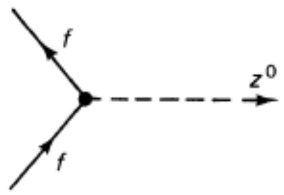
Lecture 10 : Weak interactions - 3

Neutral weak interactions of the leptons and quarks via Z^0 boson exchange can be summarized as follows :



$$\frac{-ig_z}{2}\gamma^{\mu}(c_V^f - c_A^f\gamma^5)$$

Here f is any quark or lepton and the corresponding vector and axial vector weak coupling coefficients are tabulated as :

f	c_V	c_A
$\nu_e \quad \nu_\mu \quad \nu_\tau$	$\frac{1}{2}$	$\frac{1}{2}$
$e^- \quad \mu^- \quad \tau^-$	$-\frac{1}{2}+2\sin^2\theta_W$	$-\frac{1}{2}$
$u \quad c \quad t$	$\frac{1}{2}-\frac{4}{3}\sin^2\theta_W$	$\frac{1}{2}$
$d \quad s \quad b$	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_W$	$-\frac{1}{2}$

The weak coupling constant g_W and electric charge are related to each other by Weinberg angle θ_W . The weak mixing angle θ_W has an experimental value

$$\sin^2\theta_W(M_Z) = 0.231$$

Study carefully the examples from the D.Griffiths' textbook Int.to Elementary Particles Wiley Pub. Chapter 10. p.325-330.

Example 1 : Elastic neutrino-electron sctattering $\nu_\mu + e \to \nu_e + \mu$

Example 2 : Electron-positron scattering near Z^0 pole $e^- + e^+ \to f + \bar{f}$

Electroweak Unification :

Chiral fermion states (chiral spinors) : u_L and u_R

Chiral projection operators : $P_R = \frac{1}{2}(1+\gamma_5)$ $P_L = \frac{1}{2}(1-\gamma_5)$

Chirality versus helicity

Weak isospin and hypercharge quantum numbers

Glashow-Weinberg-Salam theory for the electroweak mixing :

The full Lagrangian gor the first generation is :

$$
\begin{aligned}
\mathcal{L} \; = \; & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{8}tr(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}) - \frac{1}{2}tr(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}) \\
& +(\bar{\nu}_L, \bar{e}_L)\,\tilde{\sigma}^\mu iD_\mu \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} + \bar{e}_R\sigma^\mu iD_\mu e_R + \bar{\nu}_R\sigma^\mu iD_\mu \nu_R + (\text{h.c.}) \\
& -\frac{\sqrt{2}}{v}\left[(\bar{\nu}_L, \bar{e}_L)\,\phi M^e e_R + \bar{e}_R \bar{M}^e \bar{\phi}\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}\right] \\
& -\frac{\sqrt{2}}{v}\left[(-\bar{e}_L, \bar{\nu}_L)\,\phi^* M^\nu \nu_R + \bar{\nu}_R \bar{M}^\nu \phi^T\begin{pmatrix} -e_L \\ \nu_L \end{pmatrix}\right] \\
& +(\bar{u}_L, \bar{d}_L)\,\tilde{\sigma}^\mu iD_\mu \begin{pmatrix} u_L \\ d_L \end{pmatrix} + \bar{u}_R\sigma^\mu iD_\mu u_R + \bar{d}_R\sigma^\mu iD_\mu d_R + (\text{h.c.}) \\
& -\frac{\sqrt{2}}{v}\left[(\bar{u}_L, \bar{d}_L)\,\phi M^d d_R + \bar{d}_R \bar{M}^d \bar{\phi}\begin{pmatrix} u_L \\ d_L \end{pmatrix}\right] \\
& -\frac{\sqrt{2}}{v}\left[(-\bar{d}_L, \bar{u}_L)\,\phi^* M^u u_R + \bar{u}_R \bar{M}^u \phi^T\begin{pmatrix} -d_L \\ u_L \end{pmatrix}\right] \\
& +\overline{(D_\mu\phi)}D^\mu\phi - m_h^2[\bar{\phi}\phi - v^2/2]^2/2v^2.
\end{aligned}
$$

The covariant derivative acting on left-handed lepton spinors is given by :

$$D_\mu \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \left[\partial_\mu - \frac{ig_1}{2}B_\mu + \frac{ig_2}{2}\mathbf{W}_\mu\right]\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

For the other derivatives on the right handed fields, on the spinors of the quark sector and on the scalar Higgs field please examine the textbooks by D.Griffiths’ “Introduction to Elementary Particles, JohnWiley and Sons Pub.(2008)” and also F.Halzen and Alan D.Martin’s “Quarks And Leptons: An Introductory Course In Modern Particle Physics John Wiley and Sons (1984)”

Homework and Study Problems :

Examine and try to solve the problems listed from the same textbook by D.Griffiths.

Solve Problem 10.16

Solve Problem 10.17

Solve Problem 10.18

Solve Problem 10.19

Solve Problem 10.20

Solve Problem 10.21

Solve Problem 10.22

Solve Problem 10.23

Solve Problem 10.24

Solve Problem 10.25