Lecture 13 : **Dirac Equation - 2**

Maxwell equations :

$$\nabla \cdot \mathbf{E} = 4\pi\rho \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0 \qquad \nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi}{c}\mathbf{J}$$

Electric and magnetic fields unify in the covariant formalism of special relativity

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

Also the charge and current densities can be written in a covariant form :

$$J^{\mu} = (c\rho, \mathbf{J})$$

The inhomegenous Maxwell equations take the following form in this notation :

$$\partial_{\mu} F^{\mu\nu} = \frac{4\pi}{c} J^{\nu}$$

The homogenous equations can be expressed with the use of the dual tensor $G^{\mu\nu}$

$$\partial_{\nu} G^{\mu\nu} = 0$$

Mawwell field tensor can be expressed in terms of the potentials in four vector formalism :

$$F^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}$$

In Lorentz gauge choice the four potential A^{μ} satisfies the following equation which is equvalent to the inhomegenous Maxwell equations.

$$\partial_\mu \partial^\mu A^\nu - \partial^\nu (\partial_\mu A^\mu) = \frac{4\pi}{c} J^\nu$$

becomes =>

$$\Box A^\mu = \frac{4\pi}{c} J^\mu$$

for the Lorentz choice $\partial_\mu A^\mu = 0$.

Feynman rules :

Electrons are represented by :

$$\psi(x) = a e^{-(i/\hbar) p\cdot x} u^{(s)}(p)$$

Positrons are represented by :

$$\psi(x) = a e^{(i/\hbar) p\cdot x} v^{(s)}(p)$$

$u^{(s)}$ and $v^{(s)}$ spinors satisfy the Dirac equations :

$$(\gamma^\mu p_\mu - mc)u = 0 \qquad (\gamma^\mu p_\mu + mc)v = 0$$

Their adjoints satisfy :

$$\bar{u}(\gamma^\mu p_\mu - mc) = 0 \qquad \bar{v}(\gamma^\mu p_\mu + mc) = 0$$

Also they have the orthogonality relations and normalization conditions :

$$\bar{u}^{(1)} u^{(2)} = 0 \qquad \bar{v}^{(1)} v^{(2)} = 0$$

and

$$\bar{u}u = 2mc \qquad \bar{v}v = -2mc$$

Photons are represented by

$$A_\mu(x) = ae^{-(i/\hbar)p\cdot x}\epsilon_\mu^{(s)}$$

Polarization vector satisfies

$$p^\mu \epsilon_\mu = 0$$

a) Identify the momenta of the incoming, outgoing and internal particles
b) Specify the external lines and their contributions
c) Specify the vertex factors
d) Specify the propagators for the electrons and photons

$$\frac{i(\gamma^\mu q_\mu + mc)}{q^2 - m^2c^2} \qquad \frac{-ig_{\mu\nu}}{q^2}$$

e) Confirm the energy-momentum conservation at each vertex
f) Integrate over the internal momenta
g) Obtain the scattering amplitude M
h) Antisymmetrization

For the details examine the textbook D.Griffiths "Int. to Elementary Particles" J.Wiley

Homework :

Solve the following problems from the textbook D.Griffiths' "Int. to Elementary Particles"

Solve Problem 7.7

Solve Problem 7.8

Solve Problem 7.9

Solve Problem 7.10

Solve Problem 7.11