FLUID MECHANICS

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4. ELEMENTARY FLUID DYNAMICS -THE BERNOULLI EQUATION

4.5. Examples of Use of the Bernoulli Equation

In this section we illustrate various additional applications of the Bernoulli equation. Between any two points, (1) and (2), on a streamline in steady, inviscid, incompressible flow the Bernoulli equation can be applied in the form.

$$P_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

Obviously if five of the six variables are known, the remaining one can be determined. In many instances it is necessary to introduce other equations, such as the conservation of mass.

4.5.1.Free Jet

One of the oldest equations in fluid mechanics deals with the flow of a liquid from a large reservoir. A modern version of this type of flow involves the flow of coffee from a coffee urn as indicated by the below Fig.4.8. The exit pressure for an incompressible fluid jet is equal to the surrounding pressure.

Figure 4.8. The flow of coffee from a coffee urn

The basic principles of this type of flow are shown in the below Fig.4.9 where a jet of liquid of diameter *d* flows from the nozzle with velocity *V*. (A nozzle is a device shaped to accelerate a fluid.). Application of the above Equation between points (1) and (2) on the streamline shown gives

$$V = \sqrt{2gh}$$

Which is the modern version of a result obtained in 1643 by Torricelli 11608–16472, an Italian physicist.

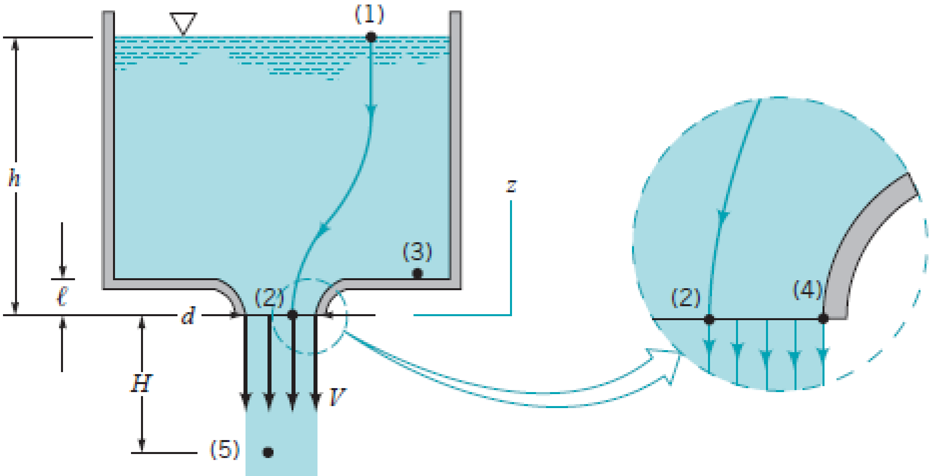


Figure 4.9. Vertical flow from a tank

For the horizontal nozzle of Fig.4.10*a*, the velocity of the fluid at the centerline, V_2 will be slightly greater than that at the top, V_1, and slightly less than that at the bottom, V_3, due to the differences in elevation. In general, $d \ll h$ as shown in Fig.4.10*b* and we can safely use the centerline velocity as a reasonable "***average velocity***." From another assumption ***a velocity factor*** can be used for real velocity.

Velocity factor $C_v = \dfrac{real\ velocity}{theoretical\ velocity} = \dfrac{V_r}{V_t} = \dfrac{V_r}{\sqrt{2gh}}$

If the exit is not a smooth, well-contoured nozzle, but rather a flat plate as shown in Fig.4.10c, the diameter of the jet, d_j, will be less than the diameter of the hole, d_h. This phenomenon, called **a *vena contracta*** effect, is a result of the inability of the fluid to turn the sharp 90° corner indicated by the dotted lines in the figure.

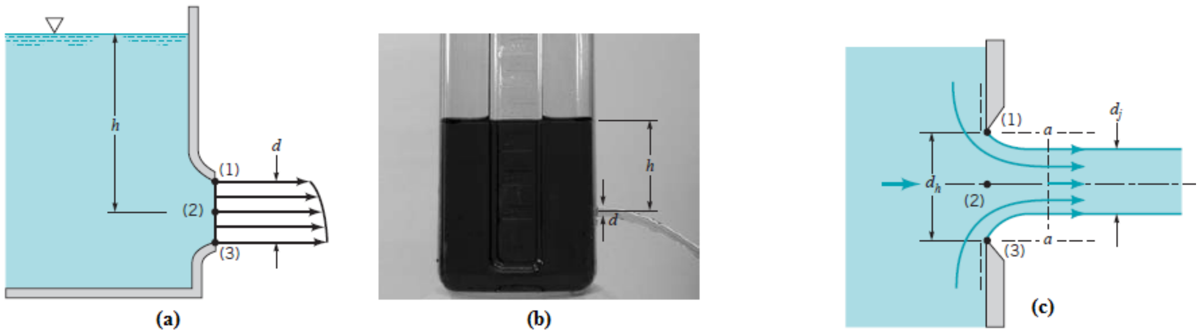


Figure 4.10. Horizontal flow from a tank (a and b) and Vena contracta effect for a sharp-edged orifice (c).

The vena contracta effect is a function of the geometry of the outlet. Some typical configurations are shown in the below Fig.4.11 along with typical values of the experimentally obtained ***contraction coefficient***, $C_c = \frac{A_j}{A_h} = (\frac{d_j}{d_h})^2$. *Where A_j and A_h* are the areas of the jet at the vena contracta and the area of the hole, respectively. $A_j = \frac{\pi d_j^2}{4}$ and $A_h = \frac{\pi d_h^2}{4}$. Then the flow rate for free jet $Q = C_v C_c A_h \sqrt{2gh} = C_d A_h \sqrt{2gh}$. $C_d = C_c C_v$ can be taken to be 0.62 for free jet.

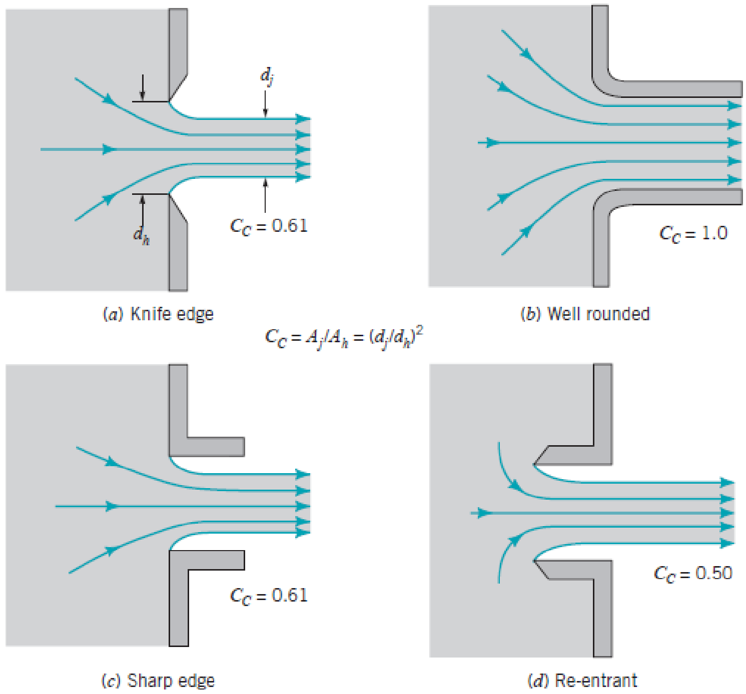


Figure 4.11. Typical flow patterns and contraction coefficients for various round exit configurations. (*a*) Knife edge, (*b*) Well rounded, (*c*) Sharp edge, (*d*) Re-entrant.

4.5.2. Confined Flows

In many cases the fluid is physically constrained within a device so that its pressure cannot be prescribed a priori as was done for the free jet examples above. Such cases include nozzles and pipes of variable diameter for which the fluid velocity changes because the flow area is different from one section to another. For these situations it is necessary to use the concept of conservation of mass (***the continuity equation***) along with the Bernoulli equation. For the needs of this chapter we can use a simplified form of the continuity equation obtained from the following intuitive arguments. Consider a fluid flowing through a fixed volume (such as a syringe) that has one inlet and one outlet as shown in Fig.4.12*a*. If the flow is steady so that there is no additional accumulation of fluid within the volume, the rate at which the fluid flows into the volume must equal the rate at which it flows out of the volume (otherwise, mass would not be conserved).

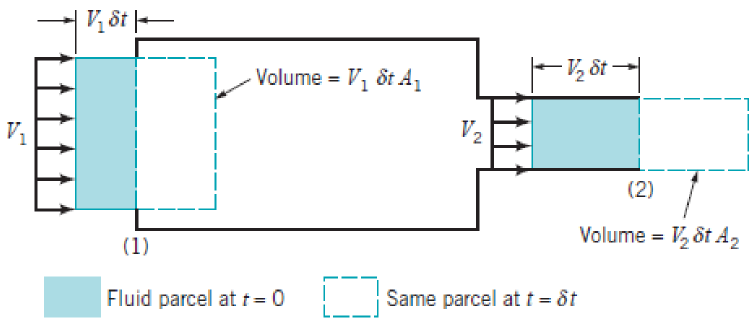
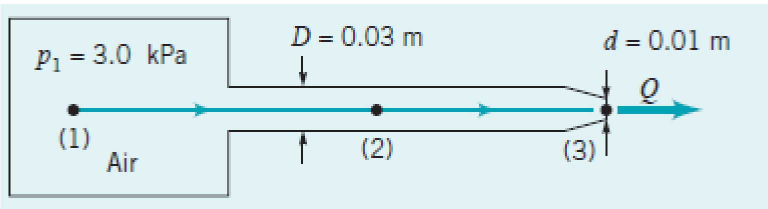


Figure 4.12. (*a*) Flow through a syringe. (*b*) Steady flow into and out of a volume.

The ***continuity equation*** for incompressible flow can be given as $Q_1=Q_2$ or $A_1V_1=A_2V_2$. Where; Q_1 is the inlet flow rate, Q_2 is the outlet flow rate, A_1 is the inlet cross section area, A_2 is the outler cross section area, V_1 is the inlet velocity of fluid, V_2 is the outlet velocity of fluid.

Example: Air flows steadily from a tank, through a hose of diameter and exits to the atmosphere from a nozzle of diameter as shown in Fig.. The pressure in the tank remains constant at 3.0 kPa (gage) and the atmospheric conditions are standard temperature and pressure. Determine the flowrate and the pressure in the hose.



Solution:

$$P_1+\frac{1}{2}\rho V_1^2+\gamma z_1=P_2+\frac{1}{2}\rho V_2^2+\gamma z_2=P_3+\frac{1}{2}\rho V_3^2+\gamma z_3$$

With the assumption that $z_1=z_2=z_3$ (horizontal hose), $V_1=0$ (large tank), and $P_3=0$ (free jet), this becomes

$$V_3=\sqrt{\frac{2P_1}{\rho}} \quad \text{and} \quad P_2=P_1-\tfrac{1}{2}\rho V_2^2 \quad (2)$$

The density of the air in the tank is obtained from the perfect gas law, using standard absolute pressure and temperature, as

$$\rho = \frac{P_1}{RT_1} == \frac{[3000 + 101000]}{286.9 \times (15 + 273)} = 1.26\ kg/m^3$$

Thus, we find that

$$V_3 = \sqrt{\frac{2 \times 3000}{1.26}} = 69\ m/s$$

$$Q = A_3 V_3 = \frac{\pi d^2}{4} V_3 = \frac{\pi}{4} (0.01)^2 \times 69 = 0.00542\ m^3/s$$

The pressure within the hose can be obtained from Eq. 1 and the continuity equation $A_2V_2=A_3V_3$ Hence

$$V_2 = \frac{A_3 V_3}{A_2} = (\frac{d}{D})^2 V_3 = (\frac{0.01}{0.03})^2 (69) = 7.67\ m/s$$

And $P_2 = 3000 - \frac{1}{2} \times 1.26 \times 7.67 = 2963\ Pa$

In general, an increase in velocity is accompanied by a decrease in pressure. For example, the velocity of the air flowing over the top surface of an airplane wing is, on the average, faster than that flowing under the bottom surface. Thus, the net pressure force is greater on the bottom than on the top—the wing generates a lift.