Physics 122: Electricity & Magnetism – Lecture 6 Gauss’s Law

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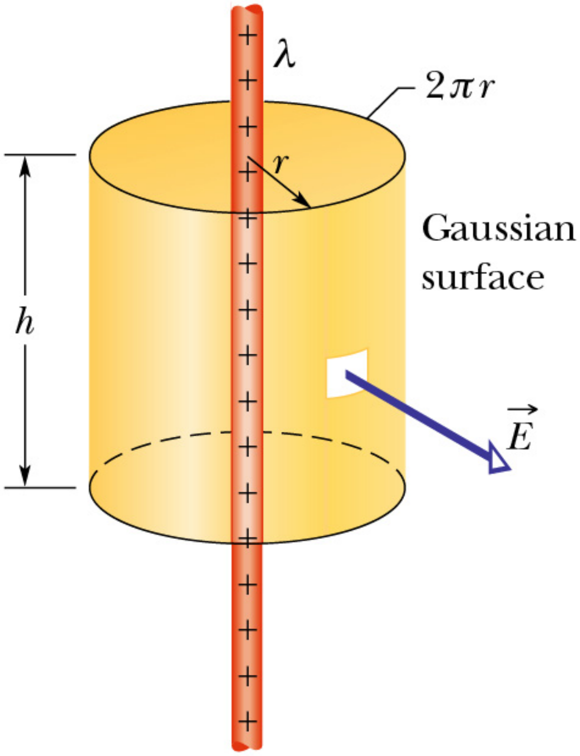
Line of Charge

- In the previous chapter, we calculated the E field on the axis of a line of charge, but with Gauss' Law we can now handle finding E off the line axis.
- Here is a line geometry, where the charges are evenly distributed on a long line. If the total charge on the line is Q, and the line has a total length L_{tot}, then the <u>*linear charge density*</u> is

$$\lambda = Q / L_{tot} \quad \mathrm{C/m}$$



- The E field is everywhere perpendicular to the line (again, if not, the charges will move until the part parallel to the line is nullified).
- Use a cylindrical gaussian surface that is parallel to $\vec{E}$ on the top and bottom (so no flux through those surfaces), and is perpendicular to $\vec{E}$ elsewhere.
- The area vector $d\vec{A}$ is parallel to $\vec{E}$, and the total area is $2\pi rh$ so

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 E 2\pi rh = \lambda h$$

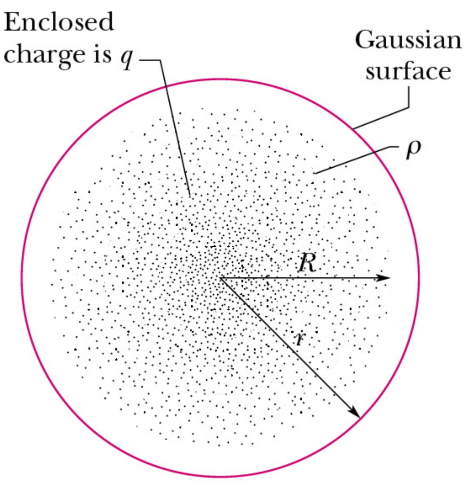
or

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Line of Charge

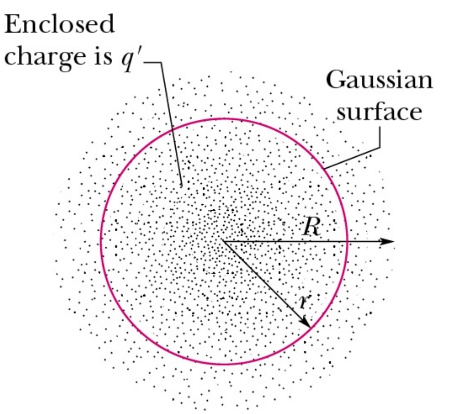
Uniform Sphere of Charge

- Here is a spherical geometry, where the charges are evenly distributed throughout the volume. If the total charge in the sphere is Q, and the sphere has a radius R, then the *volume charge density* is $$\rho = \frac{Q}{\frac{4}{3}\pi R^3} \quad \text{C/m}^3$$



(a)

- By symmetry, the E field is everywhere radial from the center of the sphere.
- Use a spherical gaussian surface, which is perpendicular to $\vec{E}$ everywhere.
- The area vector $\vec{A}$ is parallel to $\vec{E}$, and the total area is $4\pi r^2$ so when the gaussian surface radius is $r < R$, then $$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 E 4\pi r^2 = \rho \frac{4}{3}\pi r^3$$ or $$E = \frac{\rho}{3\varepsilon_0} r$$

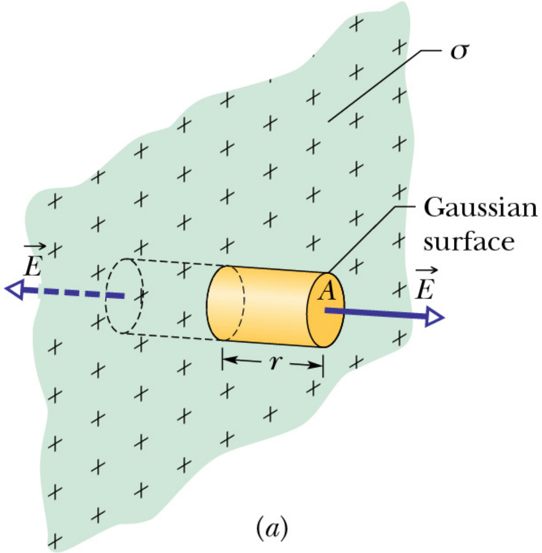


- When $r>R$, then the charge enclosed is just Q, so $$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 E 4\pi r^2 = Q$$ or $$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Coulomb's Law again

Nonconducting Sheet

- A nonconducting sheet with a uniform surface charge density has the same geometry as for the conducting plate, so use the same gaussian surface.
- The only difference is that now one end cannot close in a conductor, so there is electric flux out both ends.
- As you may expect, the resulting electric field is half of what we got before.



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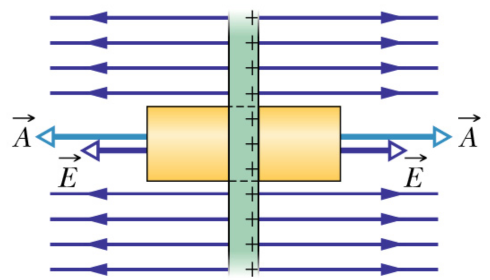
$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 2EA = \sigma A$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

Sheet of Charge



Two Parallel Conducting Plates

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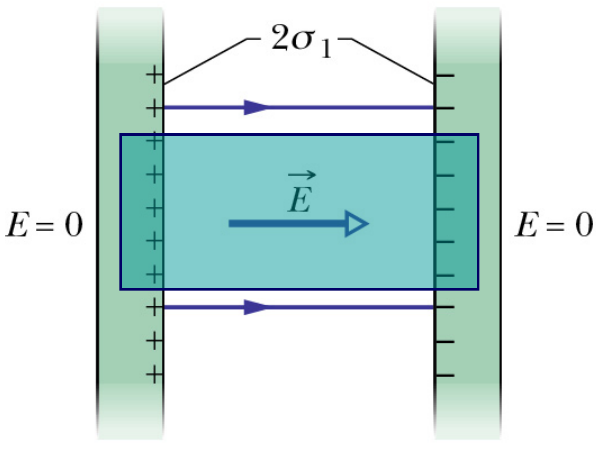
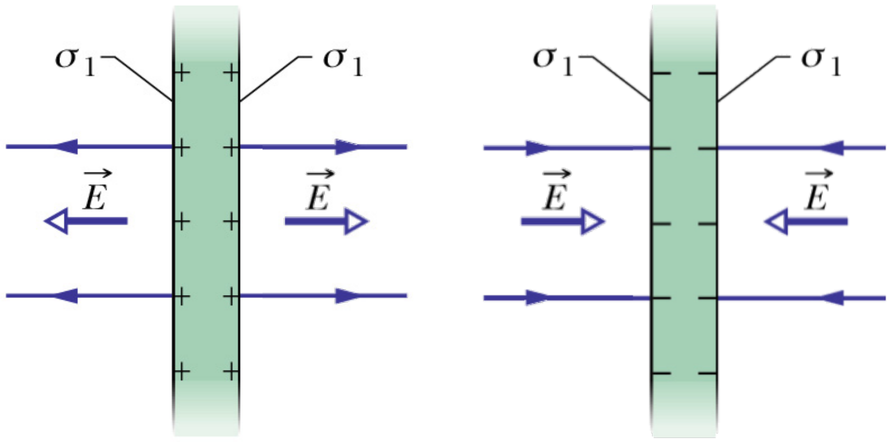
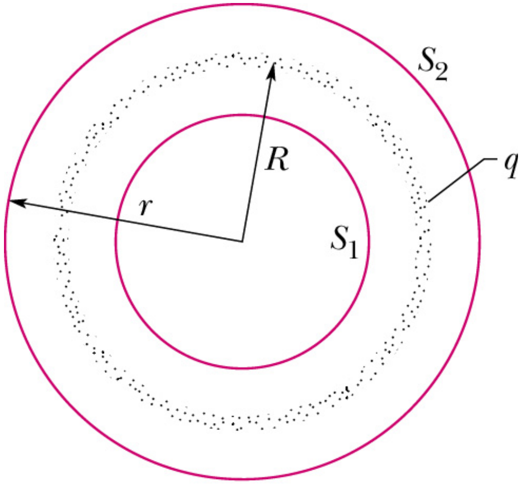


Two Parallel Nonconducting Sheets



- The result is much the same as before, with the electric field in between being twice what it was previously.

Spherical Symmetry



Spherical shell

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad (r \geq R)$$

$$E = 0 \quad (r < R)$$

We earlier said that a shell of uniform charge attracts or repels a charged particle that is outside the shell as if the shell's charge were concentrated at the center of the shell. We can now prove this using Gauss' Law.

We also said that a shell of uniform charge exerts no electrostatic force on a charged particle that is located inside the shell. Again, Gauss' Law can be used to prove this.

Summary

- Electric flux is the amount of electric field passing through a closed surface.
- Flux is positive when electric field is outward, and negative when electric field is inward through the closed surface.
- Gauss' Law states that the electric flux is proportional to the net charge enclosed by the surface, and the constant of proportionality is ε_0. In symbols, it is

$$\varepsilon_0 \Phi = q_{enc}$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$

- There are three geometries we typically deal with:

Geometry	Charge Density	Gaussian surface	Electric field
Linear	$\lambda = q/L$	Cylindrical, with axis along line of charge	$E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$ $\boxed{\text{Line of Charge}}$
Sheet or Plane	$\sigma = q/A$	Cylindrical, with axis along E.	$E = \dfrac{\sigma}{\varepsilon_0}$ $\boxed{\text{Conducting}}$ $E = \dfrac{\sigma}{2\varepsilon_0}$ $\boxed{\text{Nonconducting}}$
Spherical	$\rho = q/V$	Spherical, with center on center of sphere	$E = \dfrac{q}{4\pi\varepsilon_0 r^2}$ $\boxed{r \geq R}$ $E = \left(\dfrac{q}{4\pi\varepsilon_0 R^3}\right) r$ $\boxed{r < R}$

Summary, cont'd

- The electric field is zero inside a conductor.
- The electric field is zero inside a cavity within a conductor, unless there is a charge inside that is not in contact with the walls.
- The electric field at the surface of a conductor is always perpendicular to that surface.
- Note, none of this is true for insulators.