Lecture 9 : Bound states

a) **Schrödinger equation and hydrogen type atoms :**

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\Psi = i\hbar\frac{\partial}{\partial t}\Psi$$

For the Hydrogen atom the Coulomb interaction

$$V(r) = -\frac{e^2}{r}$$

The solution leads to the well known Lyman, Balmer etc. series for the transitions. Energy values are given by

$$E_n = -\frac{me^4}{2\hbar^2 n^2} = -\alpha^2 mc^2\left(\frac{1}{2n^2}\right) = -13.6\ \mathrm{eV}/n^2$$

where n takes on integer values n= 1, 2, 3, 4 … and

$$\alpha \equiv \frac{e^2}{\hbar c} = \frac{1}{137.036}$$

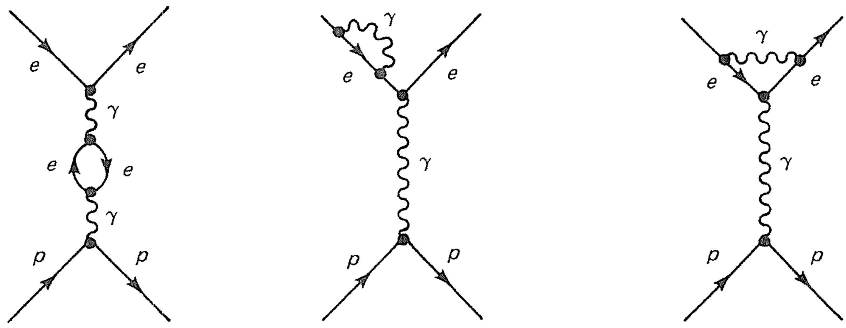
is the fine structure constant.

b) **Fine structure :** More precise spectral measurements show that the structure of the Hydrogen atom deviate slightly from the Rydberg formula. Relativistic kinetic energy correction for the electron, spin-orbit interaction and Darwin term constitute the physical origins for the fine structure of the Hydrogen atom. Energy shifts are given by :

$$\Delta E_{\mathrm{fs}} = -\alpha^4 mc^2 \frac{1}{4n^4}\left(\frac{2n}{(j+\frac{1}{2})} - \frac{3}{2}\right)$$

as can be seen from the above expression it is at the order of α^4 .

c) **Lamb shift :** According to the fine structure formula the states $2S_{1/2}$ and $2P_{1/2}$ must be degenerate because they have the same n=2 and j=1/2 quantum eigenvalues. However the famous experiment performed by Lamb and Retherford in 1947 showed that they do not have the same energies. The explanation based on the field quantization of the elctromagnetism was later given by Bethe, Feynman, Schwinger and Tomonoga. The main Feynman diagrams contributing the the Lamb shift are shown below :



Vacuum polarixation | Electron mass renormalization | Anomalous magnetic moment

Lamb shift for the state is given by

$$\Delta E_{\text{Lamb}} = \alpha^5 mc^2 \frac{1}{4n^3}\left\{k(n,l) \pm \frac{1}{\pi(j+\frac{1}{2})(l+\frac{1}{2})}\right\}, \quad \text{for } j = l \pm \tfrac{1}{2}$$

where k(n,l) is a small number less than 0.05.

d) Hyperfine structure : Lamb shift does not constitute the smallest correction to the Hydrogen atomic energy levels**.** The interaction between the electon's and proton's magnetic moments further gives rise to the hyperfine structure causing an energy shift as

$$\Delta E_{\text{hf}} = \left(\frac{m}{m_p}\right)\alpha^4 mc^2 \frac{\gamma_p}{2n^3}\frac{\pm 1}{(f+\frac{1}{2})(l+\frac{1}{2})}, \quad \text{for} \quad f = j \pm \tfrac{1}{2}$$

Thus for any state with $l = 0$, the singlet (spins opposite) and triplet (spins parallel) states are shifted down and up so that the corrsponding transition wavelength is $\lambda = 21.1$ cm, well known in microwave astronomy.

e) Positronium, quarkonium : Bound state of the electron and the positron is another strange composite system which requires the consideration of the field quantization also. Its reduced mass is obviously $m_{red} = m/2$, and the positronium allows the precision tests of atomic theory and quantum electrodynamics.

Bound states of quarks (and antiquarks) constitute a very rich and active field of research. Is is fully a reserach field quantum chromodyanmics and requires non-petrurbative calculational techniques. The binding force is strong nuclear force, colour degree of freedom plays an essential role.

- Pseudoscalar and vector mesons
- Charmonium
- Bottomonium
- Baryons

Study Exercise 1 : Work out the Example 5.2 from the textbook "Introduction to Elementary Particles by D.Griffiths" in order to understand how to construct proton's total wavefunction

$$\psi_{proton} = \psi(\text{space}) \times \psi(\text{spin}) \times \psi(\text{flavor}) \times \psi(\text{color})$$

considering its space, spin, flavour and color degrees of freedom.

Study Exercise 2 : Examine the Figure 5.6 from the same textbook for the energy spectrum of positronium and charmonium.

Homework

Solve the following problems at the end of the Chapter V of the textbook by D.Griffiths "Introduction to Elementary Particles J.Wiley)

Solve Problem 5.1

Solve Problem 5.2

Solve Problem 5.10

Solve Problem 5.11

Solve Problem 5.12

Solve Problem 5.15

Solve Problem 5.16

Solve Problem 5.17

Solve Problem 5.18

Solve Problem 5.19