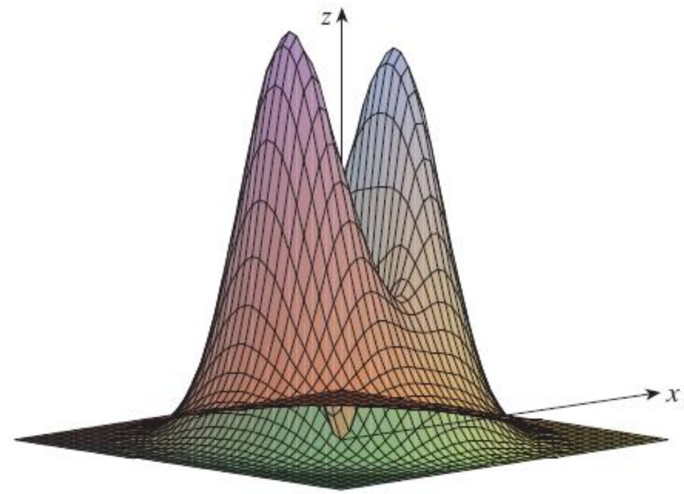


This figure shows the graph of a surface

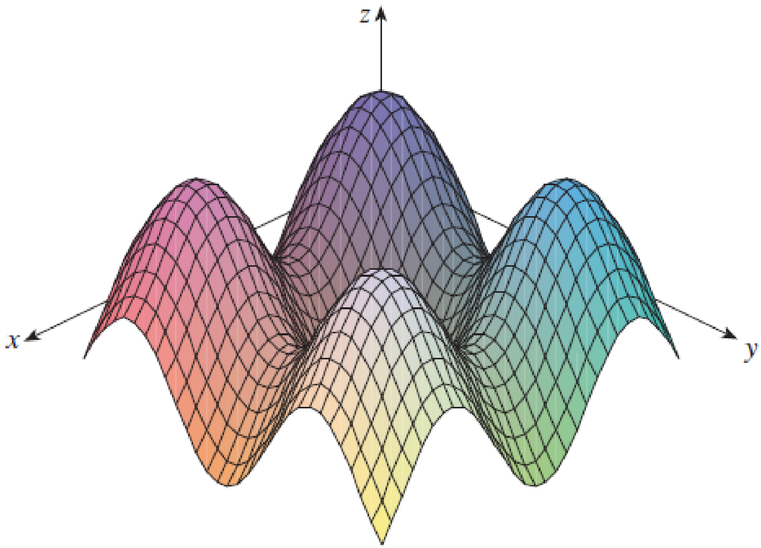
You can see the graph of several functions as follows:



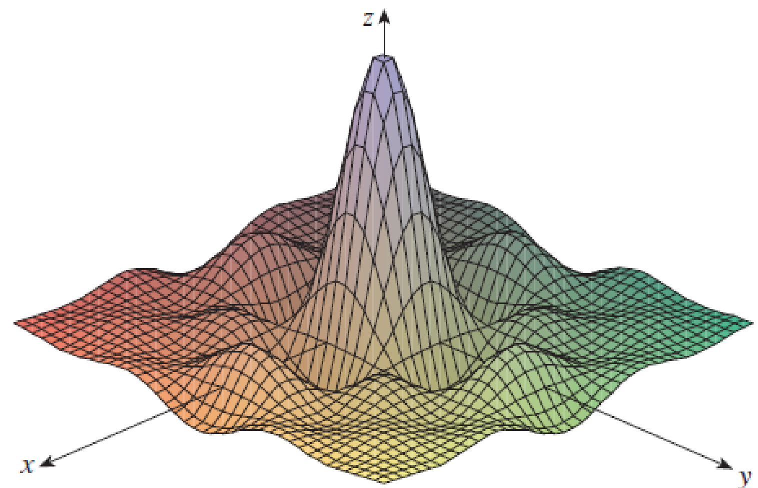
(a) $f(x, y) = (x^2 + 3y^2)e^{-x^2-y^2}$



(b) $f(x, y) = (x^2 + 3y^2)e^{-x^2-y^2}$



(c) $f(x, y) = \sin x + \sin y$



(d) $f(x, y) = \dfrac{\sin x \, \sin y}{xy}$

Limits and Continuity

Definition (Limit) Let f be a function of two variables whose domain D includes points arbitrarily close to (a, b). Then we say that the limit of $f(x, y)$ as (x, y) approaches (a, b) is L and we write

$$\lim_{(x,y)\to(a,b)} f(x, y) = L$$

if for every number $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that
if $(x, y) \epsilon D$ and $0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta$ then $|f(x, y) - L| < \varepsilon$

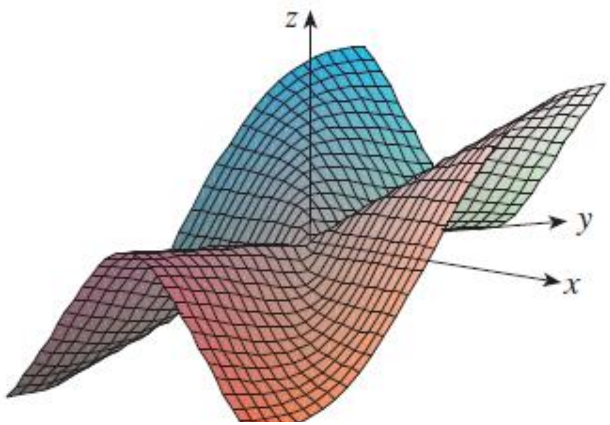
> If $f(x, y) \to L_1$ as $(x, y) \to (a, b)$ along a path C_1 and $f(x, y) \to L_2$ as $(x, y) \to (a, b)$ along a path C_2, where $L_1 \neq L_2$, then $\lim_{(x, y)\to(a, b)} f(x, y)$ does not exist.

Definition (Continuity):

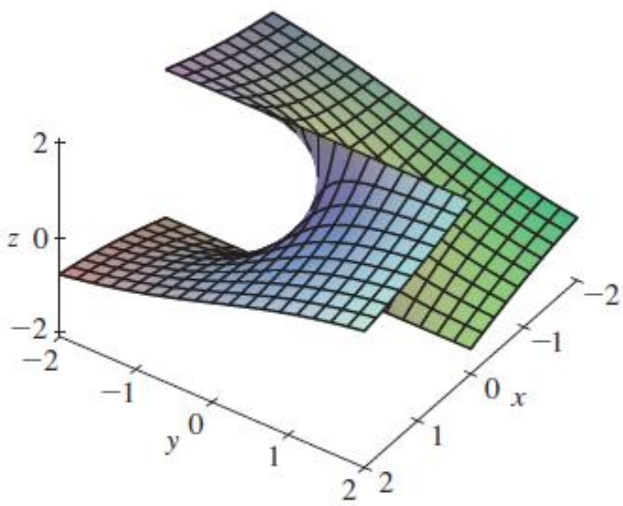
A function f of two variables is called continuous at (a, b) if

$$\lim_{(x,y)\to(a,b)} f(x, y) = f(a, b)$$

We say f is continuous on D if f is continuous at every point (a, b) in D.



Graph of a continuous function



Graph of a discontinuous function