

CHE/CEN I 38

COMPUTER PROGRAMMING

MATRIX OPERATIONS

References

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2. Hunt, B.R., Lipsman, L.R. and Rosemberg J. M. "A guide to MATLAB for Beginners and Experienced Users" Cambridge University Press, 2001.
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CONTENT

- Definitions and Terms
- Special Matrices
- Matrix Algebra
- Determinant and Inverse

- **Definitions and Terms**

- Vector, matrix
- Element, subscript
- "Matrices of the same kind"

Vector

A *VECTOR* IS A COLUMN OF NUMBERS
CONSISTING OF N NUMBERS

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \\ \dots \\ \mathbf{a}_n \end{bmatrix}$$

The values shown by \mathbf{a}_i are the **elements** of vector \mathbf{a} .

Matrix (Pl.: Matrices)

MATRIX

is a rectangular array of numbers consisting of m **rows** and n **columns**.

$$\begin{bmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \dots & \mathbf{a}_{1m} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \dots & \mathbf{a}_{2m} \\ \dots & \dots & \dots & \dots \\ \mathbf{a}_{n1} & \dots & \dots & \mathbf{a}_{nm} \end{bmatrix}$$

The values shown by \mathbf{a}_{ij} are the **elements** of vector \mathbf{a} .

Matrix example

$$\begin{bmatrix} 2 & 5 & 7 & 8 \\ 5 & 6 & 8 & 3 \\ 1 & 6 & 4 & 0 \end{bmatrix}$$

This matrix has 3 rows
rows and 4 columns.

We say it is a **3 x 4**
(3 by 4) matrix.

We denote the element on the second row
and fourth column with **$a_{2,4}$**

- **Special Matrices**

- Row / column matrices
- Zero matrix
- Square matrix
- Diagonal matrix
- Unit matrix
- Symmetric matrix
- Transpose
- Orthogonal matrix

Row matrix / column matrix

A matrix with
one row is called
a **row matrix**.

$$\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

A matrix with one column
is called a **column matrix**.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Also called: **row-vector** and **column-vector**.

Zero Matrix

**A ZERO
MATRIX**

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Square Matrix

$$\begin{bmatrix} 2 & 5 & 7 \\ 5 & 6 & 8 \\ 1 & 6 & 4 \end{bmatrix}$$

A SQUARE
MATRIX
HAS THE SAME
NUMBER OF ROWS
AND COLUMNS.

The elements $a_{i,i}$, with $i = 1, 2, 3, \dots$
are called **diagonal** elements.

Diagonal matrix

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

**A DIAGONAL MATRIX IS
A SQUARE MATRIX WITH
ALL NON-DIAGONAL
ELEMENTS ARE 0.**

Unit matrix

A **unit matrix** is a diagonal matrix with all diagonal elements **1**.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Also called: **identity matrix - I**.

Symmetric matrix

A SYMMETRIC MATRIX IS A SQUARE MATRIX WHERE $A_{I,J} = A_{J,I}$ FOR ALL ELEMENTS.

$$\begin{bmatrix} 1 & -1 & 4 \\ -1 & 2 & 3 \\ 4 & 3 & 6 \end{bmatrix}$$

Transpose

THE TRANSPOSE OF THE $M \times N$ MATRIX A IS SHOWN BY A^T .

- A^T is an $n \times m$ matrix,
- row i of A = column i of A^T
for ($i = 1, 2, 3, \dots, n$)

Transpose (2)

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

$$A^T =$$

$$\begin{bmatrix} 1 & 5 & 9 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \\ 4 & 8 & 12 \end{bmatrix}$$

Orthogonal matrix

AN **ORTHOGONAL MATRIX** IS A SQUARE MATRIX WHICH PRODUCES A UNIT MATRIX IF IT IS MULTIPLIED BY ITS OWN TRANSPOSE:

$$\mathbf{A} \times \mathbf{A}^T = \mathbf{I}$$

Properties of special matrices

$$\mathbf{A} = (\mathbf{A}^T)^T = (((\mathbf{A}^T)^T)^T)^T$$

$$(\mathbf{A} + \mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$$

If \mathbf{A} is a symmetric matrix, then

$$\mathbf{A} = \mathbf{A}^T$$

If \mathbf{A} is an orthogonal matrix, then

$$\mathbf{A} \times \mathbf{A}^T = \mathbf{I}$$