Lecture 7 : Symmetries – 1

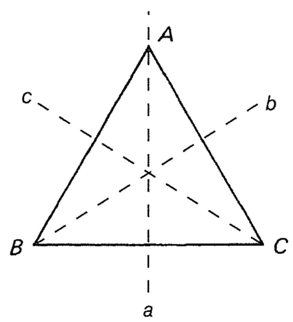
Symmetries, groups, and conservation laws

Symmetry idea in physics : Simple example even and odd functions

Continuous symmetries < = > Conservation laws (Noether's Theorem)

<u>Symmetries</u>		<u>Conservation laws</u>
Translation in time	= >	Energy conservation
Translation in space	= >	Linear monetum conservation
Rotation in space	= >	Angular momentum conservation
Gauge transformation	= >	Charge (in general)
$U_{em}(1)$ global gauge trans.	= >	Electric charge

Some geometric examples for symmetry :



Symmetries of an eqilateral triangle

Point and Space summetries play an important role in crystallography.

Groups

Axioms of groups

Examples : Rotation group SO(3)

Classifications og the groups :

Matrix representations of the groups

Role of the group theory and quantum mechanics

Angular momentum and Spin in Quantum Mechanics

Addition of two angular momentum operators

Clebsch-Gordon coefficients

$$|j_1 m_1\rangle |j_2 m_2\rangle = \sum_{j=|j_1-j_2|}^{(j_1+j_2)} C^{jj_1j_2}_{mm_1m_2} |jm\rangle, \quad \text{with } m = m_1 + m_2$$

or

$$|jm\rangle = \sum_{j_1 j_2} C^{jj_1j_2}_{mm_1m_2} |j_1 m_1\rangle |j_2 m_2\rangle$$

2 × 1/2		5/2									
		+5/2	5/2	3/2							
+2	1/2	1	3/2	+3/2							
	+2	−1/2	1/5	4/5	5/2	3/2					
	+1	+1/2	4/5	−1/5	+1/2	+1/2					
			1	−1/2	2/5	3/5	5/2	3/2			
			0	+1/2	3/5	−2/5	−1/2	−1/2			
					0	−1/2	3/5	2/5	5/2	3/2	
					−1	+1/2	2/5	−3/5	−3/2	−3/2	
							−1	−1/2	4/5	1/5	5/2
							−2	+1/2	1/5	−4/5	−5/2
									−2	−1/2	1

Clebsch-Gordon coefficients for the case of j_1=2 and j_2=1/2

Note : A square root sign is implied for every matrix element

Examples : Possible spin values of the mesons and baryons

(look at their quark structures)

Spin ½ systems :

Pauili spim matrices

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{S}_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Singlet and triplet states.

<u>A simple problem which teaches the very fundamentles of quantum mechanics :</u>

a) An electron is prepared in spin up direction along the z-exis. If one measures its spin along the x-axis what would be the possible results and their probabilities ?

b) Assume that the result of this measurement turns out to be $+\hbar/2$. Now if you measure the z component of the electron's spin what would be your expectations ?

Homework

Solve the following problems at the end of the Chapter IV of the textbook by D.Griffiths "Introduction to Elementary Particles J.Wiley)

Solve Problem 4.1

Solve Problem 4.2

Solve Problem 4.3

Solve Problem 4.4

Solve Problem 4.5

Solve Problem 4.6

Solve Problem 4.7

Solve Problem 4.8

Solve Problem 4.9

Solve Problem 4.10