Lecture 2 : Quantum Electrodynamics (QED) -2

- Dirac, Weyl and Majorana spinors :

a) Dirac fermions :
Lorentz transformations

$$x'^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu}.$$

Poincaré algebra

$$[M^{\mu\nu}, M^{\rho\sigma}] = -i\left(\eta^{\mu\rho} M^{\nu\sigma} - \eta^{\mu\sigma} M^{\nu\rho} - \eta^{\nu\rho} M^{\mu\sigma} + \eta^{\nu\sigma} M^{\mu\rho}\right).$$

$$[M^{\mu\nu}, P^{\rho}] = -i\left(\eta^{\mu\rho} P^{\nu} - \eta^{\nu\rho} P^{\mu}\right)$$

$$[P_{\mu}, P_{\nu}] = 0$$

There are two Casimir operators

1) $P_{\mu} P^{\mu} = P^2$ and
2) Pauli - Lubanski vector

$$W_{\mu} := \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} P^{\nu} M^{\rho\sigma}$$

One additional property is

$$W_{\mu} P^{\mu} = 0$$

Unitary irreducible representations of the Poincaré group can be split into **two** :

$$P^2 = P_{\mu} P^{\mu} = m^2 > 0; \quad W^2 = -m^2 s(s+1)$$

where s is the spin and it can take the values $s = 0,\ 1/2, 1,\ 3/2 \quad ....$

and

$$P^2 = 0; \quad W^2 = 0 \qquad W_\mu = \lambda P_\mu$$

where the parameter λ is the helicity and is equal to $\pm s$

and furthermore one has the relation

$$\lambda = \frac{\mathbf{P} \cdot \mathbf{J}}{P_0}$$

so that the helicity is the projection of the spin along the three momentum.

Chirality projection operators P_R and P_L

$$P_R \equiv \frac{1+\gamma^5}{2} \qquad P_L \equiv \frac{1-\gamma^5}{2}$$

So that one may write any Dirac spinor as a sum of left and right handed spinors.

$$\psi = \psi_R + \psi_L \,.$$

where

$$\psi_R = \frac{1+\gamma^5}{2}\psi \qquad \psi_L = \frac{1-\gamma^5}{2}\psi$$

A Dirac spinor consists od two separate Weyl spinor so it 2x4 = 8 degrees of freedom.

$$\psi = \begin{bmatrix} \xi_\alpha \\ \overline{\eta}^{\dot{\alpha}} \end{bmatrix}$$

It implies that one may also write

$$\psi = \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix}$$

On the other hand a Majorana spinor is formed only by a one Weyl spinor. The second spinor needed in the construction is simply the adjoint of the first one :

$$\psi_M = \begin{bmatrix} \xi_\alpha \\ \overline{\xi}^{\dot{\alpha}} \end{bmatrix}$$

Other references for this section :

1) Fundamentals of the Neutrino Physics and Astrophysics
   C.Giunti and C.W.Kim, Oxford Pub. 2007
2) Introduction to Supersymmetry by H.Müller-Kirsten
   World – Scientific 2005

Homework problems :

Solve the following problems from the textbook D.Griffiths' "Int. to Elementary Particles"

Solve Problem 7.8

Solve Problem 7.9

Solve Problem 7.11

Solve Problem 7.12

Solve Problem 7.13

Solve Problem 7.16