

# FLUID MECHANICS



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## 2. PROPERTIES OF FLUIDS

### 2.4. Viscosity

The viscosity of a fluid is a measure of its “*resistance to deformation.*”

Viscosity is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other.

Viscosity is caused by the cohesive forces between the molecules in liquids and by the molecular collisions in gases, and it varies greatly with temperature.

When two solid bodies in contact move relative to each other, a friction force develops at the contact surface in the direction opposite to motion.

To move a table on the floor, for example, we have to apply a force to the table in the horizontal direction large enough to overcome the friction force.

The magnitude of the force needed to move the table depends on the *friction coefficient* between the table and the floor.

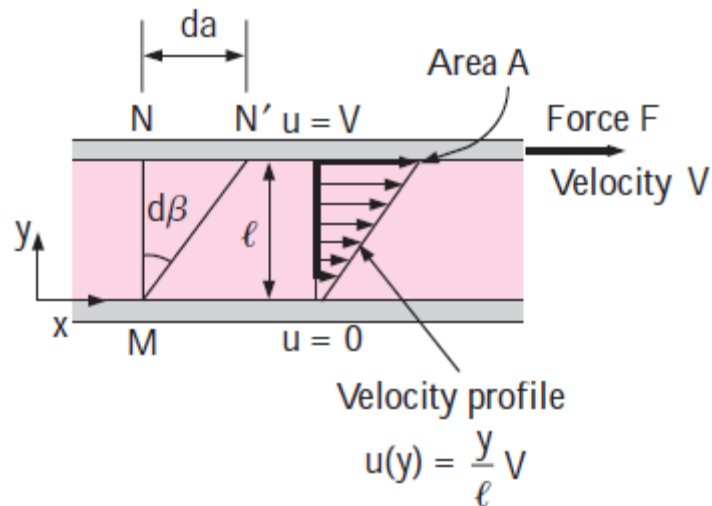
To obtain a relation for viscosity, consider a fluid layer between two very large parallel plates separated by a distance (Fig.).

Now a constant parallel force  $F$  is applied to the upper plate while the lower plate is held fixed. It is observed that the upper plate moves continuously under the influence of this force at a constant velocity  $V$ .

The fluid in contact with the upper plate sticks to the plate surface and moves with it at the same velocity, and the shear stress  $\tau$  acting on this fluid layer is

$$\tau = \frac{F}{A}$$

Where;  $A$  is the contact area between the plate and the fluid. Note that the fluid layer deforms continuously under the influence of shear stress.



**Figure 2.2. The behavior of a fluid in laminar flow between two parallel plates when the upper plate moves with a constant velocity.**

The fluid in contact with the lower plate assumes the velocity of that plate, which is zero. In steady laminar flow, the fluid velocity between the plates varies linearly between 0 and  $V$ , and thus the *velocity profile* and the *velocity gradient* are

Velocity profile:  $u(y) = \frac{y}{l} V$

Velocity gradient:  $\frac{du}{dy} = \frac{V}{l}$  (the rate of gradient, rate of deformation)

Where;  $y$  is the vertical distance from the lower plate.

In one-dimensional shear flow of Newtonian fluids, shear stress can be expressed by the linear relationship.

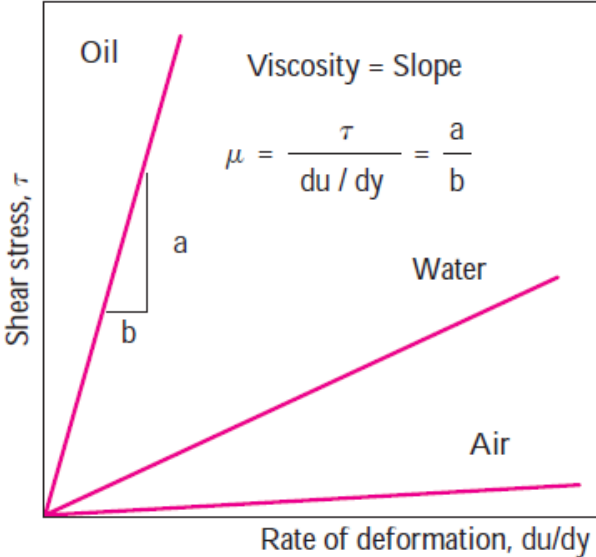
*Shear stress:*  $\tau = \mu \frac{du}{dy}$

Where; the constant of proportionality  $\mu$  (mü) is called the **coefficient of viscosity** or **the dynamic (or absolute) viscosity** of the fluid, whose unit is  $\text{kg/m} \cdot \text{s}$ , or equivalently,  $\text{N} \cdot \text{s/m}^2$  (Pa. s).

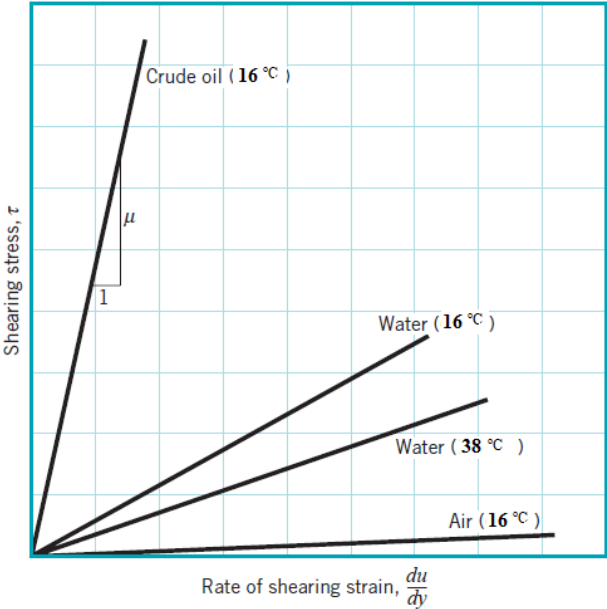
A common viscosity unit is **poise**, which is equivalent to 0.1 Pa. s (or *centipoise*, which is one-hundredth of a poise).

The viscosity of water at 20°C is 1 centipoise, and thus the unit centipoise serves as a useful reference.

A plot of shear stress versus the rate of deformation (velocity gradient) for a Newtonian fluid is a straight line whose slope is the viscosity of the fluid, as shown in Fig.2.3 and Fig.2.4. Note that viscosity is independent of the rate of deformation.



**Figure 2.3. The rate of deformation (velocity gradient) of a Newtonian fluid is proportional to shear stress, and the constant of proportionality is the viscosity.**



**Figure 2.4. Linear variation of shearing stress with rate of shearing strain for common fluids.**

The *shear force* acting on a Newtonian fluid layer (or, by Newton’s third law, the force acting on the plate) is

$$F = \tau A = \mu A \frac{du}{dy} \quad (N)$$

Where;  $A$  is the contact area between the plate and the fluid. Then the force  $F$  required to move the upper plate at a constant velocity of  $V$  while the lower plate remains stationary is

$$F = \mu A \frac{V}{l}$$

This relation can alternately be used to calculate  $\mu$  when the force  $F$  is measured. Therefore, the experimental setup just described can be used to measure the viscosity of fluids. Note that under identical conditions, the force  $F$  will be very different for different fluids.

Fluids for which the rate of deformation is proportional to the shear stress are called **Newtonian fluids** after Sir Isaac Newton, who expressed it first in 1687. Most common fluids such as water, air, gasoline, honey, and oils are Newtonian fluids. When a fluid's viscosity is constant it is referred to as a Newtonian fluid.

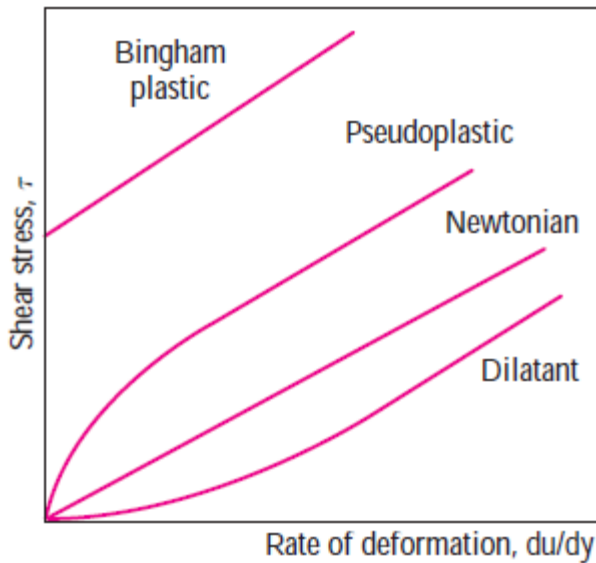
For non-Newtonian fluids, the relationship between shear stress and rate of deformation is not linear, as shown in Fig.2.5. Blood, corn starch and liquid plastics are examples of non-Newtonian fluids.

The slope of the curve on the  $\tau$  versus  $du/dy$  chart is referred to as the **apparent viscosity** of the fluid.

Fluids for which the apparent viscosity increases with the rate of deformation (such as solutions with suspended starch or sand) are referred to as **dilatant** or **shear thickening fluids**.

Fluids for which the apparent viscosity decreases with the rate of deformation (the fluid becoming less viscous as it is sheared harder, such as some paints, polymer solutions, and fluids with suspended particles) are referred to as **pseudoplastic** or **shear thinning fluids**.

Some materials such as toothpaste can resist a finite shear stress and thus behave as a solid, but deform continuously when the shear stress exceeds the yield stress and thus behave as a fluid. Such materials are referred to as Bingham plastics after **Eugene Cook Bingham**, who did pioneering work on fluid viscosity for the U.S. National Bureau of Standards in the early twentieth century.



**Figure 2.5. Variation of shear stress with the rate of deformation for Newtonian and non-Newtonian fluids (the slope of a curve at a point is the apparent viscosity of the fluid at that point).**

Honey is much more viscous than water. *When a fluid's viscosity is constant it is referred to as a Newtonian fluid.*

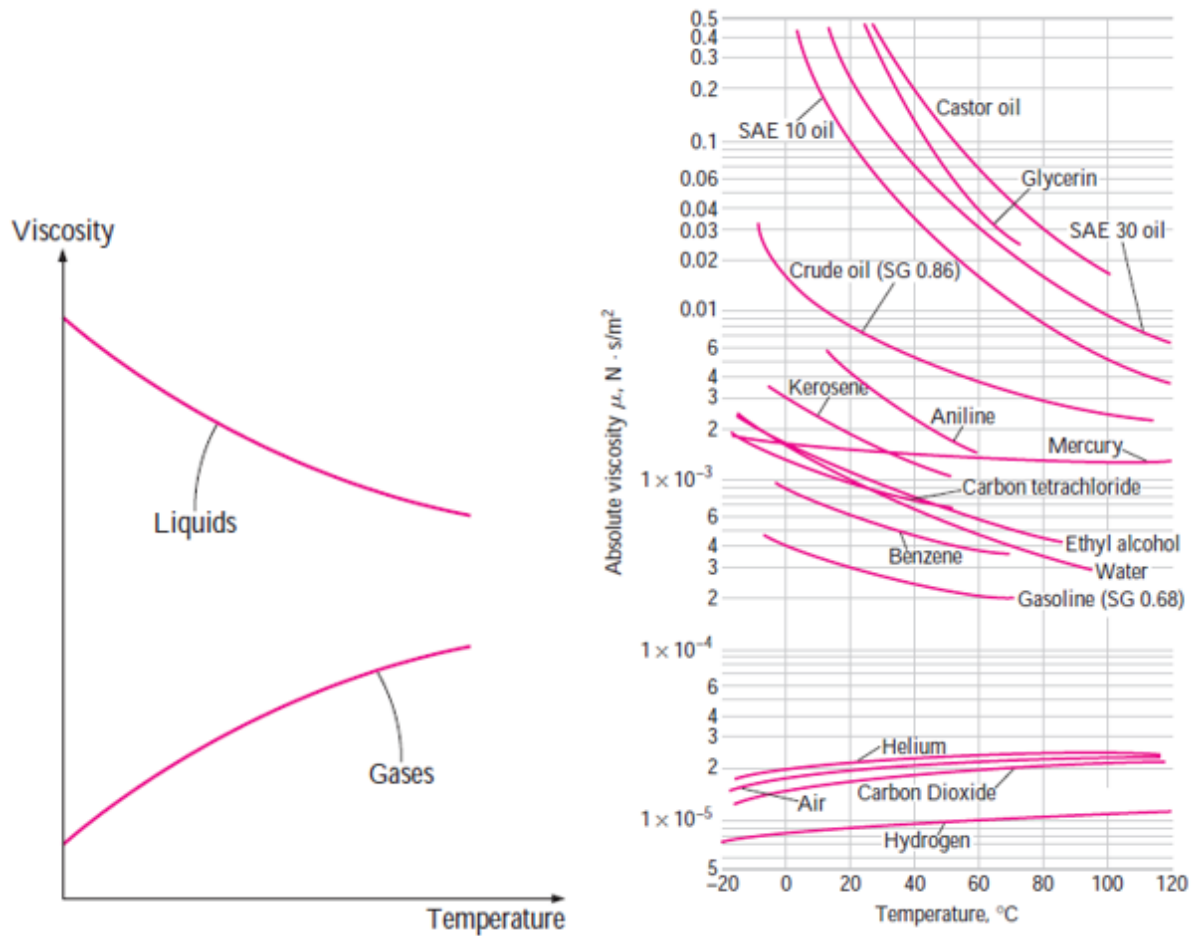
Oobleck, a suspension of cornstarch and water, is an example of a fluid whose viscosity is not constant, it changes depending on the stress or forces applied to it. If you poke it with your finger and apply a large force, it becomes very viscous and stays in place. If you gently pour it, applying little force, it will flow like water. This kind of fluid is called a dilatant material or a shear thickening fluid. It becomes more viscous when agitated or compressed.

Another non-Newtonian liquid is ketchup. Ketchup behaves in just the opposite way from oobleck. It becomes less viscous when agitated. Liquids like this are called thixotropic. If you leave a bottle of ketchup on a shelf, it becomes thicker or more viscous. Nearly everyone has experienced this while trying to pour the liquid from a new bottle – it refuses to move. If you shake the bottle or stir it up it becomes less viscous and pours easily.

The ratio of dynamic viscosity to density appears frequently. This ratio is given the name **kinematic viscosity,  $\nu$  (nü)** and is expressed as  $\nu = \mu/\rho$ . Two common units of kinematic viscosity are  $\text{m}^2/\text{s}$  and stoke (1 stoke =  $1 \text{ cm}^2/\text{s} = 0.0001 \text{ m}^2/\text{s}$ ). The dimensions of kinematic viscosity are  $\text{L}^2/\text{T}$ .

The viscosity of liquids decreases with temperature, whereas the viscosity of gases increases with temperature (Fig.2.6). This is because in a liquid the molecules possess more energy at higher temperatures, and they can oppose the large cohesive intermolecular forces more strongly. As a result, the energized

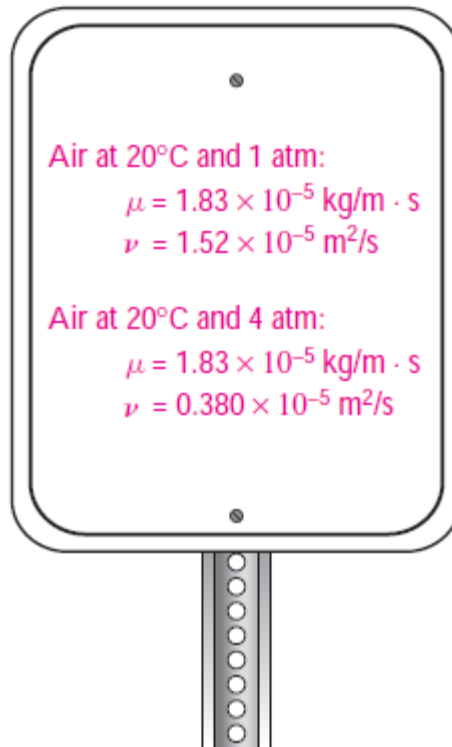
liquid molecules can move more freely. As the temperature of the gas increases, the random molecular activity increases with a corresponding increase in viscosity



**Figure 2. 6. The viscosity of liquids decreases and the viscosity of gases increases with temperature**

For *liquids*, both the dynamic and kinematic viscosities are practically independent of pressure, and any small variation with pressure is usually disregarded, except at extremely high pressures.

For *gases*, this is also the case for dynamic viscosity (at low to moderate pressures), but not for kinematic viscosity since the density of a gas is proportional to its pressure (Fig.2.7).



**Figure 2.7. Dynamic viscosity, in general, does not depend on pressure, but kinematic viscosity does.**

The kinetic theory of gases predicts the viscosity of gases to be proportional to the square root of temperature. This prediction is confirmed by practical observations, but deviations for different gases need to be accounted for by incorporating some correction factors. The viscosity of *gases* is expressed as a function of temperature by the Sutherland correlation (from The U.S. Standard Atmosphere) as

$$\mu = \frac{CT^{3/2}}{T + S}$$

Where;  $C$  and  $S$  are empirical constants, and  $T$  is absolute temperature. Thus, if the viscosity is known at two temperatures,  $C$  and  $S$  can be determined or, if more than two viscosities are known, the data can be correlated with the above equation by using some type of curve-fitting scheme.

For liquids an empirical equation that has been used is

$$\mu = De^{B/T}$$

Where;  $D$  and  $B$  are constants and  $T$  is absolute temperature. This equation is often referred to as *Andrade's equation*. As was the case for gases, the viscosity must be known at least for two temperatures so the two constants can be determined.



**Example:** A Newtonian fluid having a viscosity of 0.29 Pa.s, and a specific gravity of 0.80 flows through a 20-mm diameter pipe with a velocity of 2 m/s. Determine the value of the Reynolds number.

**Solution:** Reynolds number (Re) is a dimensionless combination of variables that is important in the study of viscous flow through pipes.

$$Re = \frac{\rho V D}{\mu} \quad Re = \frac{0.80 \times 1000 \times 2 \times 0.020}{0.29} = 110.34$$

## 2.5. Compressibility of Fluids

### 2.5.1. Bulk Modulus

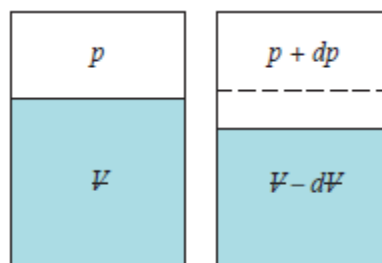
A property that is commonly used to characterize compressibility is the **bulk modulus**, defined as

$$E_v = \frac{\text{Stress}}{\text{Strain}} = -\frac{dP}{dV/V} = -V \frac{dP}{dV} = -V_0 \frac{(P - P_0)}{(V - V_0)}$$

Where;  $dP$  is the differential change in pressure needed to create a differential change in volume  $dV$ , of a volume  $V$ .  $P$  is the last pressure.  $P_0$  is the first pressure.  $V$  is the last volume, and  $V_0$  is the first volume. This is illustrated by the figure. The negative sign is included since an increase in pressure will cause a decrease in volume (Fig.2.8).

**Strain** describes relative deformation or change in shape and size of elastic, plastic, and fluid materials under applied forces.

**Stress** is defined as force per unit area and is determined by dividing the force by the area upon which it acts.



**Figure 2.8. The parameters of bulk modulus**

Since a decrease in volume of a given mass,  $m = \rho V$ , will result in an increase in density. The above equation can also be expressed as

$$E_v = \frac{dP}{d\rho/\rho} = \rho \frac{(P - P_0)}{(\rho - \rho_0)}$$

Where;  $\rho$  is the last density.  $\rho_0$  is the first density.

The bulk modulus (also referred to as the *bulk modulus of elasticity*) has dimensions of pressure  $F L^{-2}$ . In SI units The values of  $E_v$  are usually given as Pa ( $N/m^2$ ).

Large values for the bulk modulus indicate that the fluid is relatively incompressible—that is, it takes a large pressure change to create a small change in volume. As expected, values of for common liquids are large. Since such large pressures are required to effect a change in volume, we conclude that liquids can be considered as *incompressible* for most practical engineering applications. As liquids are compressed the bulk modulus increases, but the bulk modulus near atmospheric pressure is usually the one of interest. The use of bulk modulus as a property describing compressibility is most prevalent when dealing with liquids, although the bulk modulus can also be determined for gases.