Lecture 14 : **Basics of Gauge Theories - 2**

Non-Abelian Groups : Also known as non-commutative groups, at least two group members of an nonabalian group do not commute. Examples ; SU(2), SO(3), Poncaré Group, …

Yang-Mills Theory : A local gauge theory of quantum fields based on the group SU(2).

Let the ψ_1 and ψ_2 be two spin ½ fields. Let us construct a two-component column vector ψ :

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad \text{and its adjoint} \quad \bar{\psi} = \begin{pmatrix} \bar{\psi}_1 & \bar{\psi}_2 \end{pmatrix}$$

If the spinor fields ψ_1 and ψ_2 have the same masses namely $m_1 = m_2 = m$ then the following Lagrangian describes a two-component field with a masss m

$$\mathcal{L} = i\hbar c\bar{\psi}\gamma^{\mu}\,\partial_{\mu}\psi - mc^2\bar{\psi}\psi$$

The field ψ admits a more general symmetry :

$$\psi \longrightarrow U\psi$$

where U is 2x2 unitary matrix with unity determinant. In trems of Pauli matrices one may write the transformation matrix U as

$$U = e^{i\theta}e^{i\tau\cdot\mathbf{a}}$$

The first factor is a phase transformation and has already been examined in abelian U(1) case so let us continue with the following form only :

$$\psi \longrightarrow e^{i\tau\cdot\mathbf{a}}\psi$$

Thus the Lagrangian is invariant under the above <u>global</u> SU(2) transformation.

If we demand the invariance to be local then one can immediately use the receipe of replacing the ordinary derivatives by the gauge covariant derivatives :

$$\mathcal{D}_\mu \equiv \partial_\mu + i\frac{q}{\hbar c}\boldsymbol{\tau}\cdot\mathbf{A}_\mu$$

where $A_\mu = (A^{(1)}{}_\mu \quad A^{(2)}{}_\mu \quad A^{(3)}{}_\mu)$ are three gauge fields needed in the Lagrangian in order to ensure the local invariance.

The total Lagrangian becomes :

$$\mathcal{L} = [i\hbar c\bar{\psi}\gamma^\mu \partial_\mu\psi - mc^2\bar{\psi}\psi] - \frac{1}{16\pi}\mathbf{F}^{\mu\nu}\cdot\mathbf{F}_{\mu\nu} - (q\bar{\psi}\gamma^\mu\boldsymbol{\tau}\psi)\cdot\mathbf{A}_\mu$$

Chromodynamics : If one starts with column vector having components

$$\psi \equiv \begin{pmatrix} \psi_r \\ \psi_b \\ \psi_g \end{pmatrix}$$

$$\bar{\psi} = (\bar{\psi}_r\,\bar{\psi}_b\,\bar{\psi}_g)$$

Here ψ_r , ψ_b , and ψ_g are three independent Dirac spinors.

Again considering a transformation on the column vector

$$U = e^{i\theta}e^{i\boldsymbol{\lambda}\cdot\mathbf{a}}$$

where λ matrices are SU(3) generators. Finally we get the following local gauge invariant total Lagrangian of QCD :

$$\mathcal{L} = [i\hbar c\bar{\psi}\gamma^\mu \partial_\mu\psi - mc^2\bar{\psi}\psi] - \frac{1}{16\pi}\mathbf{F}^{\mu\nu}\cdot\mathbf{F}_{\mu\nu} - (q\bar{\psi}\gamma^\mu\boldsymbol{\lambda}\psi)\cdot\mathbf{A}_\mu$$

Feynman Rules :

1) External Lines
2) Propagators
3) Vertex factors

Reference : See "D.Griffiths textbook Int.to Elementary Particles 2nd Ed. J.Wiley"

Higgs Mechanism :

a) Mass term
b) Spontaneous symmetry breaking
c) The Higgs mechanism

The total Lagrangian of the Standard Model of the elementary particles :

$$\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{8}tr(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}) - \frac{1}{2}tr(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}) \\
& +(\bar{\nu}_L, \bar{e}_L)\,\tilde{\sigma}^\mu iD_\mu \begin{pmatrix}\nu_L \\ e_L\end{pmatrix} + \bar{e}_R\sigma^\mu iD_\mu e_R + \bar{\nu}_R\sigma^\mu iD_\mu \nu_R + (\text{h.c.}) \\
& -\frac{\sqrt{2}}{v}\left[(\bar{\nu}_L, \bar{e}_L)\,\phi M^e e_R + \bar{e}_R\bar{M}^e\bar{\phi}\begin{pmatrix}\nu_L \\ e_L\end{pmatrix}\right] \\
& -\frac{\sqrt{2}}{v}\left[(-\bar{e}_L, \bar{\nu}_L)\,\phi^* M^\nu \nu_R + \bar{\nu}_R\bar{M}^\nu\phi^T\begin{pmatrix}-e_L \\ \nu_L\end{pmatrix}\right] \\
& +(\bar{u}_L, \bar{d}_L)\,\tilde{\sigma}^\mu iD_\mu \begin{pmatrix}u_L \\ d_L\end{pmatrix} + \bar{u}_R\sigma^\mu iD_\mu u_R + \bar{d}_R\sigma^\mu iD_\mu d_R + (\text{h.c.}) \\
& -\frac{\sqrt{2}}{v}\left[(\bar{u}_L, \bar{d}_L)\,\phi M^d d_R + \bar{d}_R\bar{M}^d\bar{\phi}\begin{pmatrix}u_L \\ d_L\end{pmatrix}\right] \\
& -\frac{\sqrt{2}}{v}\left[(-\bar{d}_L, \bar{u}_L)\,\phi^* M^u u_R + \bar{u}_R\bar{M}^u\phi^T\begin{pmatrix}-d_L \\ u_L\end{pmatrix}\right] \\
& +\overline{(D_\mu\phi)}D^\mu\phi - m_h^2[\bar{\phi}\phi - v^2/2]^2/2v^2.
\end{aligned}$$

For all the details of the calculations and interpretation of the results see the textbook by D.Griffiths' "Intr. to Elementary Particles, J.Wiley"

Homework problems :

Solve the following problems from the textbook D.Griffiths' "Int. to Elementary Particles"

Solve Problem 11.6

Solve Problem 11.7

Solve Problem 11.12

Solve Problem 11.14

Solve Problem 11.17

Solve Problem 11.21

Solve Problem 11.23