Physics 121: Electricity & Magnetism – Lecture 9
Capacitance

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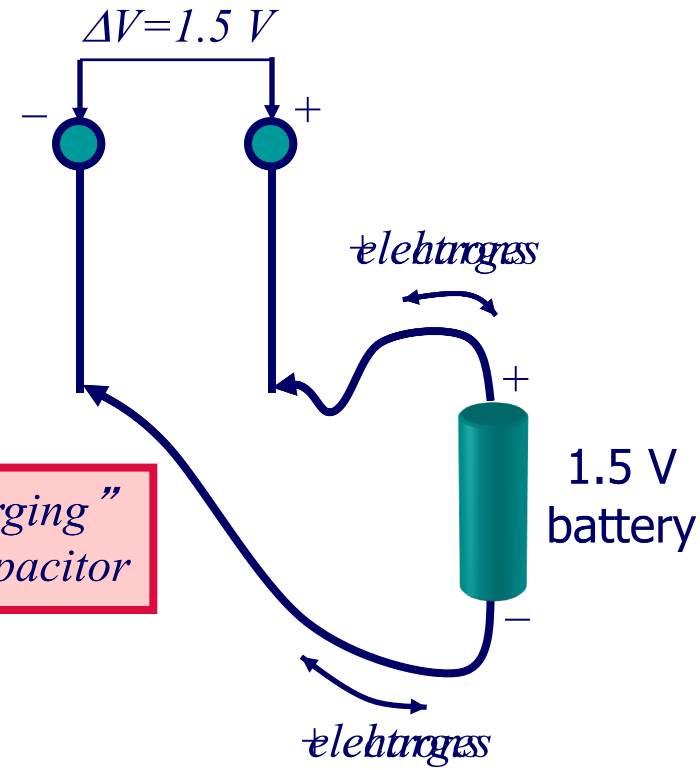
What is Capacitance?



- Charges will flow until the right conductor's potential is the same as the + side of the battery, and the left conductor's potential is the same as the – side of the battery.

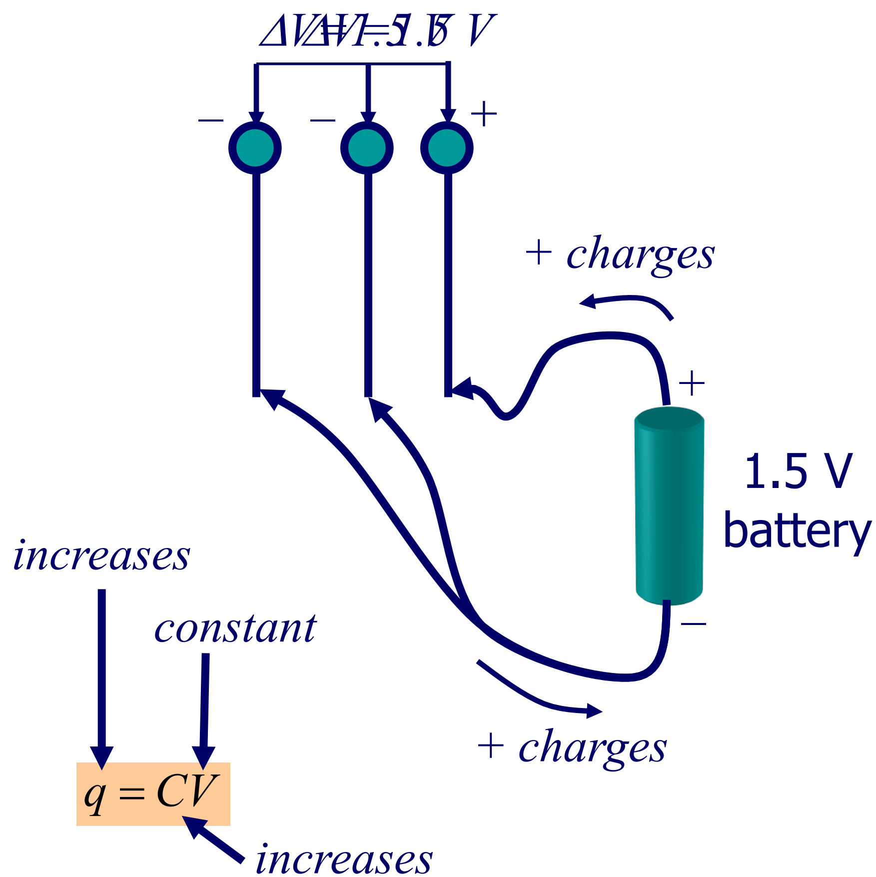
"Charging" the capacitor

- How much charge is needed to produce an electric field whose potential difference is 1.5 V?
- Depends on capacitance: $q = CV$

definition of capacitance

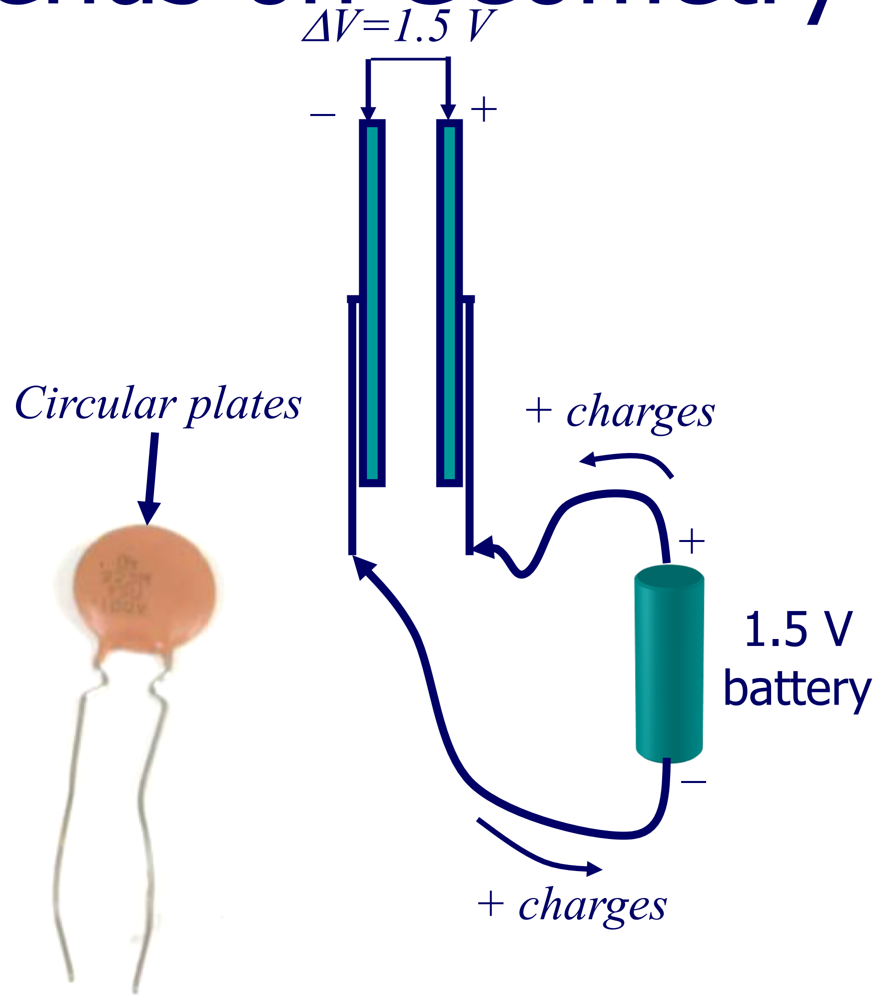
Capacitance Depends on Geometry

- What happens when the two conductors are moved closer together?
- They are still connected to the battery, so the potential difference cannot change.
- But recall that $V = -\int \vec{E} \cdot d\vec{s}$.
- Since the distance between them decreases, the E field has to increase.
- Charges have to flow to make that happen, so now these two conductors can hold more charge. I.e. the capacitance increases.

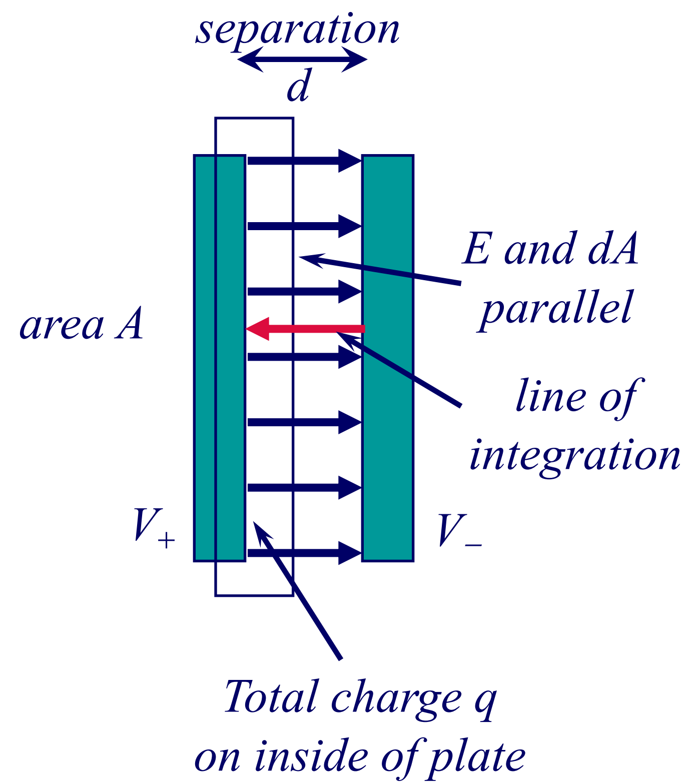


Capacitance Depends on Geometry

- What happens if we replace the small conducting spheres with large conducting plates?
- The plates can hold a lot more charge, so the capacitance goes way up.



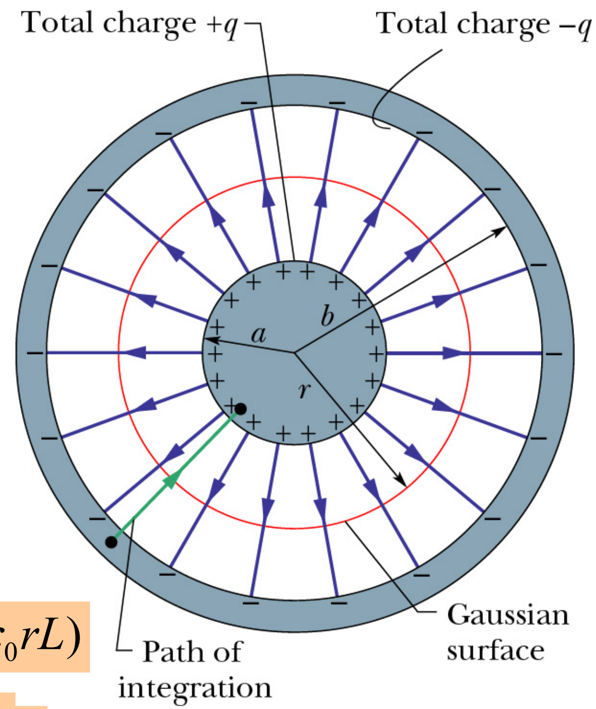
Capacitance for Parallel Plates



$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q$ so $q = \varepsilon_0 E A$

- Need to find potential difference $V = V_+ - V_- = -\int \vec{E} \cdot d\vec{s}$
- Since $E = constant$, we have $V = Ed$, so the capacitance is $C = q/V = \frac{\varepsilon_0 E A}{Ed} = \frac{\varepsilon_0 A}{d}$

Capacitance for Other Configurations (Cylindrical)



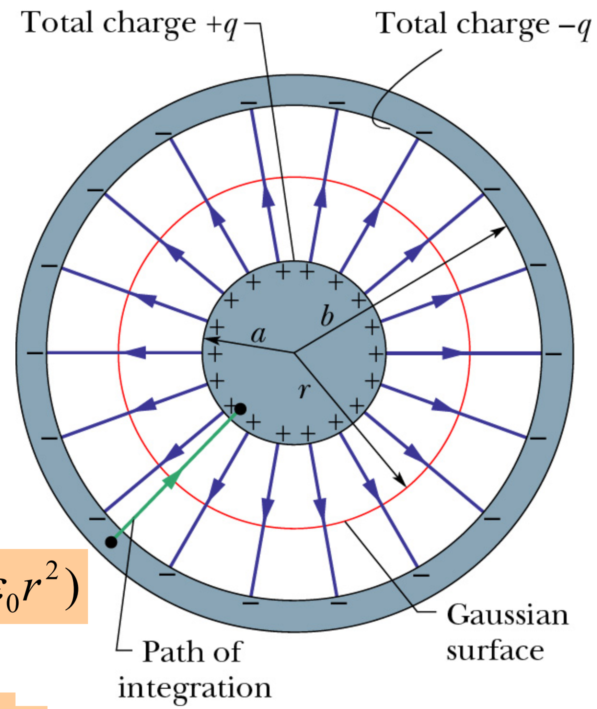
- Cylindrical capacitor
  - The E field falls off as $1/r$.
  - The geometry is fairly simple, but the V integration is slightly more difficult.
- To calculate capacitance, we first need to determine the E field between the plates. We use Gauss' Law, with a cylindrical gaussian surface closed in the region between the plates (neglect fringing at ends):

$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q$ So $q = \varepsilon_0 EA = \varepsilon_0 E(2\pi rL)$ or $E = q/(2\pi\varepsilon_0 rL)$

- Need to find potential difference $V = V_+ - V_- = -\int \vec{E} \cdot d\vec{s}$
- Since $E \sim 1/r$, we have $V = \dfrac{q}{2\pi\varepsilon_0 L}\int_b^a \dfrac{dr}{r} = \dfrac{q}{2\pi\varepsilon_0 L}\ln\left(\dfrac{b}{a}\right)$, so the capacitance is

$$C = q/V = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$

Capacitance for Other Configurations (Spherical)



$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q$ So $q = \varepsilon_0 EA = \varepsilon_0 E(4\pi r^2)$ or $E = q/(4\pi\varepsilon_0 r^2)$

- Need to find potential difference $V = V_+ - V_- = -\int \vec{E} \cdot d\vec{s}$
- Since $E \sim 1/r^2$, we have $V = \frac{q}{4\pi\varepsilon_0} \int_b^a \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$, so the capacitance is

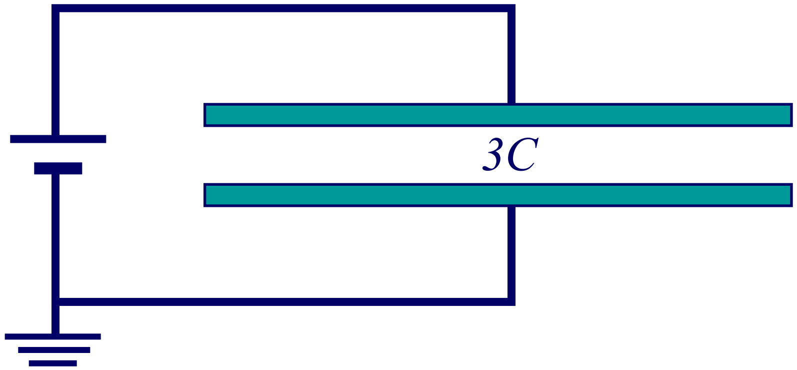
$$C = q/V = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

Capacitance Summary

- Parallel Plate Capacitor $$C = \frac{\varepsilon_0 A}{d}$$
- Cylindrical (nested cylinder) Capacitor $$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$
- Spherical (nested sphere) Capacitor $$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$
- Capacitance for isolated Sphere $$C = 4\pi\varepsilon_0 R$$
- Units: $\varepsilon_0 \times$length = C^2/Nm = F (farad), named after Michael Faraday. [note: ε_0 = 8.85 pF/m]

Capacitors in Parallel

- No difference between



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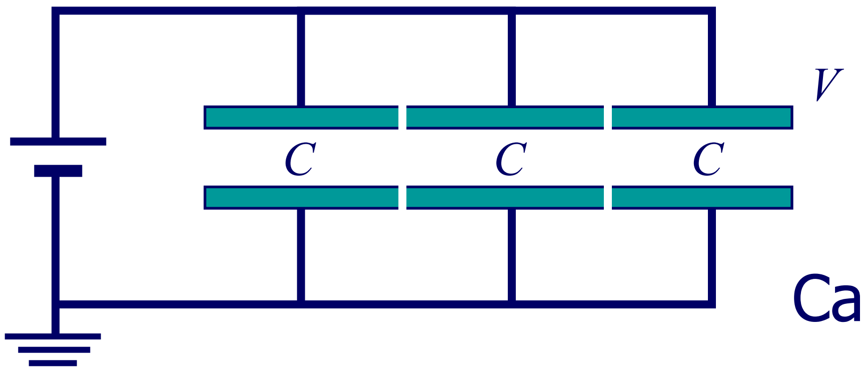
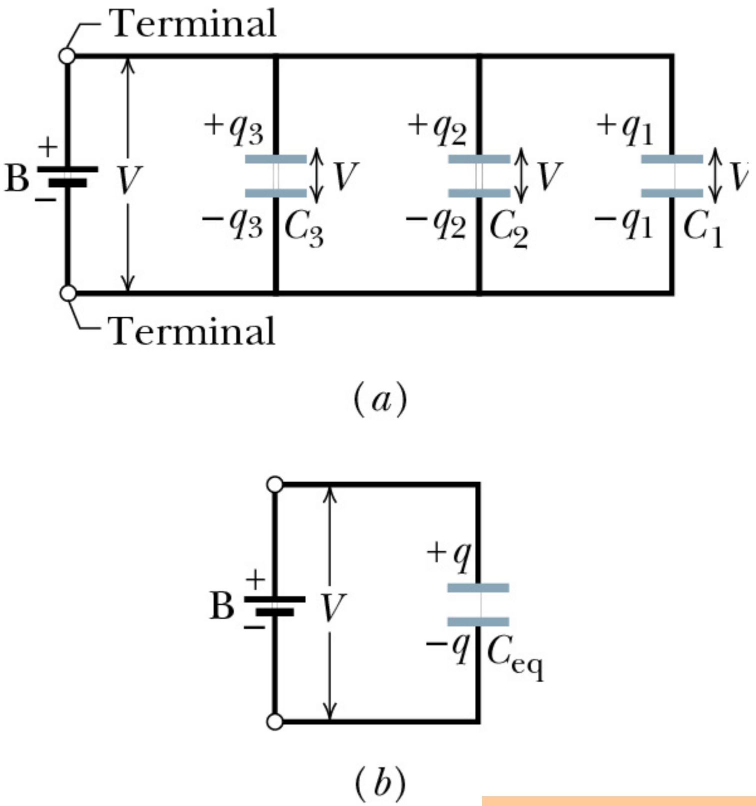
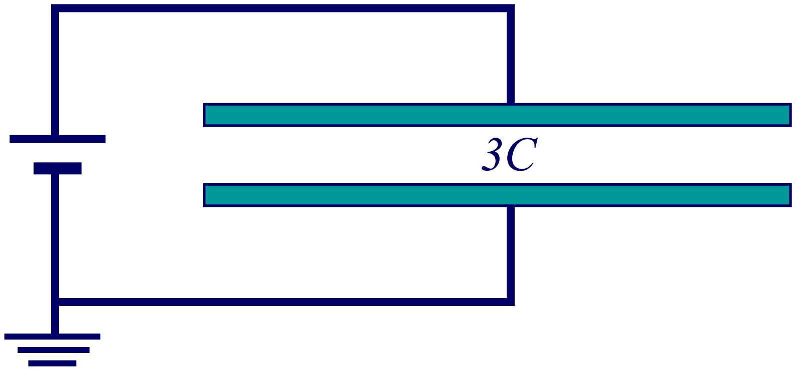




Capacitors in parallel: $C_{eq} = \sum_{j=1}^{n} C_j$

Capacitors in Series

- There *is* a difference between



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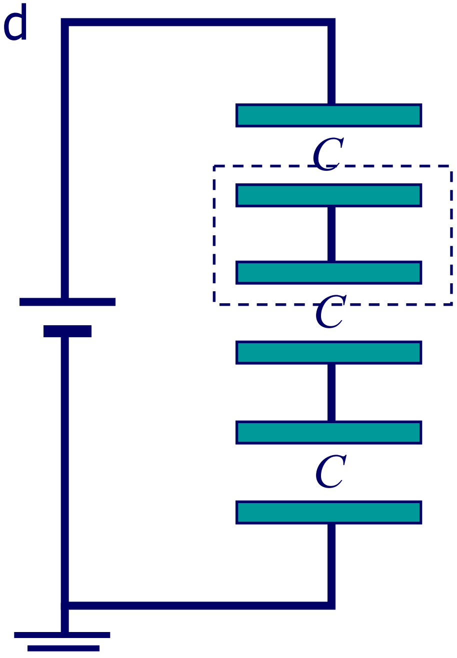

- Charge on lower plate of one and upper plate of next are equal and opposite. (show by gaussian surface around the two plates).
- Total charge is q, but voltage on each is only $V/3$.

Capacitors in series: $$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$$

Capacitors in Series

- To see the series formula, consider the individual voltages across each capacitor

$$V_1 = \frac{q}{C_1},\ V_2 = \frac{q}{C_2},\ V_3 = \frac{q}{C_3}$$

- The sum of these voltages is the total voltage of the battery, V

$$V = V_1 + V_2 + V_3 = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

- Since $V/q = 1/C_{eq}$, we have

$$\frac{V}{q} = \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$