# **CHAPTER 5**

# EQUILIBRIUM PROBLEMS IN COMPLEX SYSTEMS:

APPLICATION OF EQUILIBRIUM CALCULATIONS TO COMPLEX SYSTEMS, SYSTEMATIC METHOD FOR SOLVING MULTIPLE EQUILIBRIUM PROBLEMS, SOLUBILITY EQUILIBRIUM CALCULATIONS

### In this chapter,

Chemical equilibrium including calculations of chemical composition and of equilibrium concentrations for monoprotic acid/base systems will be considered.

buffer solutions, which are extremely important in many areas of science, and describe the properties of these solutions will be discussed.

# THE CHEMICAL COMPOSITION OF AQUEOUS SOLUTIONS

**Electrolytes** form ions when dissolved in water (or certain other solvents) and thus produce solutions that conduct electricity.

Strong electrolytes ionize essentially completely in a solvent, Weak electrolytes ionize only partially.

# Conducting electricity by weak electrolytes <<< strong electrolyte

### TABLE 9-1

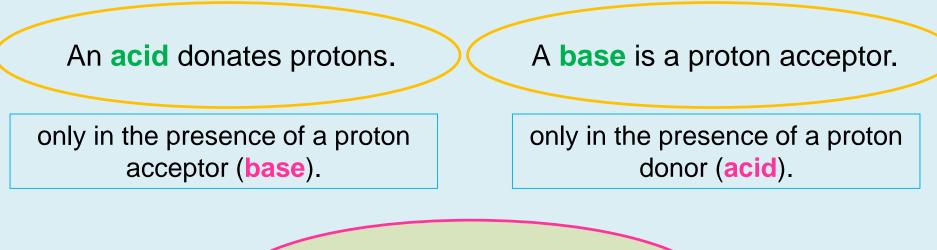
### Classification of Electrolytes

Strong	Weak
1. Inorganic acids such as HNO <sub>3</sub> , HClO <sub>4</sub> ,	1. Many inorganic acids, including H <sub>2</sub> CO <sub>3</sub> ,
H <sub>2</sub> SO <sub>4</sub> *, HCl, HI, HBr, HClO <sub>3</sub> , HBrO <sub>3</sub>	H <sub>3</sub> BO <sub>3</sub> , H <sub>3</sub> PO <sub>4</sub> , H <sub>2</sub> S, H <sub>2</sub> SO <sub>3</sub>
2. Alkali and alkaline-earth hydroxides	2. Most organic acids
3. Most salts	3. Ammonia and most organic bases
	4. Halides, cyanides, and thiocyanates of
	Hg, Zn, and Cd

 $^{*}H_{2}SO_{4}$  is completely dissociated into  $HSO_{4}^{2-}$  and  $H_{3}O^{+}$  ions and for this reason is classified as a **strong electrolyte**.

 $HSO_4^{2-}$  ion is a weak electrolyte and is only partially dissociated into  $SO_4^{2-}$  and  $H_3O^+$ .

### **Brønsted-Lowry Theory**

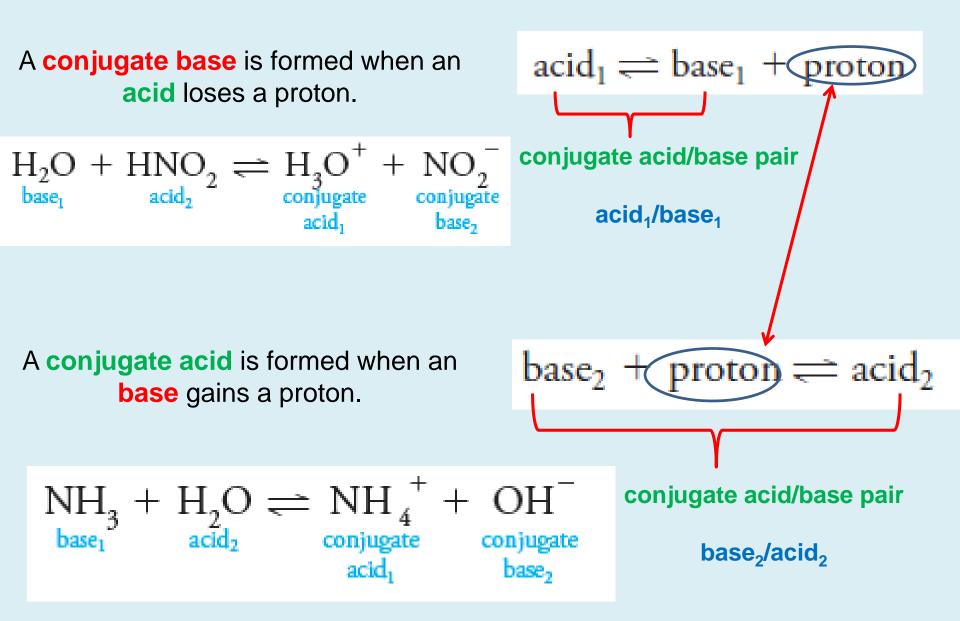


A **salt** is produced in the reaction of an acid with a base. NaCl, Na<sub>2</sub>SO<sub>4</sub>, NaOOCCH<sub>3</sub>

Arrhenius acid-base definition

### (acid produces H<sub>3</sub>O<sup>+</sup> and base forms OH<sup>-</sup> in water.) Lewis acid-base definition

(acid accepts and shares electron and bases share their electrons)



A substance acts as an acid only in the presence of a base and vice versa.

# <u>Neutralization:</u> $acid_1 + base_2 \Longrightarrow base_1 + acid_2$

 $HNO_2 + NH_3 \Leftrightarrow NH_4^+ + NO_2^-$ 

$$NO_2^- + H_2O \Leftrightarrow HNO_2 + OH^-$$
  
base<sub>1</sub> acid<sub>2</sub> conjugate conjugate  
acid<sub>1</sub> base<sub>2</sub>

# **Amphiprotic Species**

Species that have both **acidic** and **basic** properties are **amphiprotic**.

behaves as a base in the presence of a proton donor.

$$H_2PO_4^- + H_3O^+ \rightleftharpoons H_3PO_4 + H_2O$$

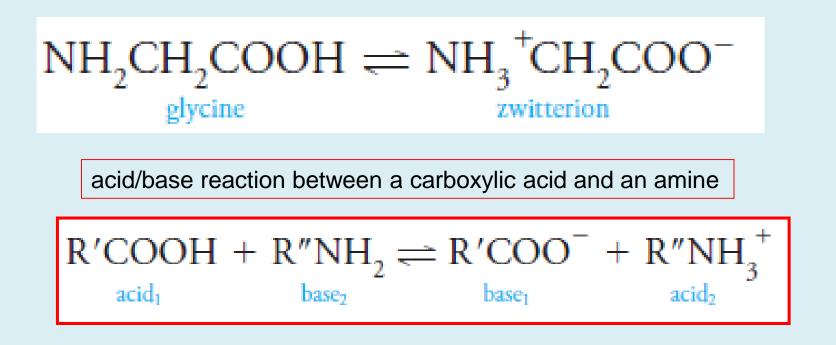
$$acid_2 acid_1 base_2$$

behaves as an acid in the presence of a proton acceptor.

$$H_2PO_4^{-} + OH^{-} \rightleftharpoons HPO_4^{2-} + H_2O_4^{2-}$$
  
acid<sub>1</sub> base<sub>2</sub> base<sub>1</sub> acid<sub>2</sub>

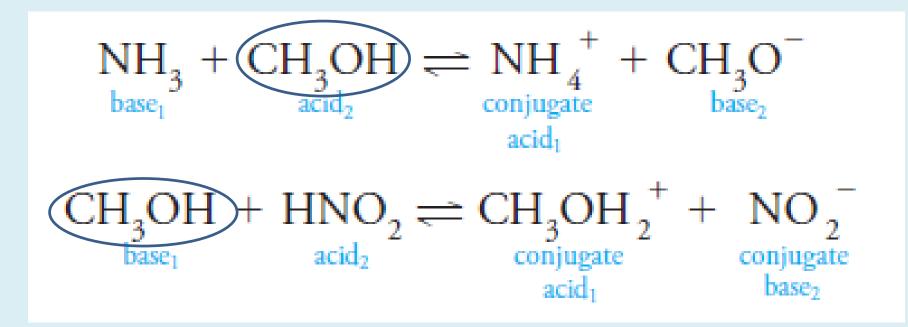
Water acts as a proton acceptor (base) and as a proton donor (acid): Amphiprotic solvent The simple **amino acids** are an important class of **amphiprotic compounds** that contain both a weak acid and a weak base functional group.

an amino acid, undergoes a kind of internal acid/base reaction to produce a zwitterion—a species that has both a positive and a negative charge.



Water acts as a proton acceptor (base) and as a proton donor (acid): Amphiprotic solvent

Other common amphiprotic solvents are methanol, ethanol, and anhydrous acetic acid.



# Autoprotolysis (autoionization)

Amphiprotic solvents undergo **self-ionization**, or **autoprotolysis**, to form a pair of ionic species.

base <sub>1</sub>	+ acid <sub>2</sub>	$\rightleftharpoons$ acid <sub>1</sub>	+ base <sub>2</sub>
$H_2O$	$+ H_2O$	$\rightleftharpoons H_3O^+$	$+ OH^{-}$
CH <sub>3</sub> OH	+ CH <sub>3</sub> OH	$\rightleftharpoons$ CH <sub>3</sub> OH <sub>2</sub> <sup>+</sup>	$+ CH_3O^-$
HCOOH	+ HCOOH	$I \rightleftharpoons HCOOH_2^+$	$+ HCOO^{-}$
NH <sub>3</sub>	$+ NH_3$	$\rightleftharpoons NH_4^+$	$+NH_2^-$

Water slightly undergoes autoprotolysis at room temperature. Thus, the hydronium ( $H_3O^+$ ) and hydroxide ion( $OH^-$ ) concentrations in pure water are only about 10<sup>-7</sup> M.

### **Strengths of Acids and Bases**

strong acids: reaction with the solvent is sufficiently complete that no undissociated solute molecules are left in aqueous solution.

weak acids: react incompletely with water to give solutions containing significant quantities of both the parent acid and its conjugate base.

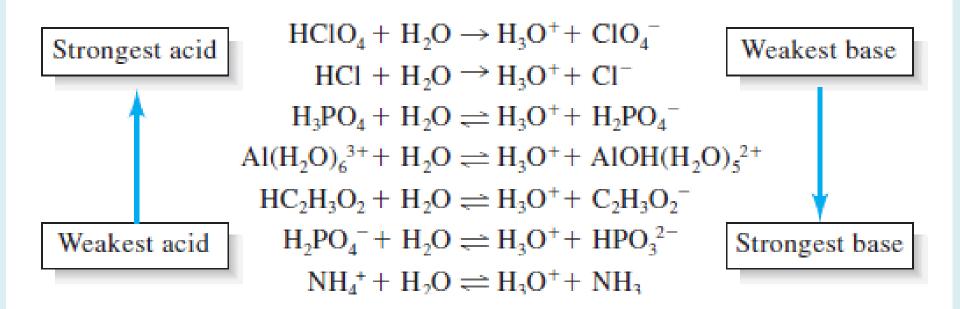
Note that acids can be cationic, anionic, or electrically neutral. The same holds for bases.

The common strong acids include HCI, HBr, HI, HCIO<sub>4</sub>, HNO<sub>3</sub>, the first proton in  $H_2SO_4$ , and the organic sulfonic acid RSO<sub>3</sub>H. The common strong bases include NaOH, KOH,  $Ba(OH)_2$ , and the quaternary ammonium hydroxide  $R_4NOH$ , where R is an alkyl group such as  $CH_3$  or  $C_2H_5$ . Conjugate acids/bases of strong acids/bases do not undergo a hydrolysis reaction. They are not strong enough to produce hydronium (H<sub>3</sub>O<sup>+</sup>) and hydroxide ion(OH<sup>-</sup>).

**Conjugate acids/bases of weak acids are strong enough to undergo a hydrolysis** reaction. They are produce hydronium (H<sub>3</sub>O<sup>+</sup>) or hydroxide ion(OH<sup>-</sup>) and change the pH of the solution.

Note that acids can be cationic, anionic, or electrically neutral. The same holds for bases. The tendency of a solvent to accept or donate protons determines the strength of a solute acid or base dissolved in it.

Water, is a leveling solvent for perchloric, hydrochloric, and nitric acids because all three are completely ionized in this solvent and show no differences in strength.



In a leveling solvent, several acids are completely dissociated and show the same strength.

Anhydrous acetic acid acts as a differentiating solvent toward the two acids by revealing the inherent differences in their acidities.

$$CH_{3}COOH + HClO_{4} \rightleftharpoons CH_{3}COOH_{2}^{+} + ClO_{4}^{-}$$

$$acid_{2} = CH_{3}COOH_{2}^{+} + bloc_{4}^{-}$$

$$base_{2}$$

Perchloric acid is, however, about 5000 times stronger than hydrochloric acid in this solvent.

We conclude that :

anhydrous acetic acid, is a weaker proton acceptor than water,
 neither of HCI and HCIO<sub>4</sub> undergoes complete dissociation.
 Instead, equilibria such as the above mentioned are established.

In a differentiating solvent, various acids dissociate to different degrees and have different strengths.

"Only perchloric acid is a strong acid in methanol and ethanol. Therefore, these two alcohols are also differentiating solvents."

# **CHEMICAL EQUILIBRIUM**

Many reactions used in analytical chemistry never result in complete conversion of reactants to products.

They proceed to a state of **chemical equilibrium** in which the ratio of concentrations of reactants and products is **constant**.

### **Equilibrium- constant expressions:**

- describe the concentration relationships among reactants and products at equilibrium.
- permit calculation of the error in an analysis resulting from the quantity of unreacted analyte that remains when equilibrium has been reached.

### The Equilibrium State

# $H_{3}AsO_{4} + 3I^{-} + 2H^{+} \rightleftharpoons H_{3}AsO_{3} + I_{3}^{-} + H_{2}O$ $H_{3}AsO_{3} + I_{3}^{-} + H_{2}O \rightleftharpoons H_{3}AsO_{4} + 3I^{-} + 2H^{+}$

We can follow the rate of this reaction and the extent to which it proceeds to the right by monitoring the appearance of the **orange-red** color of the triiodide ion  $I_3^-$ .

A solution of **identical color** intensity can be produced by using appropriate amount of reactants for both reactions.

The final position of a chemical equilibrium is **independent** of the route to the equilibrium state.

the **concentration relationship** at chemical equilibrium (that is, the *position of equilibrium) is independent of the route to the equilibrium state.* 

This relationship is altered by applying stress to the system.

Such stresses include changes in temperature, in pressure (if one of the reactants or products is a gas), or in total concentration of a reactant or a product.

These effects can be predicted qualitatively from the Le Châtelier's principle.

The mass-action effect is a shift in the position of an equilibrium caused by adding one of the reactants or products to a system.

This principle states that the position of chemical equilibrium always shifts in a direction that tends to relieve the effect of an applied stress. Equilibrium is a dynamic process.

Although chemical reactions appear to stop at equilibrium, in fact, the amounts of reactants and products are constant because the **rates of the forward and reverse processes are exactly the same.** 

**Chemical thermodynamics** is a branch of chemistry that **concerns the flow of heat and energy** in chemical reactions. The position of a chemical equilibrium is related to these energy changes.

### **Equilibrium-Constant Expressions**

$$wW + xX \rightleftharpoons yY + zZ$$

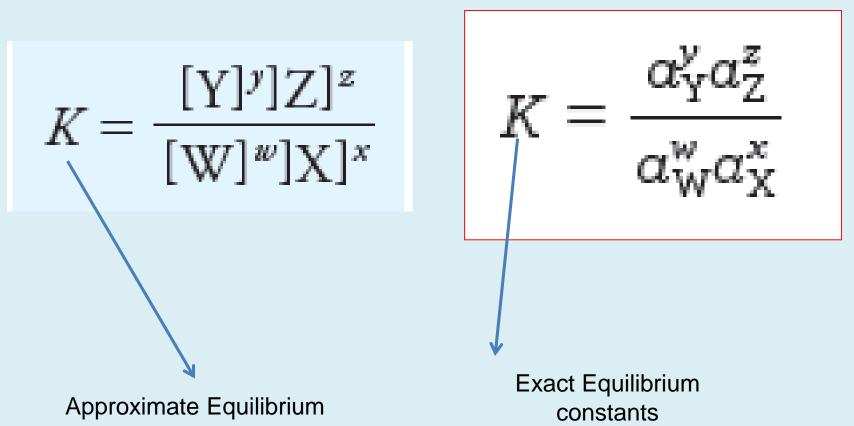
where the square-bracketed terms are:

- 1. molar concentrations if they represent dissolved solutes.
- **2. partial pressures in atmospheres** if they are gas-phase reactants or products.

In such an instance, we will often replace the square bracketed terms with the symbol *p*, which stands for the partial pressure of the gases in atmospheres.

If a reactant or product is a **pure liquid**, a **pure solid**, or the **solvent** present in excess, **no term** for this species appears in the equilibrium-constant expression.

 $K = \frac{[Y]^{y}]Z]^{z}}{[W]^{w}]X]^{x}}$ 



constants

(thermodynamic)

### **Types of Equilibrium Constants in Analytical Chemistry**

#### **TABLE 9-2**

Equilibria and Equilibrium Constants Important in Analytical Chemistry					
Type of Equilibrium	Name and Symbol of Equilibrium-Constant	Typical Example	Equilibrium-Constant Expression		
Dissociation of water Heterogeneous equilibrium between a slightly soluble substance and its ions in a saturated solution	Ion-product constant, <i>K</i> <sub>w</sub> Solubility product, <i>K</i> <sub>sp</sub>	$2H_2O \rightleftharpoons H_3O^+ + OH^-$ BaSO <sub>4</sub> (s) \rightleftharpoons Ba <sup>2+</sup> + SO <sub>4</sub> <sup>2-</sup>	$K_{\rm w} = [{\rm H}_3{\rm O}^+][{\rm OH}^-]$ $K_{\rm sp} = [{\rm Ba}^{2+}][{\rm SO}_4^{-2-}]$		
Dissociation of a weak acid or base	Dissociation constant, <i>K</i> <sub>a</sub> or <i>K</i> <sub>b</sub>	$CH_{3}COOH + H_{2}O \rightleftharpoons$ $H_{3}O^{+} + CH_{3}COO^{-}$ $CH_{3}COO^{-} + H_{2}O \rightleftharpoons$ $OH^{-} + CH_{3}COOH$	$K_{a} = \frac{[H_{3}O^{+}][CH_{3}COO^{-}]}{[CH_{3}COOH]}$ $K_{b} = \frac{[OH^{-}][CH_{3}COOH]}{[CH_{3}COO^{-}]}$		
Formation of a complex ion	Formation constant, $\beta_n$	$Ni^{2+} + 4CN^{-} \rightleftharpoons Ni(CN)_{4}^{2-}$	$\beta_4 = \frac{[\text{Ni}(\text{CN})_4^{2-}]}{[\text{Ni}^{2+}][\text{CN}^{-}]^4}$		
Oxidation/reduction equilibrium	K <sub>redox</sub>	$MnO_4^- + 5Fe^{2+} + 8H^+ \rightleftharpoons$ $Mn^{2+} + 5Fe^{3+} + 4H_2O$	$K_{\text{redox}} = \frac{[\text{Mn}^{2+}][\text{Fe}^{3+}]^5}{[\text{Mn}\text{O}_4^{-}][\text{Fe}^{2+}]^5[\text{H}^{+}]^8}$		
Distribution equilibrium for a solute between immiscible solvents	K <sub>d</sub>	$I_2(aq) \rightleftharpoons I_2(org)$	$K_{\rm d} = \frac{[I_2]_{\rm org}}{[I_2]_{\rm aq}}$		

The formation of Ni(CN)<sub>4</sub><sup>2-</sup> is typical in that it occurs in steps as shown. **stepwise formation constants are:**  $K_1$ ,  $K_2$ , ...

$$\operatorname{Ni}^{2^{+}} + \operatorname{CN}^{-} \rightleftharpoons \operatorname{Ni}(\operatorname{CN})^{+} \qquad K_{1} = \frac{[\operatorname{Ni}(\operatorname{CN})^{+}]}{[\operatorname{Ni}^{2^{+}}][\operatorname{CN}^{-}]}$$
$$\operatorname{Ni}(\operatorname{CN})^{+} + \operatorname{CN}^{-} \rightleftharpoons \operatorname{Ni}(\operatorname{CN})_{2} \qquad K_{2} = \frac{[\operatorname{Ni}(\operatorname{CN})_{2}]}{[\operatorname{Ni}(\operatorname{CN})^{+}][\operatorname{CN}^{-}]}$$
$$\operatorname{Ni}(\operatorname{CN})_{2} + \operatorname{CN}^{-} \rightleftharpoons \operatorname{Ni}(\operatorname{CN})_{3}^{-} \qquad K_{3} = \frac{[\operatorname{Ni}(\operatorname{CN})_{3}^{-}]}{[\operatorname{Ni}(\operatorname{CN})_{2}][\operatorname{CN}^{-}]}$$
$$\operatorname{Ni}(\operatorname{CN})_{3}^{-} + \operatorname{CN}^{-} \rightleftharpoons \operatorname{Ni}(\operatorname{CN})_{4}^{2^{-}} \qquad K_{4} = \frac{[\operatorname{Ni}(\operatorname{CN})_{4}^{2^{-}}]}{[\operatorname{Ni}(\operatorname{CN})_{3}^{-}][\operatorname{CN}^{-}]}$$

Overall constants are designated by the symbol  $\beta_n$ .

$$Ni^{2+} + 2CN^{-} \rightleftharpoons Ni(CN)_{2} \qquad \beta_{2} = K_{1}K_{2} = \frac{[Ni(CN)_{2}]}{[Ni^{2+}][CN^{-}]^{2}}$$
$$Ni^{2+} + 3CN \rightleftharpoons Ni(CN)_{3}^{-} \qquad \beta_{3} = K_{1}K_{2}K_{3} = \frac{[Ni(CN)_{3}^{-}]}{[Ni^{2+}][CN^{-}]^{3}}$$
$$Ni^{2+} + 4CN^{-} \rightleftharpoons Ni(CN)_{4}^{2-} \qquad \beta_{4} = K_{1}K_{2}K_{3}K_{4} = \frac{[Ni(CN)_{4}^{2-}]}{[Ni^{2+}][CN^{-}]^{4}}$$

### **Applying the Ion-Product Constant for Water**

Aqueous solutions contain small concentrations of **hydronium** and **hydroxide ions** as a result of the dissociation reaction.

$$2H_{2}O \rightleftharpoons H_{3}O^{+} + OH^{-}$$

$$K[H_{2}O]^{2} = K_{w} = [H_{3}O^{+}][OH^{-}]$$
ion-product constant for water  
-log  $K_{w} = -\log[H_{3}O^{+}] - \log[OH^{-}]$   
 $M_{w} = pH + pOH$  At 25°C,  $pK_{w} = 14.00$ .  

$$K = \frac{[H_{3}O^{+}][OH^{-}]}{[H_{2}O]^{2}}$$

$$K = \frac{[H_{3}O^{+}][OH^{-}]}{[H_{2}O]^{2}}$$

$$K = \frac{[H_{3}O^{+}][OH^{-}]}{[H_{2}O]^{2}}$$

$$K = \frac{[H_{3}O^{+}][OH^{-}]}{[H_{2}O]^{2}}$$

### Why [H<sub>2</sub>O] Does Not Appear in Equilibrium-Constant Expressions for Aqueous Solutions

In a dilute aqueous solution, the molar concentration of water is

$$[H_2O] = \frac{1000 \text{ g} \text{ H}_2O}{\text{L} \text{ H}_2O} \times \frac{1 \text{ mol } \text{H}_2O}{18.0 \text{ g} \text{ H}_2O} = 55.6 \text{ M}$$

If we have 0.1 mol of HCl in 1 L of water. The presence of this acid will shift the equilibrium to the left.

$$2H_2O \rightleftharpoons H_3O^+ + OH^-$$

Originally, however, there was only 10<sup>-7</sup> mol/L OH<sup>-</sup> to consume the added protons.

Therefore, even if all the  $OH^{-}$  ions are converted to  $H_2O$ , the water concentration will increase to only

$$[H_2O] = 55.6 \frac{\text{mol } H_2O}{\text{L } H_2O} + 1 \times 10^{-7} \frac{\text{mol } OH^-}{\text{L } H_2O} \times \frac{1 \text{ mol } H_2O}{\text{mol } OH^-} \approx 55.6 \text{ M}$$

 $\frac{10^{-7} \,\mathrm{M}}{55.6 \,\mathrm{M}} \times 100\% = 2 \times 10^{-7}\%$ 

The percent change in water concentration is insignificant.

 $K(55.6)^2 = K_w = 1.00 \times 10^{-14} \text{ at } 25^{\circ}\text{C}$ 

The ion-product constant for water permits us to **easily find the hydronium and hydroxide ion concentrations** of aqueous solutions.

### Applying the ION-PRODUCT constant for water

Calculate the hydronium and hydroxide ion concentrations of pure water at 25°C and 100°C.

Answers:

At 25°C $[H_3O^+] = [OH^-] = 1.00 \times 10^{-7} M$ At 100°C $[H_3O^+] = [OH^-] = 7.00 \times 10^{-7} M$ 

**EXAMPLE :** Calculate the hydronium and hydroxide ion concentrations and the pH and pOH of 0.013 M aqueous KOH at 25 °C.

**EXAMPLE :** Calculate the hydronium and hydroxide ion concentrations and the pH and pOH of **1.30×10**<sup>-7</sup> M aqueous KOH at 25 °C.

**EXAMPLE :** Calculate the hydronium and hydroxide ion concentrations and the pH and pOH of 0.023 M aqueous HNO<sub>3</sub> at 25 °C.

**EXAMPLE :** Calculate the hydronium and hydroxide ion concentrations and the pH and pOH of  $2.30 \times 10^{-7}$  M aqueous HNO<sub>3</sub> at 25 °C.

# **USING ACID/BASE DISSOCIATION CONSTANTS**

## 1. Dissociation Constants for Conjugate Acid/Base Pairs

# 2. Hydronium Ion Concentration of Solutions of Weak Acids

## 3. Hydronium Ion Concentration of Solutions of Weak Bases

# **Using Solubility-Product Constants**

## **1.** The Solubility of a Precipitate in Pure Water

# 2.The Effect of a Common Ion on the Solubility of a Precipitate