EEE328
Digital Signal Processing

Ankara University

Faculty of Engineering

Electrical and Electronics Engineering Department

Fourier Transform and Its Properties

EEE328 Digital Signal Processing

Lecture 6

Agenda

- Fourier Transform Representation of Sequences
- Properties of Fourier Transform
- Fourier Transform Theorems

D-T FT

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

$$X(e^{j\omega}) = X_R(e^{j\omega}) + jX_I(e^{j\omega})$$

$$X(e^{j\omega}) = |X(e^{j\omega})|e^{j\sphericalangle X(e^{j\omega})}$$

D-T FT

Impulse Response by Inverse D-T FT

$$h[n] = \frac{1}{2\pi} \int_{2\pi} H(e^{j\omega}) e^{j\omega n} d\omega$$

Convergence

$$\left|X\left(e^{j\omega}\right)\right| = \left|\sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}\right| \leq \sum_{n=-\infty}^{\infty} |x[n]|\, e^{-j\omega}|$$

$$\leq \sum_{n=-\infty}^{\infty} |x[n]| < \infty$$

Symmetry Property of D-T FT

$$x[n] = x_e[n] + x_o[n]$$

$$x_e[n] = \frac{1}{2}(x[n] + x^*[-n]) = x_e^*[-n]$$

$$x_o[n] = \frac{1}{2}(x[n] - x^*[-n]) = -x_o^*[-n]$$

$$X(e^{j\omega}) = X_e(e^{j\omega}) + X_o(e^{j\omega})$$

$$X_e(e^{j\omega}) = \frac{1}{2}[X(e^{j\omega}) + X^*(e^{-j\omega})]$$

$$X_o(e^{j\omega}) = \frac{1}{2}[X(e^{j\omega}) - X^*(e^{-j\omega})]$$

⇒

$$X_e(e^{j\omega}) = X_e^*(e^{j\omega})$$

$$X_o(e^{j\omega}) = -X_o^*(e^{j\omega})$$

Fourier Transform Theorems

$$X(e^{j\omega}) = \mathcal{F}\{x[n]\}$$

$$x[n] = \mathcal{F}^{-1}\{X(e^{j\omega})\}$$

$$x[n] \overset{\mathcal{F}}{\longleftrightarrow} X(e^{j\omega})$$

$$y[n] \overset{\mathcal{F}}{\longleftrightarrow} Y(e^{j\omega})$$

$$ax[n] + by[n] \overset{\mathcal{F}}{\leftrightarrow} X(e^{j\omega}) + Y(e^{j\omega})$$

$$x[n - n_d] \overset{\mathcal{F}}{\leftrightarrow} e^{-j\omega n_d} X(e^{j\omega})$$

$$x[-n] \overset{\mathcal{F}}{\leftrightarrow} X(e^{-j\omega})$$

$$x[n] * y[n] \overset{\mathcal{F}}{\leftrightarrow} X(e^{j\omega}) Y(e^{j\omega})$$

$$x[n] y[n] \overset{\mathcal{F}}{\leftrightarrow} \frac{1}{2\pi} \int_{2\pi} X(e^{j\theta}) Y(e^{j(\omega-\theta)}) d\theta$$

- Parseval's Theorem

$$x[n] \overset{\mathcal{F}}{\leftrightarrow} X(e^{j\omega})$$

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\omega})|^2 \, d\omega$$

References

- Signals & Systems, Second Edition, A. V. Oppenheim, A. S. Willsky with S. H. Nawab, Prentice Hall, 1997
- Discrete-Time Signal Processing, Second Edition, A. V. Oppenheim, R. W. Schafer with J. R. Buck, Prentice Hall, 1999