# EEE104
# Circuit Analysis I

Ankara University

Faculty of Engineering

Electrical and Electronics Engineering Department

# Natural and Step Responses of RLC Circuits

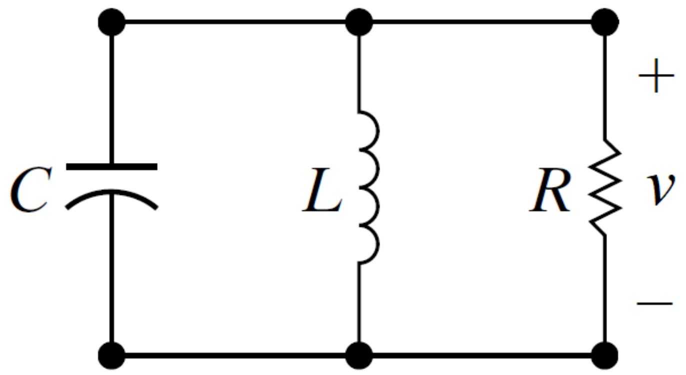
EEE104 Circuit Analysis I

Lecture 12

# Agenda

- Natural Response of a Parallel RLC Circuit
- Forms of Natural Response of a Parallel RLC Circuit
- The Overdamped Response

- Natural Response of a Parallel RLC Circuit



- Natural Response of a Parallel RLC Circuit

General Solution of the Second-Order Differential Equation

$$v = Ae^{st}$$

$$As^2e^{st} + \frac{As}{RC}e^{st} + \frac{Ae^{st}}{LC} = 0$$

$$Ae^{st}\left(s^2 + \frac{s}{RC} + \frac{1}{LC}\right) = 0$$

- Natural Response of a Parallel RLC Circuit

General Solution of the Second-Order Differential Equation

$$s^2 + \frac{s}{RC} + \frac{1}{RC} = 0$$

$$s_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$s_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$v = A_1 e^{s_1 t}$$

$$v = A_2 e^{s_2 t}$$

- Natural Response of a Parallel RLC Circuit

General Solution of the Second-Order Differential Equation

$$v = v_1 + v_2 = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$A_1 e^{s_1 t}\left(s_1^2 + \frac{1}{RC}s_1 + \frac{1}{LC}\right) + A_2 e^{s_2 t}\left(s_2^2 + \frac{1}{RC}s_2 + \frac{1}{LC}\right) = 0$$

$$v = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

Characteristic Roots

Neper Frequency: $\alpha = \frac{1}{2RC}$

Resonant Radian Frequency: $\omega_0 = \frac{1}{\sqrt{LC}}$

- Forms of the Natural Response of a Parallel RLC Circuit

- The Overdamped Response

$$\omega_0^2 < \alpha^2$$

$$v = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$v(0^+) = A_1 + A_2$$

$$\frac{dv(0^+)}{dt} = s_1 A_1 + s_2 A_2$$

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C}$$

- Forms of the Natural Response of a Parallel RLC Circuit

- The Overdamped Response

$$i_C(0^+) = \frac{-V_0}{R} - I_0$$

- Forms of the Natural Response of a Parallel RLC Circuit

- The Underdamped Response

$\omega_0^2 > \alpha^2$

$$s_1 = -\alpha + \sqrt{-(\omega_0^2 - \alpha^2)}$$
$$= -\alpha + j\sqrt{\omega_0^2 - \alpha^2}$$
$$= -\alpha + j\omega_d$$
$$s_2 = -\alpha - j\omega_d$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

★ $$v(t) = B_1 e^{-\alpha t} cos\omega_d t + B_2 e^{-\alpha t} sin\omega_d t$$

$$v(0^+) = V_0 = B_1$$

$$\frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C} = -\alpha B_1 + \omega_d B_2$$

- Forms of the Natural Response of a Parallel RLC Circuit

- The Critically Damped Response

$$s_1 = s_2 = -\alpha = -\frac{1}{2RC}$$

$$v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$ ★

$$v(0^+) = V_0 = D_2$$

$$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = D_1 - \alpha D_2$$

# Reference

- Electric Circuits, Tenth Edition, James W. Nilsson, Susan A. Riedel Pearson, 2015