Lecture 1 : Quantum Electrodynamics (QED) - 1

"PHYS 438 High Energy Physics II" course will mainly cover the following titles :

1. Quick overview of the materials studied in the previous course PHYS 437. D.Griffiths' well-known textbook "Introduction to Elementary Particles 2nd Ed." is adopted as our main textbook. Its Chapters 7-12 will be our focus of examination. The students are expected to learn all these chapters well enough, which constitute our sole basic aim.
2. Basics of quantum electrodynamics.
3. Physics of quarks and hadrons
4. Basics of quantum chromodynamics
5. Weak interactions
6. Electroweak unification.
7. Elementary introduction to gauge field theories
8. Neutrino physics

All the necessary derivations and the details of the mathematical calculations will be presented on the board in the class.

Nonrelativistic quantum mechanics >>>> Schrödinger's equation

In order to obtain a first order relativistic wave eqaution Dirac tried to factor the relativistic energy-momentum relation

$$p^{\mu}p_{\mu} - m^2c^2 = 0$$

$$(p^{\mu}p_{\mu} - m^2c^2) = (\beta^{\kappa}p_{\kappa} + mc)(\gamma^{\lambda}p_{\lambda} - mc)$$

One gets the Dirac equation

$$\gamma^{\mu}p_{\mu} - mc = 0$$

Dirac matrices γ^{μ} satisfy the Dirac algebra :

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu}$$

where $g^{\mu\nu}$ is the Minkowski metric and the gamma matrices have the standard representations :

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

where σ^i are the 2x2 Pauli matrices.

The wavefunction is the Dirac spinor with four complex elements in general.

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

For the zero momentum case, $\vec{p}=0$ one gets four independent solutions :

$$\psi^{(1)} = e^{-i(mc^2/\hbar)t} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad \psi^{(2)} = e^{-i(mc^2/\hbar)t} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\psi^{(3)} = e^{+i(mc^2/\hbar)t} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \qquad \psi^{(4)} = e^{+i(mc^2/\hbar)t} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

For the non-zero three momentum case $\vec{p} \neq 0$:

First express the Dirac spinor as composed of two-spinors :

$$\begin{pmatrix}\psi_A\\ \psi_B\end{pmatrix} \qquad \psi_A = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \qquad \psi_B = \begin{pmatrix}\psi_3\\ \psi_4\end{pmatrix}$$

The four independent solutions are :

$$u_A = \begin{pmatrix}1\\0\end{pmatrix} : u_B = \frac{\mathbf{p}\cdot\boldsymbol{\sigma}}{p^0+mc}\begin{pmatrix}1\\0\end{pmatrix} = \frac{c}{E+mc^2}\begin{pmatrix}p_z\\ p_x+ip_y\end{pmatrix}$$

$$u_A = \begin{pmatrix}0\\1\end{pmatrix} : u_B = \frac{\mathbf{p}\cdot\boldsymbol{\sigma}}{p^0+mc}\begin{pmatrix}0\\1\end{pmatrix} = \frac{c}{E+mc^2}\begin{pmatrix}p_x-ip_y\\ -p_z\end{pmatrix}$$

$$u_B = \begin{pmatrix}1\\0\end{pmatrix} : u_A = \frac{\mathbf{p}\cdot\boldsymbol{\sigma}}{p^0+mc}\begin{pmatrix}1\\0\end{pmatrix} = \frac{c}{E+mc^2}\begin{pmatrix}p_z\\ p_x+ip_y\end{pmatrix}$$

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The first two of them are the particle solutions with spin up and down respectively. The other two represent the corresponding anti-particle solutions.

The bilinear covariants are :

$$\overline{\psi}\psi \quad \overline{\psi}\gamma^5\psi \quad \overline{\psi}\gamma^\mu\psi \quad \overline{\psi}\gamma^\mu\gamma^5\psi \quad \overline{\psi}\sigma^{\mu\nu}\psi$$

which have scalar, pseudoscalar, vector, pseudovector and tensor characters under transformations. Here $\sigma^{\mu\nu}$ matrices are :

$$\sigma^{\mu\nu} \equiv \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$$

Sixteen different products od gamma matrices and their transformation behaviours :

Scalar	(one component)	$\bar{\psi}\,\psi$
Pseudoscalar	(one component)	$\bar{\psi}\,\gamma^5\psi$
Vector	(four components)	$\bar{\psi}\,\gamma^\mu\psi$

Pseudovector (four components) $\bar{\psi}\,\gamma^{\mu}\gamma^{5}\psi$

Antisymmetric tensor (six components) $\bar{\psi}\,\sigma^{\mu\nu}\psi$

The photon : Electromagnetic field tensor $F^{\mu\nu}$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

Current four vector and the inhomogenous Maxwell equations are :

$$J^{\mu} = (c\rho, \mathbf{J}) \qquad \partial_{\mu}F^{\mu\nu} = \frac{4\pi}{c}J^{\nu}$$

Photon wavefunction is written in terms of the polarization four vector as :

$$A^{\mu}(x) = ae^{-(i/\hbar)p\cdot x}\epsilon^{\mu}(p)$$

with the Lorentz condition

$$p^{\mu}\epsilon_{\mu} = 0$$

The polarization four vectors satisfy the completeness relation :

$$\sum_{s=1,2} (\epsilon_{(s)})_i(\epsilon^{*}_{(s)})_j = \delta_{ij} - \hat{p}_i\hat{p}_j$$