**Lecture 3 : Quantum Electrodynamics (QED) -3**

Feynman Rules for QED :

Electrons are represented by :

$$\psi(x) = ae^{-(i/\hbar)p\cdot x}u^{(s)}(p)$$

Positrons are represented by :

$$\psi(x) = ae^{(i/\hbar)p\cdot x}v^{(s)}(p)$$

$u^{(s)}$ and $v^{(s)}$ spinors satisfy the Dirac equations :

$$(\gamma^{\mu}p_{\mu} - mc)u = 0 \qquad (\gamma^{\mu}p_{\mu} + mc)v = 0$$

Their adjoints satisfy :

$$\bar{u}(\gamma^{\mu}p_{\mu} - mc) = 0 \qquad \bar{v}(\gamma^{\mu}p_{\mu} + mc) = 0$$

Also we have the orthogonality relations and normalization conditions :

$$\bar{u}^{(1)}u^{(2)} = 0 \qquad \bar{v}^{(1)}v^{(2)} = 0$$

and

$$\bar{u}u = 2mc \qquad \bar{v}v = -2mc$$

Photons are represented by

$$A_{\mu}(x) = ae^{-(i/\hbar)p\cdot x}\epsilon_{\mu}^{(s)}$$

Polarization vector satisfies

$$p^{\mu}\epsilon_{\mu} = 0$$

**Rules :**

**a)** Identify the momenta of the incoming, outgoing and internal particles
**b)** Specify the external lines and their contributions
**c)** Specify the vertex factors
**d)** Specify the propagators for the electrons and photons

$$\frac{i(\gamma^{\mu} q_{\mu} + mc)}{q^2 - m^2 c^2} \qquad \frac{-i g_{\mu\nu}}{q^2}$$

**e)** Confirm the energy-momentum conservation at each vertex
**f)** Integrate over the internal momenta

**g)** Obtain the scattering amplitude $\mathcal{M}$

**h)** Antisymmetrization

For the details examine the textbook D.Griffiths "Int. to Elementary Particles" J.Wiley

Homework :

Solve the following problems from the textbook D.Griffiths' "Int. to Elementary Particles"

Solve Problem 7.1

Solve Problem 7.2

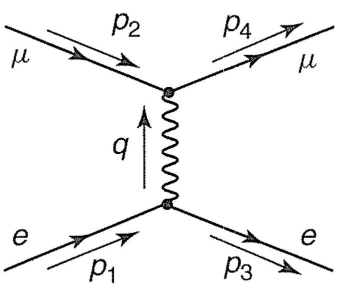
Solve Problem 7.3

Solve Problem 7.4

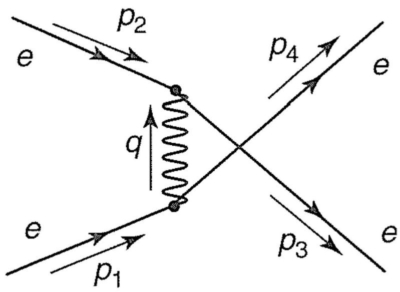
Solve Problem 7.5

**Examples for the scattering processes :** See the D.Griffiths textbook "Int.Elementary Particles J.Wiley) for all the calculational details.

**1. Electron-muon scattering** $e^- + \mu^- \to e^- + \mu^-$



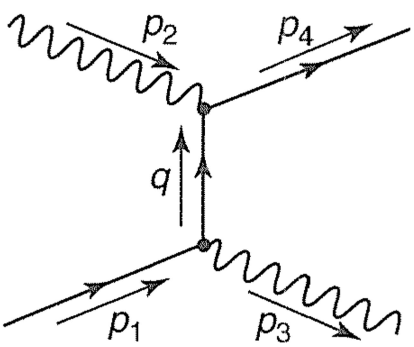

Feynman diagram for
the process $e^- + \mu^- \to e^- + \mu^-$

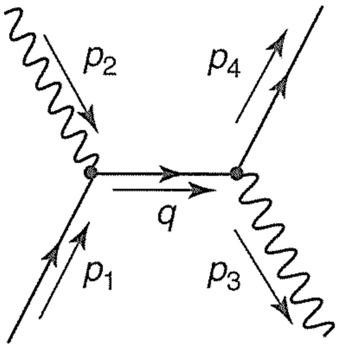



Twisted Feynman diagram

**2. Compton scattering** $e^- + \gamma \to e^- + \gamma$

Tree level diagrams




**3. Casimir trick :** Sum over the spins using completeness relations and trace techniques.

4. **Trace techniques :**

$$\mathrm{Tr}(A+B) = \mathrm{Tr}(A) + \mathrm{Tr}(B)$$

$$\mathrm{Tr}(\alpha A) = \alpha\,\mathrm{Tr}(A)$$

$$\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$$

$$g_{\mu\nu} g^{\mu\nu} = 4$$

$$\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2g^{\mu\nu}$$

$$\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2g^{\mu\nu}$$

$$\gamma_{\mu}\gamma^{\nu}\gamma^{\lambda}\gamma^{\mu} \quad = \quad 4g^{\nu\lambda}$$

$$\gamma_{\mu}\gamma^{\nu}\gamma^{\lambda}\gamma^{\sigma}\gamma^{\mu} \quad = \quad -2\gamma^{\sigma}\gamma^{\lambda}\gamma^{\nu}$$

$$\mathrm{Tr}(\gamma^{\mu}\gamma^{\nu}\gamma^{\lambda}\gamma^{\sigma}) = 4(g^{\mu\nu}g^{\lambda\sigma} - g^{\mu\lambda}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\lambda})$$

$$\mathrm{Tr}(\gamma^{5}\gamma^{\mu}\gamma^{\nu}\gamma^{\lambda}\gamma^{\sigma}) \quad = \quad 4i\epsilon^{\mu\nu\lambda\sigma}$$

Trace of a product of odd number of the gamma matrices is zero.

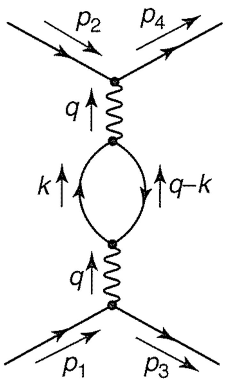
5. **Cross sections and lifetimes**

6. **Mott and Rutherford scattering**

7. **Pair annihilation**

8. **Renormalization concept**

Example : Vacuum polarization diagram ( a fourth order correction)




For all the details of the calculations and interpretaion of the results see the textbook by D.Griffiths' "Intr. to Elementary Particles, J.Wiley"

For the details examine the textbook D.Griffiths "Int. to Elementary Particles" J.Wiley

Homework Problems :

Solve the following problems from the textbook D.Griffiths' "Int. to Elementary Particles"

Solve Problem 7.29 Solve Problem 7.30

Solve Problem 7.31 Solve Problem 7.32

Solve Problem 7.33 Solve Problem 7.34

Solve Problem 7.35 Solve Problem 7.36

Solve Problem 7.38 Solve Problem 7.41

Solve Problem 7.43 Solve Problem 7.44