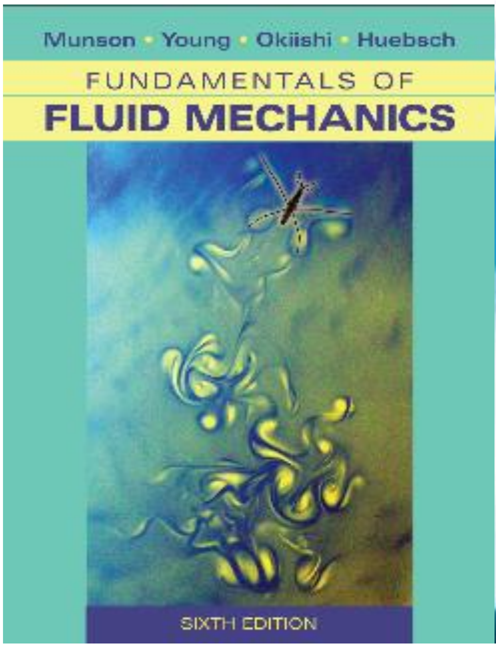
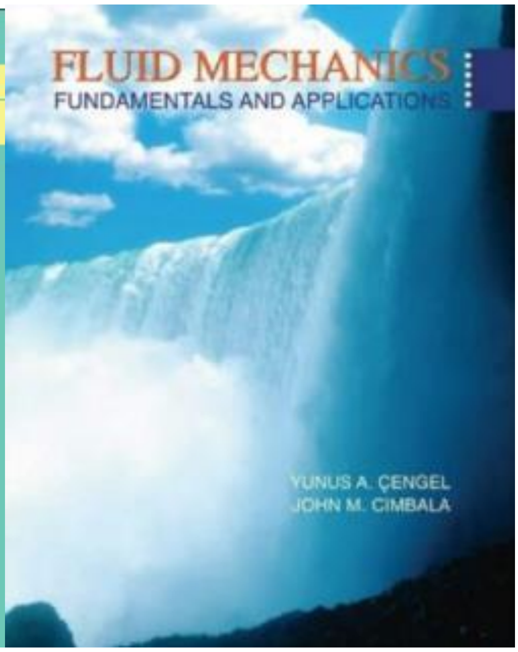
FLUID MECHANICS





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4. ELEMENTARY FLUID DYNAMICS -THE BERNOULLI EQUATION

4.1. Newton's Second Law

As a fluid particle moves from one location to another, it usually experiences an acceleration or deceleration. According to Newton's second law of motion, the net force acting on the fluid particle under consideration must equal its mass times its acceleration, ***F=ma***

In this chapter we consider the motion of inviscid fluids. That is, the fluid is assumed to have zero viscosity. *If the viscosity is zero, then the thermal conductivity of the fluid is also zero and there can be no heat transfer (except by radiation).*

In practice there are no inviscid fluids, since every fluid supports shear stresses when it is subjected to a rate of strain displacement. For many flow situations the viscous effects are relatively small compared with other effects. As a first approximation for such cases it is often possible to ignore viscous effects. For example, often the viscous forces developed in flowing water may be several orders of magnitude smaller than forces due to other influences, such as gravity or pressure differences. For other water flow situations, however, the viscous effects may be the dominant ones. Similarly, the viscous effects associated with the flow of a gas are often negligible, although in some circumstances they are very important.

We assume that the fluid motion is governed by pressure and gravity forces only and examine Newton's second law as it applies to a fluid particle in the form:

(Net pressure force on a particle) + *(net gravity force on particle)* = *(particle mass)* × *(particle acceleration)*

The results of the interaction between the pressure, gravity, and acceleration provide numerous useful applications in fluid mechanics.

In this chapter we will be concerned with two-dimensional motion like that confined to the x–z plane as is shown in Fig.4.1*a*. Clearly we could choose to describe the flow in terms of the components of acceleration and forces in the x and z coordinate directions. The resulting equations are frequently referred to as a two-dimensional form of the *Euler equations* of motion in rectangular Cartesian coordinates.

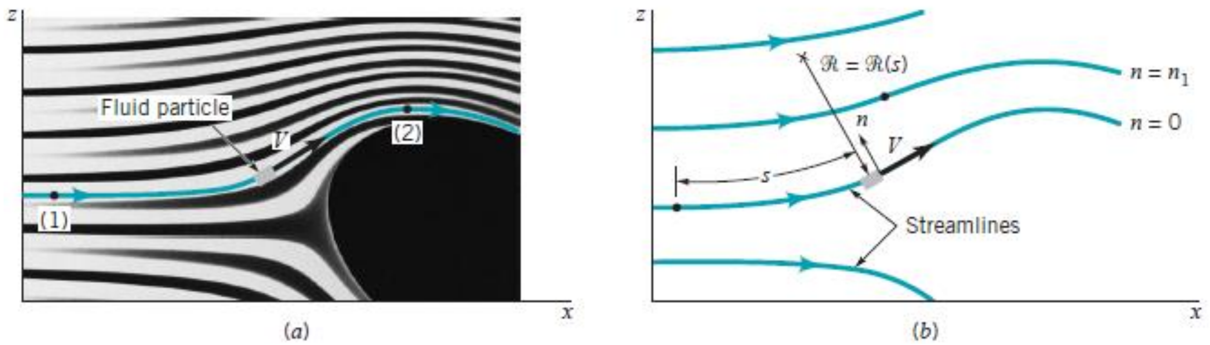


Figure 4.1. (*a*) Flow in the *x–z* plane. (*b*) Flow in terms of streamline and normal coordinates.

The motion of each fluid particle is described in terms of its velocity vector, **V**, which is defined as the time rate of change of the position of the particle. The particle's velocity is a vector quantity with a magnitude (the speed, $\boldsymbol{V = |V|}$) and direction. As the particle moves about, it follows a particular path, the shape of which is governed by the velocity of the particle. The location of the particle along the path is a function of where the particle started at the initial time and its velocity along the path. If it is ***steady flow (***i.e., nothing changes with time at a given location in the flow field), each successive particle that passes through a given point [such as point (1) in above Fig.4.1*a*] will follow the same path. For such cases the path is a fixed line in the *x–z* plane. Neighboring particles that pass on either side of point (1) follow their own paths, which may be of a different shape than the one passing through (1). The entire *x–z* plane is filled with such paths.

For steady flows each particle slides along its path, and its velocity vector is everywhere tangent to the path. The lines that are tangent to the velocity vectors throughout the flow field are called ***streamlines.*** For many situations it is easiest to describe the flow in terms of the "streamline" coordinates based on the streamlines as are illustrated in last Fig.4.1*b*. The particle motion is described in terms of its distance, $s=s(t)$, along the streamline from some convenient origin and the local radius of curvature of the streamline, $R=R(t)$ The distance along the streamline is related to the particle's speed by $V=ds/dt$ and the radius of curvature is related to the shape of the streamline. In addition to the coordinate along the streamline, *s*, the coordinate normal to the streamline, *n*, as is shown in the last Fig. 4.1*b*, will be of use.

To apply Newton's second law to a particle flowing along its streamline, we must write the particle acceleration in terms of the streamline coordinates. By definition, the acceleration is the time rate of change of the velocity of the particle, ***a=dV/dt***. For two-dimensional flow in the *x–z* plane, the acceleration has two components—one along the streamline, a_s, the streamwise acceleration, and one normal to the streamline, a_n, ***the*** normal acceleration.

The streamwise acceleration results from the fact that the speed of the particle generally varies along the streamline, *V=V(s)*. For example, in the last Fig.*a* the speed may be 30 m/s at point (1) and 15 m/s at point (2). Thus, by use of the chain rule of differentiation, the *s* component of the acceleration is given by $a_s = \frac{dV}{dt} = \left(\frac{\partial V}{\partial s}\right)\left(\frac{ds}{dt}\right) = \left(\frac{\partial V}{\partial s}\right)V$. We have used the fact that speed is the time rate of change of distance, *V=ds/dt*. Note that the streamwise acceleration is the product of the rate of change of speed with distance along the streamline, $\left(\frac{\partial V}{\partial s}\right)$ and the speed, *V*. Since $\left(\frac{\partial V}{\partial s}\right)$ can be positive, negative, or zero, the streamwise acceleration can, therefore, be positive (acceleration), negative (deceleration), or zero (constant speed).

The normal component of acceleration, the centrifugal acceleration, is given in terms of the particle speed and the radius of curvature of its path. Thus, $a_n=V^2/R$, where both *V* and R may vary along the streamline. These equations for the acceleration should be familiar from the study of particle motion in physics or dynamics. The components of acceleration in the *s* and *n* directions, and are given by

$$a_s = \boldsymbol{V}\frac{\partial \boldsymbol{V}}{\partial \boldsymbol{s}} \quad , \quad a_n = \frac{V^2}{R}$$

Where; R is the local radius of curvature of the streamline, and *s* is the distance measured along the streamline from some arbitrary initial point. In general there is acceleration along the streamline (because the particle speed changes along its path, $\frac{\partial V}{\partial s} \neq 0$) and acceleration normal to the streamline (because the particle does not flow in a straight line, $R \neq \infty$). Various flows and the accelerations associated with them are shown in the figure. For incompressible flow the velocity is inversely proportional to the streamline spacing. Hence, converging streamlines produce positive streamwise acceleration. To produce this acceleration there must be a net, nonzero force on the fluid particle.

To determine the forces necessary to produce a given flow (or conversely, what flow results from a given set of forces), we consider the free-body diagram of a small fluid particle as is shown in the below Fig.4.2. The particle of interest is removed from its surroundings, and the reactions of the surroundings on the particle are indicated by the appropriate forces present, $\boldsymbol{F_1}$, $\boldsymbol{F_2}$ and so forth. For the present case, the important forces are assumed to be gravity and pressure. Other forces, such as viscous forces and surface tension effects, are assumed negligible. The acceleration of gravity, *g*, is assumed to be constant and acts vertically, in the negative *z* direction, at an angle relative to the normal to the streamline.

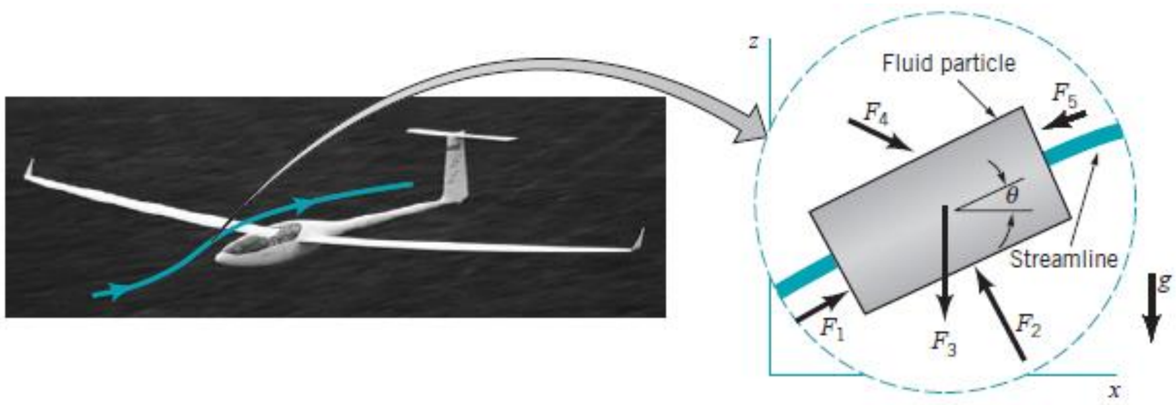


Figure 4.2. Isolation of a small fluid particle in a flow field. (Photo courtesy of Diana Sailplanes.)

4.2. F=ma Along the Streamline

Consider the small fluid particle of size $\delta s\ by\ \delta n$ in the plane of the figure and δy normal to the figure as shown in the free-body diagram of Fig.4.3. Unit vectors along and normal to the streamline are denoted by $\hat{s}$ and $\hat{n}$ respectively. For steady flow, the component of Newton's second law along the streamline direction, *s*, can be written as

$$\sum \delta F_s = \delta m\, a_s = \delta m\, V\frac{\partial V}{\partial s} = \rho \delta \forall V \frac{\partial V}{\partial s}$$

Where $\sum \delta F_s$ represents the sum of the *s* components of all the forces acting on the particle, which has mass $\delta m = \rho \delta \forall V$, and $V\ \frac{\partial V}{\partial s}$ is the acceleration in the *s* direction. Here, $\delta\forall = \delta s\ \delta n\ \delta y$ is the particle volume. The above equation is valid for both compressible and incompressible fluids. That is, the density need not be constant throughout the flow field.

After a lot of assumptios we can obtain the following equation of motion along the streamline direction:

$$-\gamma \sin\theta - \frac{\partial P}{\partial s} = \rho V \frac{\partial V}{\partial s} = \rho a_s$$

This equation can be rearranged and integrated as follows.

$dP + \frac{1}{2}\rho d(V^2) + \gamma dz = 0$ (along a streamline) which, for constant acceleration of gravity, can be integrated to give

$\int \frac{dP}{\rho} + \frac{1}{2}V^2 + gz = C$ (along a streamline)

Where C is a constant of integration to be determined by the conditions at some point on the streamline.

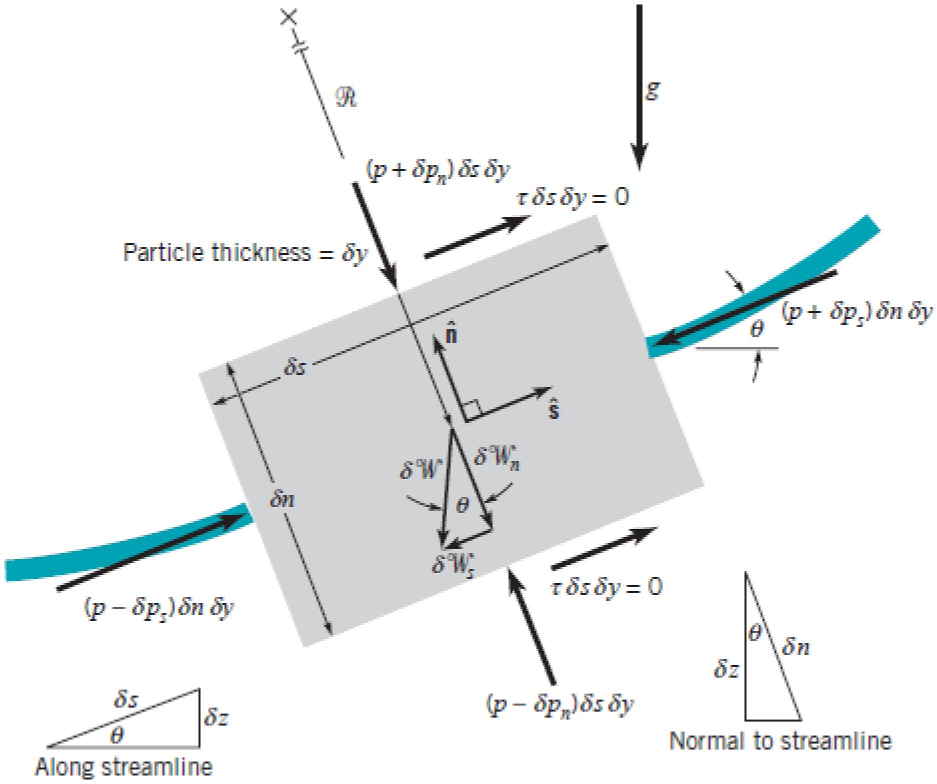


Figure 4.3. Freebody diagram of a fluid particle for which the important forces are those due to pressure and gravity.

Since the last two terms are exact differentials. In the case of incompressible flow, the first term also becomes an exact differential, and its integration gives

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \boldsymbol{constant\ (along\ a\ streamline)}$$

This is the celebrated ***Bernoulli equation***—a very powerful tool in fluid mechanics. In 1738 Daniel Bernoulli 11700–17822 published his *Hydrodynamics* in which an equivalent of this famous equation first appeared.

To use ***Bernoulli equation*** correctly we must constantly remember the basic assumptions used in its derivation:

1) Viscous effects are assumed negligible,
2) The flow is assumed to be steady,
3) The flow is assumed to be incompressible,
4) The equation is applicable along a streamline.

In the derivation of the last equation, we assume that the flow takes place in a plane (the *x–z* plane). In general, this equation is valid for both planar and nonplanar (three-dimensional) flows, provided it is applied along the streamline.

An alternate but equivalent form of the Bernoulli equation is obtained by dividing each term of the last equation by the specific weight, to obtain

$$\frac{P}{\gamma}+\frac{V^2}{2g}+z=constant$$

Each of the terms in this equation has the units of energy per weight or length (feet, meters) and represents a certain type of head.

$\frac{P}{\gamma}$ →The pressure term is called the ***pressure head*** and represents the height of a column of the fluid that is needed to produce the pressure *p*.

$\frac{V^2}{2g}$ →The velocity term, is the ***velocity head*** and represents the vertical distance needed for the fluid to fall freely (neglecting friction) if it is to reach velocity *V* from rest.

z →The elevation term is related to the potential energy of the particle and is called the ***elevation head*.**

The Bernoulli equation states that the sum of the pressure head, the velocity head, and the elevation head is constant along a streamline

The Bernoulli equation can also be written between any two points on the same streamline as

$$\frac{P_1}{\rho}+\frac{V_1^2}{2}+gz_1=\frac{P_2}{\rho}+\frac{V_2^2}{2}+gz_2$$

The Bernoulli equation is obtained from the conservation of momentum for a fluid particle moving along a streamline. It can also be obtained from the *first law of thermodynamics* applied to a steady-flow system.

4.3. F=ma Normal to a Streamline

In this section we will consider application of Newton's second law in a direction normal to the streamline. In many flows the streamlines are relatively straight, the flow is essentially one-dimensional, and variations in parameters across streamlines (in the normal direction) can often be neglected when compared to the variations along the streamline. However, in numerous other situations valuable information can be obtained from considering normal to the streamlines. For example, the devastating low-pressure region at the center of a tornado can be explained by applying Newton's second law across the nearly circular streamlines of the tornado.

A force balance in the direction *n* normal to the streamline yields the following relation applicable *across* the streamlines for steady, inviscid, incompressible flow is

$$P + \int \frac{V^2}{R} dn + \gamma z = constant\ across\ the\ streamline$$

As with the Bernoulli equation, we must be careful that the assumptions involved in the derivation of this equation are not violated when it is used.

For flow along a straight line, $R \rightarrow \infty$ and thus the last equatin reduces to P/ρ+gz= constant or P=-ρgz + constant, which is an expression for the variation of hydrostatic pressure with vertical distance for a stationary fluid body. Therefore, the variation of pressure with elevation in steady, incompressible flow along a straight line is the same as that in the stationary fluid (Fig.4.4).

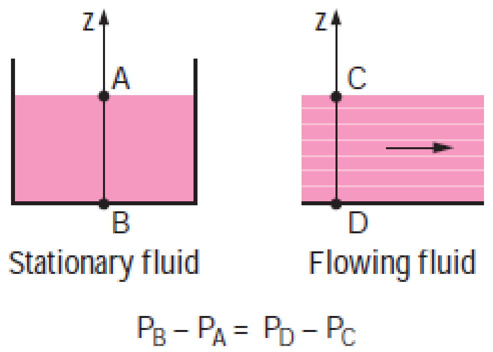


Figure 4.4. The variation of pressure with elevation in steady, incompressible flow along a straight line is the same as that in the stationary fluid (but this is not the case for a curved flow section).

Similarly, using both terms in the acceleration expression, it can be shown that the Bernoulli equation for *unsteady, compressible flow* is

$$\int \frac{dP}{\rho} + \int \frac{\partial V}{\partial t} ds + \frac{V^2}{2} + gz = constant$$

4.4. Static, Stagnation, Dynamic, and Total Pressure

The Bernoulli equation states that the sum of the flow, kinetic, and potential energies of a fluid particle along a streamline is constant. Therefore, the kinetic and potential energies of the fluid can be converted to flow energy (and vice versa) during flow, causing the pressure to change. This phenomenon can be made more visible by multiplying the Bernoulli equation by the density ρ,

$$P + \frac{1}{2}\rho V^2 + \rho g z = constant\ along\ streamline$$

Each term in this equation has pressure units, and thus each term represents some kind of pressure:

• P is the **static pressure** (it does not incorporate any dynamic effects); it represents the actual thermodynamic pressure of the fluid. This is the same as the pressure used in thermodynamics and property tables.

• $\frac{1}{2}\rho V^2$ is the **dynamic pressure;** it represents the pressure rise when the fluid in motion is brought to a stop isentropically.

• $\rho g z$ is the **hydrostatic pressure,** which is not pressure in a real sense since its value depends on the reference level selected; it accounts for the elevation effects, i.e., of fluid weight on pressure.

The sum of the static, dynamic, and hydrostatic pressures is called the ***total pressure.*** *Therefore, the Bernoulli equation states that the total pressure along a streamline is constant*

The sum of the static and dynamic pressures is called the ***stagnation pressure,*** and it is expressed as

$$P_2 = P_{stag} = P_1 + \frac{\rho V_1^2}{2}$$

The pressure at the stagnation point is greater than the static pressure, P_1, by an amount $\frac{\rho V_1^2}{2}$, the dynamic pressure. It can be shown that there is a stagnation point on any stationary body that is placed into a flowing fluid (Fig.4.5). Some of the fluid flows "over" and some "under" the object. The dividing line (or surface for two-dimensional flows) is termed the *stagnation streamline* and terminates at the stagnation point on the body. For symmetrical objects (such as a baseball) the stagnation point is clearly at the tip or front of the object.

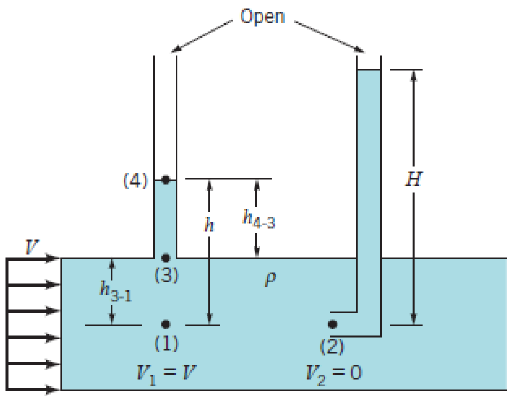


Figure 4.5. Measurement of static and stagnation pressures.

If elevation effects are neglected, the ***stagnation pressure,*** is the largest pressure obtainable along a given streamline. It represents the conversion of all of the kinetic energy into a pressure rise. The stagnation pressure represents the pressure at a point where the fluid is brought to a complete stop isentropically. The static, dynamic, and stagnation pressures are shown in the below Fig.4.6. When static and stagnation pressures are measured at a specified location, the fluid velocity at that location can be calculated from

$$V_1 = V = \sqrt{\frac{2(P_2 - P_1)}{\rho}}$$

This equation is useful in the measurement of flow velocity when a combination of a static pressure tap and a Pitot tube is used, as illustrated in Fig.4.6.

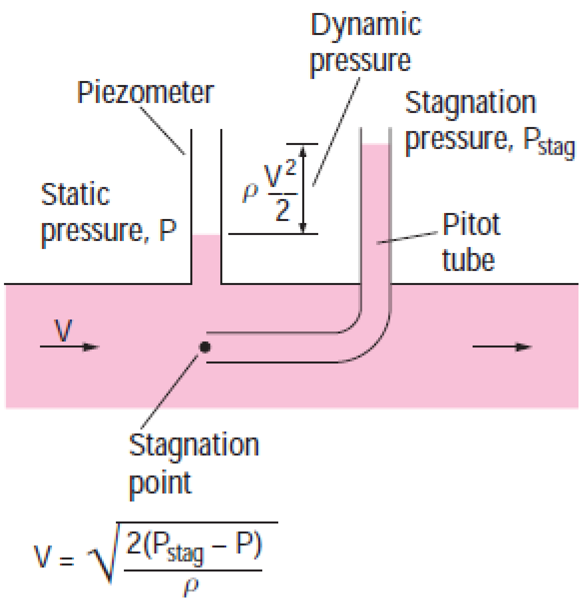


Figure 4.6. The static, dynamic, and stagnation pressures

A **static pressure tap** is simply a small hole drilled into a wall such that the plane of the hole is parallel to the flow direction. It measures the static pressure. A **Pitot tube** is a small tube with its open end aligned *into* the flow so as to sense the full impact pressure of the flowing fluid. It measures the stagnation pressure. In situations in which the static and stagnation pressure of a flowing *liquid* are greater than atmospheric pressure, a vertical transparent tube called a **piezometer tube** (or simply a **piezometer**) can be attached to the pressure tap and to the Pitot tube.The liquid rises in the piezometer tube to a column height (*head*) that is proportional to the pressure being measured. If the pressures to be measured are below atmospheric, or if measuring pressures in *gases*, piezometer tubes do not work. However, the static pressure tap and Pitot tube can still be used, but they must be connected to some other kind of pressure measurement device such as a U-tube manometer or a pressure transducer. Sometimes it is convenient to integrate static pressure holes on a Pitot probe. The result is a **Pitot-static probe,** as shown in the below Fig.4.7. A Pitot-static probe connected to a pressure transducer or a manometer measures the dynamic pressure (and thus fluid velocity) directly.

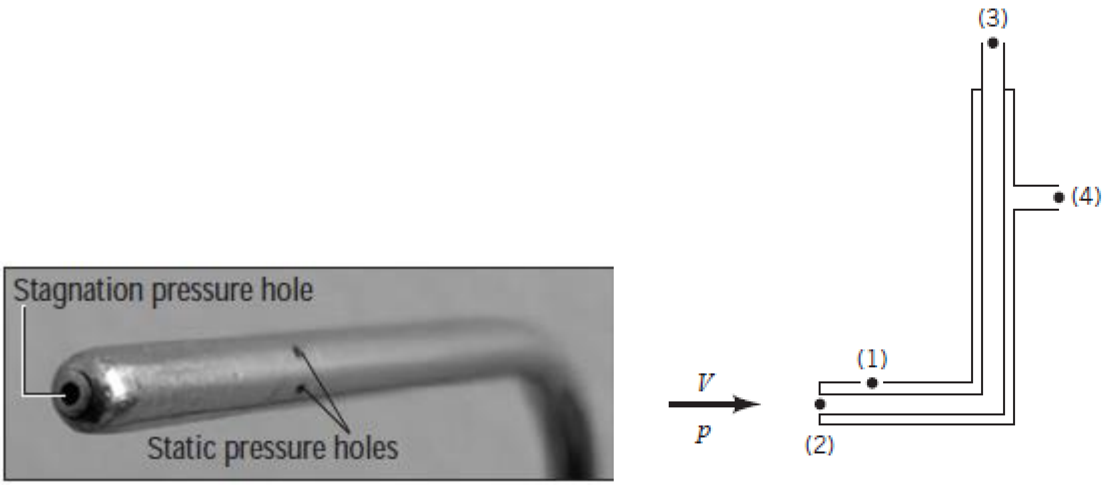


Figure 4.7. Close-up of a Pitot-static probe, showing the stagnation pressure hole and two of the five static circumferential pressure holes

As shown in Fig.4.7. two concentric tubes are attached to two pressure gages (or a differential gage) so that the values of P_3 and P_4 (or the difference P_3-P_4) can be determined. The center tube measures the stagnation pressure at its open tip. If elevation changes are negligible,

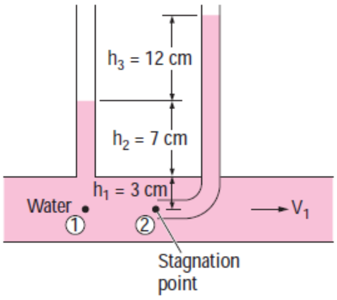
$$P_3 = P + \frac{1}{2}\rho V^2$$

Where *p* and *V* are the pressure and velocity of the fluid upstream of point (2). The outer tube is made with several small holes at an appropriate distance from the tip so that they measure the static pressure. If the effect of the elevation difference between (1) and (4) is negligible, then $P=P_4=P_1$

By combining these two equations we see that

$P_3 - P_4 = \frac{1}{2}\rho V^2$ which can be rearranged to give $V = \sqrt{\frac{2(P_3 - P_4)}{\rho}}$

Example: A piezometer and a Pitot tube are tapped into a horizontal water pipe, as shown in Fig. to measure static and stagnation (static + dynamic) pressures. For the indicated water column heights, determine the velocity at the center of the pipe. The flow is steady and incompressible.



Solution: We take points 1 and 2 along the centerline of the pipe, with point 1 directly under the piezometer and point 2 at the tip of the Pitot tube. This is a steady flow with straight and parallel streamlines, and the gage pressures at points 1 and 2 can be expressed as

$P_1 = \rho g(h_1 + h_2)$ $P_2 = \rho g(h_1 + h_2 + h_3)$

Noting that point 2 is a stagnation point and thus $V_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \rightarrow \frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g}$$

Substituting the P_1 and P_2 expressions gives

$$\frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g} = \frac{\rho g(h_1 + h_2 + h_3) - \rho g(h_1 + h_2)}{\rho g} = h_3$$

Solving for V_1 and substituting,

$$V_1 = \sqrt{2 \times 9.81 \times 0.12} = 1.53\ m/s$$

Note that to determine the flow velocity, all we need is to measure the height of the excess fluid column in the Pitot tube.