# Lecture 10 : The Feynman Calculus -1

In elementry particle physics the main experimental probes are the scatterings (collisions), decays and bound states. Astroparticle physics is another source of empirical information. In this chapter the basic notions of the quantum scattering and decay processes will be presented.

**Decay rate** : $\Gamma$ probability per unit time that a given particle (say any muon) will disintegrate. For N particles

$$dN = -\Gamma N\, dt$$

hence

$$N(t) = N(0)e^{-\Gamma t}$$

Particle might have different channels (routes) to decay, $\Gamma_i$ for each of them. Thus

$$\Gamma_{tot} = \sum_{i=1}^{n} \Gamma_i$$

Lifetime of the particle is obviously the inverse of the total decay rate :

$$\tau = \frac{1}{\Gamma_{tot}}$$

Branching ratio is defined to be :

$$\Gamma_i / \Gamma_{tot}$$

## Scattering cross section :

$$\sigma_{tot} = \sum_{i=1}^{n} \sigma_i$$

Example : Hard sphere scattering, the relation between scattering angle θ and impact parameter b is

$$\theta = 2\cos^{-1}(b/R)$$

Differential scattering cross section

$$d\sigma = D(\theta)\, d\Omega$$

$$d\sigma = |b\, db\, d\phi|, \quad d\Omega = |\sin\theta\, d\theta\, d\phi|$$

Therefore $D(\theta) = R^2/4$ and the total cross section for the hard sphere scattering is

$$\sigma = \int \frac{R^2}{4}\, d\Omega = \pi R^2$$

**Example :** Rutherford scattering. Differential cross section is

$$D(\theta) = \left(\frac{q_1 q_2}{4E\sin^2(\theta/2)}\right)^2$$

and the total cross section becomes infinity.

**Fermi's Golden Rule :** Transition rate is given by the product of the phase space and the absolute square of the scattering amplitude.

a) For the two particle decay :

$$\Gamma = \frac{S}{32\pi^2\hbar m_1}\int |\mathscr{M}|^2 \frac{\delta^4(p_1 - p_2 - p_3)}{\sqrt{\mathbf{p}_2^2 + m_2^2c^2}\sqrt{\mathbf{p}_3^2 + m_3^2c^2}}\, d^3\mathbf{p}_2 x\, d^3\mathbf{p}_3$$

S is a statistical factor and the Dirac delta function is given as

$$\delta^4(p_1 - p_2 - p_3) = \delta\left(p_1^0 - p_2^0 - p_3^0\right)\delta^3\left(\mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3\right)$$

After performing all the necessary calculations step sone can get :

$$\Gamma = \frac{S|\mathbf{p}|}{8\pi \hbar m_1^2 c} |\mathcal{M}|^2$$

b) For the particle scattering

$$\sigma = \frac{S\hbar^2}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2 c^2)^2}} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4 (p_1 + p_2 - p_3 \cdots - p_n)$$

$$\times \prod_{j=3}^{n} 2\pi \delta \left(p_j^2 - m_j^2 c^2\right) \theta \left(p_j^0\right) \frac{d^4 p_j}{(2\pi)^4}$$

In the case of two body scattering in CM frame

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1 + E_2)^2} \frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}$$

**Problem 1** : Problem 6.6 of the D.Griffiths's textbook.

**Problem 2** : Problem 6.8 of the D.Griffiths's textbook.

**Problem 3** : Problem 6.9 of the D.Griffiths's textbook.

**Problem 4** : Problem 6.10 of the D.Griffiths's textbook.