Lecture 6 : **Relativistic kinematics -2**

Spatial and temporal components of the energy-momentum four vector are :

$$\mathbf{p} = \gamma m\mathbf{v} = \frac{m\mathbf{v}}{\sqrt{1 - v^2/c^2}}$$

$$p^0 = \gamma mc$$

But since we define the relativistic total energy as

$$E \equiv \gamma mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

then energy-momentum four vector has the form

$$p^\mu = \left(\frac{E}{c}, p_x, p_y, p_z\right)$$

where the relativistic momentum is given by

$$\mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1 - v^2/c^2}},$$

Particle collisions and decays :

For a process $A + B \rightarrow C + D$

Energy momentum conservation is simply given using the four vector notation as

$$p_A^\mu + p_B^\mu = p_C^\mu + p_D^\mu$$

Example 1 : A negatively charged pion at rest decays a muon and an anti-neutrino. Find the speed of the muon

Answer :

$$v_\mu = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2} c$$

When the masses form y,the tables are inserted one finds $v_\mu = 0.271\, c$

Example 2 : Production of the antiprotons at Bevatron at Berkeley USA.

For this purpose a high energy proton is fired onto a proton at rest and an antiproton is formed as one of the products. What is the minimum energy (threshold energy) of the incoming proton so that this reaction can occur ?

Answer : Using the energy-momentum conservation princple one can get

$$E = 7mc^2$$

In princple one can solve a problem either in the lab frame or in the center of mass frame. For calculational simplicity one should prefer CM frame if the calculations in the lab frame become combersome.

Example 3 : Find the threshold energy of the incoming proton for the following reaction if the target proton is initially at rest.

$$p + p \rightarrow p + p + \pi^0$$

Answer : E = 1218 MeV

Homework

Solve the following problems at the end of the Chapter III of the textbook by D.Griffiths “Introduction to Elementary Particles J.Wiley)

Solve Problem 3.7

Solve Problem 3.8

Solve Problem 3.9

Solve Problem 3.10

Solve Problem 3.11

Solve Problem 3.12