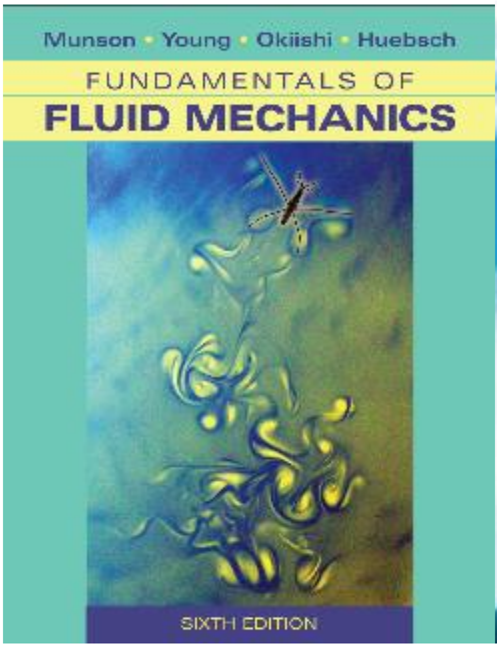
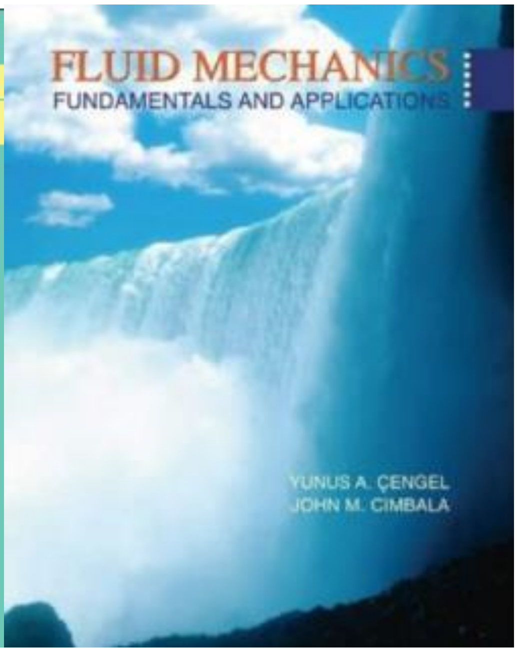
FLUID MECHANICS





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4.5.2.1. Cavitation

If the differences in velocity are considerable, the differences in pressure can also be considerable. For flows of liquids, this may result in ***cavitation,*** a potentially dangerous situation that results when the liquid pressure is reduced to the vapor pressure and the liquid "boils."

One way to produce cavitation in a flowing liquid is noted from the Bernoulli equation. If the fluid velocity is increased (for example, by a reduction in flow area as shown in Fig.4.13) the pressure will decrease. This pressure decrease 1needed to accelerate the fluid through the constriction) can be large enough so that the pressure in the liquid is reduced to its vapor pressure. A simple example of cavitation can be demonstrated with an ordinary garden hose. If the hose is "kinked," a restriction in the flow area in some ways analogous to that shown in Fig. 4.13 will result. The water velocity through this restriction will be relatively large. With a sufficient amount of restriction the sound of the flowing water will change—a definite "hissing" sound is produced. This sound is a result of cavitation.

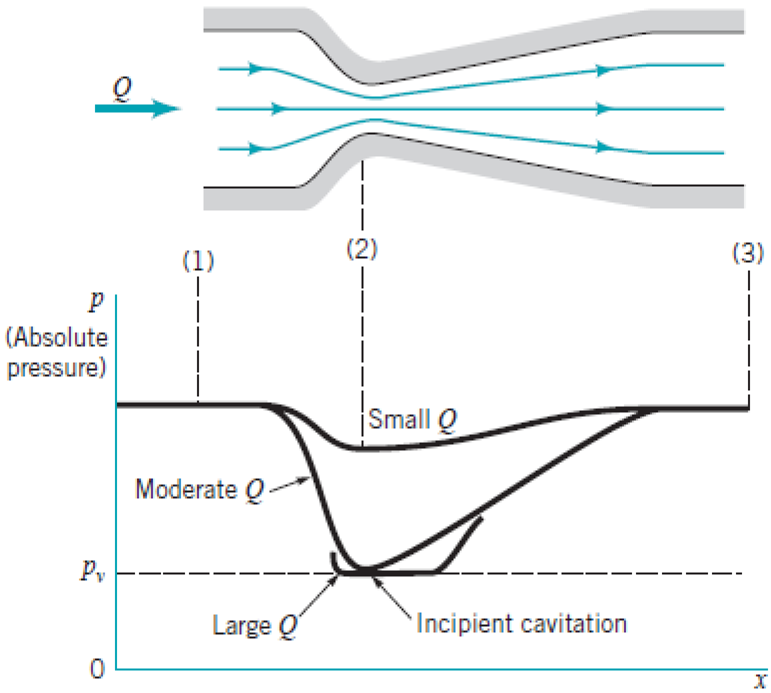


Figure 4.13. Pressure variation and cavitation in a variable area pipe.

4.5.3. Flowrate Measurement

Many types of devices using principles involved in the Bernoulli equation have been developed to measure fluid velocities and flowrates. The Pitot-static tube is an example. Other examples discussed below include devices to measure flowrates in pipes and conduits and devices to measure flowrates in open channels. In this chapter we will consider "ideal" ***flow meters***—those devoid of viscous, compressibility, and other "real-world" effects. Our goal here is to understand the basic operating principles of these simple flow meters

An effective way to measure the flowrate through a pipe is to place some type of restriction within the pipe as shown in Fig.4.14 and to measure the pressure difference between the low-velocity, high-pressure upstream section (1), and the high-velocity, low-pressure downstream section (2). Three commonly used types of flow meters are illustrated: the ***orifice meter***, the ***nozzle meter***, and the ***Venturi meter***. The operation of each is based on the same physical principles—an increase in velocity causes a decrease in pressure. The difference between them is a matter of cost, accuracy, and how closely their actual operation obeys the idealized flow assumptions. The theoretical flowrate for above flow meters can be written as the following.

$$Q = A_2 \sqrt{\frac{2(P_2 - P_1)}{\rho[1 - (A_2/A_1)^2]}}$$

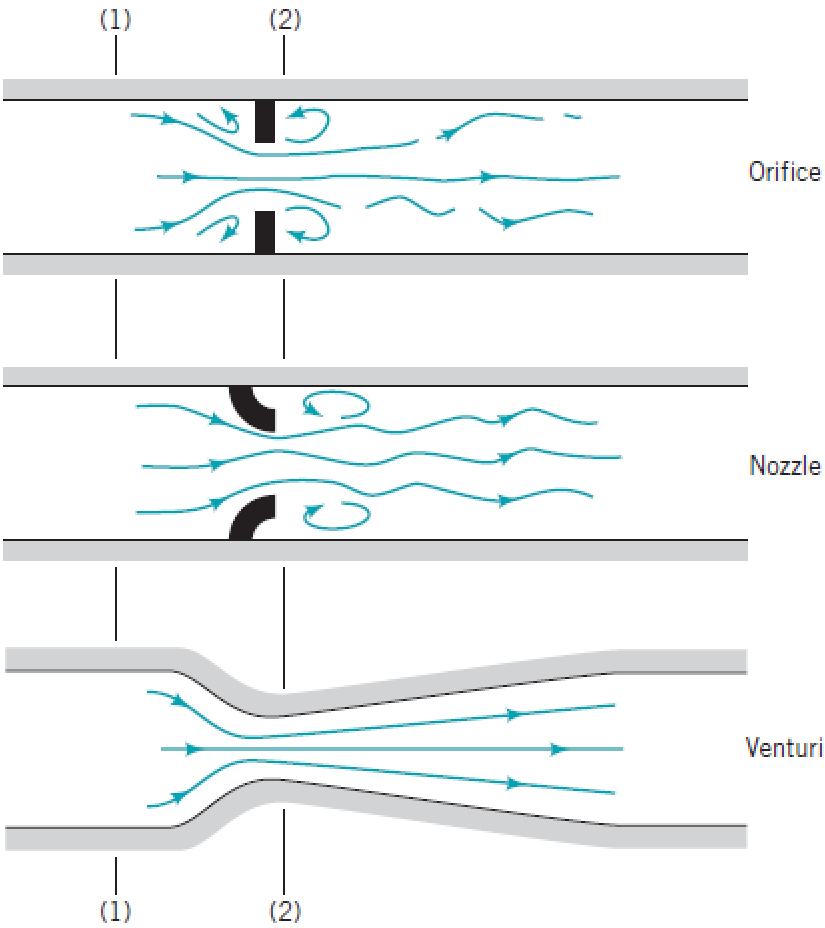


Figure 4.14. Typical devices for measuring flowrate in pipes.

4.6. Summary

Some of the important equations in this chapter are:

Streamwise and normal acceleration	$a_s = V\dfrac{\partial V}{\partial s}, \qquad a_n = \dfrac{V^2}{\mathcal{R}}$
Force balance along a streamline for steady inviscid flow	$\displaystyle\int \frac{dp}{\rho} + \frac{1}{2}V^2 + gz = C$ (along a streamline)
The Bernoulli equation	$p + \frac{1}{2}\rho V^2 + \gamma z =$ constant along streamline
Pressure gradient normal to streamline for inviscid flow in absence of gravity	$\dfrac{\partial p}{\partial n} = -\dfrac{\rho V^2}{\mathcal{R}}$

Force balance normal to a streamline for steady, inviscid, incompressible flow	$p + \rho \int \frac{V^2}{\mathfrak{R}} dn + \gamma z = \text{constant across the streamline}$
Velocity measurement for a Pitot-static tube	$V = \sqrt{2(p_3 - p_4)/\rho}$
Free jet	$V = \sqrt{2 \frac{\gamma h}{\rho}} = \sqrt{2gh}$
Continuity equation	$A_1 V_1 = A_2 V_2$, or $Q_1 = Q_2$
Flow meter equation	$Q = A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (A_2/A_1)^2]}}$
Total head	$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{constant on a streamline} = H$

EXAMPLES

Example: In cold climates, water pipes may freeze and burst if proper precautions are not taken. In such an occurrence, the exposed part of a pipe on the ground ruptures, and water shoots up to 34 m. Estimate the gage pressure of water in the pipe. State your assumptions and discuss if the actual pressure is more or less than the value you predicted. The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). The water pressure in the pipe at the burst section is equal to the water main pressure. Friction between the water and air is negligible. The irreversibilities that may occur at the burst section of the pipe due to abrupt expansion are negligible. We take the density of water to be 1000 kg/m^3.

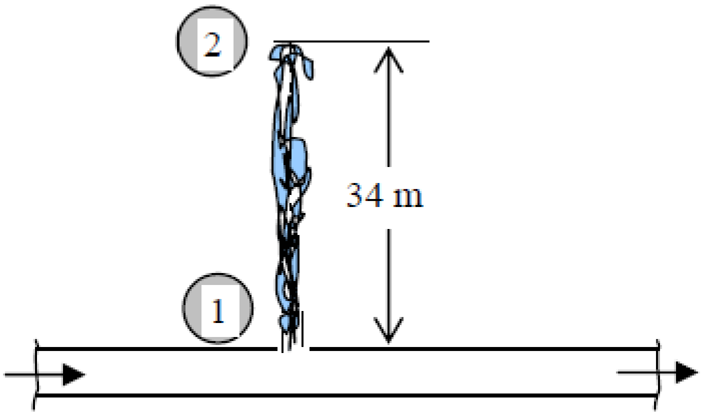
Solution: This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. The water height will be maximum under the stated assumptions. The velocity inside the hose is relatively low ($V_1 \cong 0$) and we take the burst section of the pipe as the reference level ($z_1 = 0$). At the top of the water trajectory $V_2 = 0$, and atmospheric pressure pertains. Then the Bernoulli equation simplifies to

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_1}{\rho g}=\frac{P_{atm}}{\rho g}+z_2 \quad\rightarrow\quad \frac{P_1-P_{atm}}{\rho g}=z_2 \quad\rightarrow\quad \frac{P_{1,\text{gage}}}{\rho g}=z_2$$

Solving for $P1$,gage and substituting,

$$P_{1,\text{gage}}=\rho g z_2=(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(34\ \text{m})\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)=\mathbf{334\ kPa}$$

Therefore, the pressure in the main must be at least 334 kPa above the atmospheric pressure. The result obtained by the Bernoulli equation represents a limit, since frictional losses are neglected, and should be interpreted accordingly. It tells us that the water pressure (gage) cannot possibly be less than 334 kPa (giving us a lower limit), and in all likelihood, the pressure will be much higher.



Example: A Pitot-static probe is used to measure the velocity of an aircraft flying at 3000 m. If the differential pressure reading is 3 kPa, determine the velocity of the aircraft. The air flow over the aircraft is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). Standard atmospheric conditions exist. The wind effects are negligible. The density of the atmosphere at an elevation of 3000 m is ρ= 0.909 kg/m^3.

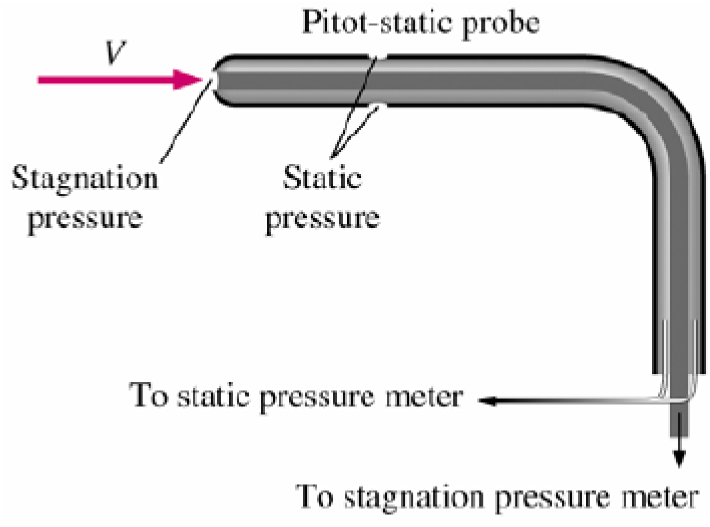
Solution: We take point 1 at the entrance of the tube whose opening is parallel to flow, and point 2 at the entrance of the tube whose entrance is normal to flow. Noting that point 2 is a stagnation point and thus $V_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{V_1^2}{2g}=\frac{P_2-P_1}{\rho g} \quad\rightarrow\quad \frac{V_1^2}{2}=\frac{P_{stag}-P_1}{\rho}$$

Solving for $V1$ and substituting,

$$V_1 = \sqrt{\frac{2(P_{stag} - P_1)}{\rho}} = \sqrt{\frac{2(3000\ \text{N/m}^2)}{0.909\ \text{kg/m}^3}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)} = \mathbf{81.2\ m/s = 292\ km/h}$$

Note that the velocity of an aircraft can be determined by simply measuring the differential pressure on a Pitot-static probe.



Example: While traveling on a dirt road, the bottom of a car hits a sharp rock and a small hole develops at the bottom of its gas tank. If the height of the gasoline in the tank is 30 cm, determine the initial velocity of the gasoline at the hole. Discuss how the velocity will change with time and how the flow will be affected if the lid of the tank is closed tightly. The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). The air space in the tank is at atmospheric pressure. The splashing of the gasoline in the tank during travel is not considered.

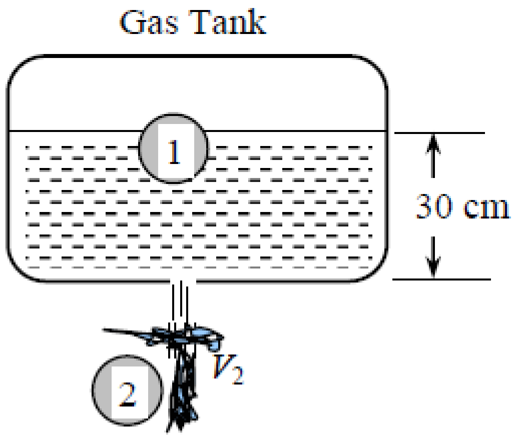
Solution: This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. We take point 1 to be at the free surface of gasoline in the tank so that $P_1 = P_{atm}$ (open to the atmosphere) $V_1 \cong 0$ (the tank is large relative to the outlet), and $z_1 = 0.3$ m and $z_2 = 0$ (we take the reference level at the hole. Also, $P_2 = P_{atm}$ (gasoline discharges into the atmosphere). Then the Bernoulli equation simplifies to

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \rightarrow \quad z_1 = \frac{V_2^2}{2g}$$

Solving for V_2 and substituting,

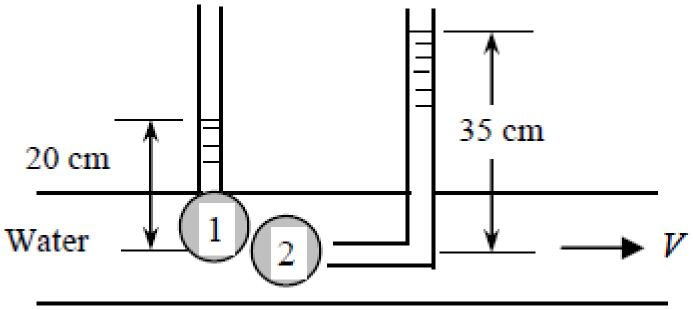
$$V_2 = \sqrt{2gz_1} = \sqrt{2(9.81 \text{ m/s}^2)(0.3 \text{ m})} = \mathbf{2.43 \text{ m/s}}$$

Therefore, the gasoline will initially leave the tank with a velocity of 2.43 m/s. The Bernoulli equation applies along a streamline, and streamlines generally do not make sharp turns. The velocity will be less than 2.43 m/s since the hole is probably sharp-edged and it will cause some head loss. As the gasoline level is reduced, the velocity will decrease since velocity is proportional to the square root of liquid height. If the lid is tightly closed and no air can replace the lost gasoline volume, the pressure above the gasoline level will be reduced, and the velocity will be decreased.



Example: A piezometer and a Pitot tube are tapped into a 3-cm diameter horizontal water pipe, and the height of the water columns are measured to be 20 cm in the piezometer and 35 cm in the Pitot tube (both measured from the top surface of the pipe). Determine the velocity at the center of the pipe. The flow is steady, incompressible, and irrotational with negligible frictional effects in the short distance between the two pressure measurement locations (so that the Bernoulli equation is applicable).

Solution: We take points 1 and 2 along the centerline of the pipe, with point 1 directly under the piezometer and point 2 at the entrance of the Pitot-static probe (the stagnation point). This is a steady flow with straight and parallel streamlines, and thus the static pressure at any point is equal to the hydrostatic pressure at that point. Noting that point 2 is a stagnation point and thus $V_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives



$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{V_1^2}{2g}=\frac{P_2-P_1}{\rho g}$$

Substituting the P_1 and P_2 expressions give

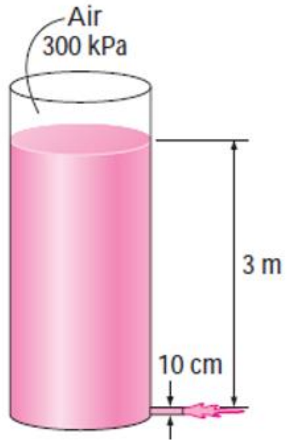
$$\frac{V_1^2}{2g}=\frac{P_2-P_1}{\rho g}=\frac{\rho g(h_{\text{pitot}}+R)-\rho g(h_{\text{piezo}}+R)}{\rho g}=\frac{\rho g(h_{\text{pitot}}-h_{\text{piezo}})}{\rho g}=h_{\text{pitot}}-h_{\text{piezo}}$$

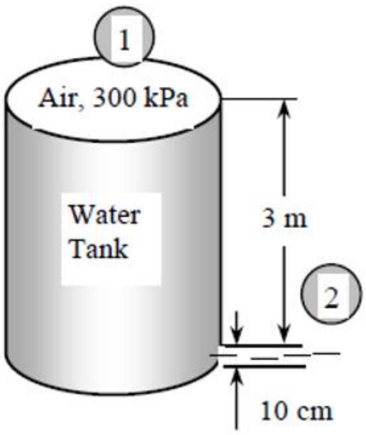
Solving for V_1 and substituting,

$$V_1=\sqrt{2g(h_{\text{pitot}}-h_{\text{piezo}})}=\sqrt{2(9.81\,\text{m/s}^2)[(0.35-0.20)\,\text{m}]}=\mathbf{1.72\ m/s}$$

Note that to determine the flow velocity, all we need is to measure the height of the excess fluid column in the Pitot-static probe.

Example: A pressurized tank of water has a 10-cm-diameter orifice at the bottom, where water discharges to the atmosphere. The water level is 3 m above the outlet. The tank air pressure above the water level is 300 kPa (absolute) while the atmospheric pressure is 100 kPa. Neglecting frictional effects, determine the initial discharge rate of water from the tank. We take the density of water to be 1000 kg/m^3.





Solution: We take point 1 at the free surface of the tank, and point 2 at the exit of orifice, which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid velocity at the free surface is very low ($V_1 \cong 0$) and water discharges into the atmosphere (and thus $P_2 = P$atm), the Bernoulli equation simplifies to

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \rightarrow \quad \frac{V_2^2}{2g} = \frac{P_1 - P_2}{\rho g} + z_1$$

Solving for $V2$ and substituting, the discharge velocity is determined to

$$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho} + 2gz_1} = \sqrt{\frac{2(300-100)\text{ kPa}}{1000\text{ kg/m}^3}\left(\frac{1000\text{ N/m}^2}{1\text{ kPa}}\right)\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) + 2(9.81\text{ m/s}^2)(3\text{ m})}$$
$$= 21.4\text{ m/s}$$

Then the initial rate of discharge of water becomes

$$\dot{V} = A_{\text{orifice}}V_2 = \frac{\pi D^2}{4}V_2 = \frac{\pi(0.10\text{ m})^2}{4}(21.4\text{ m/s}) = \mathbf{0.168\ m^3/s}$$

Note that this is the maximum flow rate since the frictional effects are ignored. Also, the velocity and the flow rate will decrease as the water level in the tank decreases.

Example: The water in a 10-m-diameter, 2-m-high aboveground swimming pool is to be emptied by unplugging a 3-cmdiameter, 25-m-long horizontal pipe attached to the bottom of the pool. Determine the maximum discharge rate of water through the pipe. Also, explain why the actual flow rate will be less.

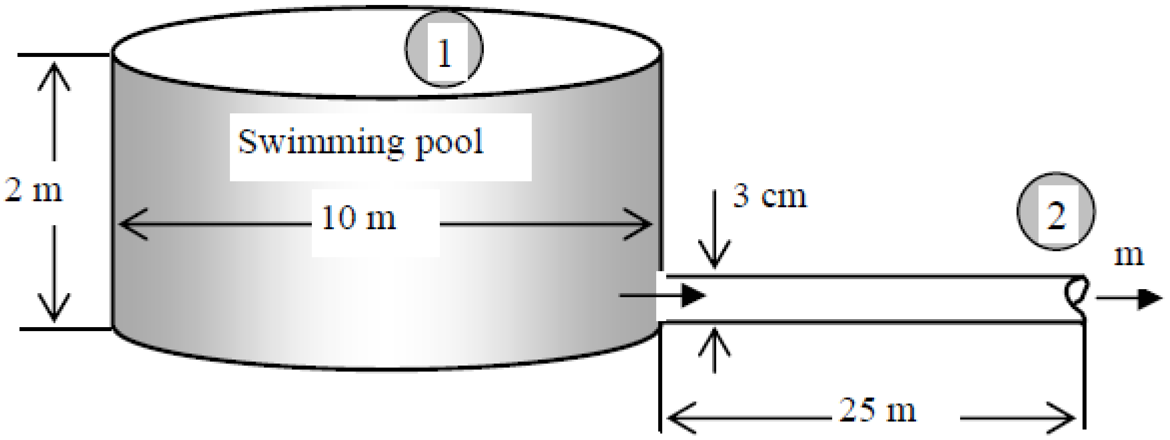
Solution: We take point 1 at the free surface of the pool, and point 2 at the exit of pipe. We take the reference level at the pipe exit ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P$atm) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the Bernoulli equation between these two points simplifies to

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \rightarrow \quad z_1 = \frac{V_2^2}{2g} \quad \rightarrow \quad V_2 = \sqrt{2gz_1}$$

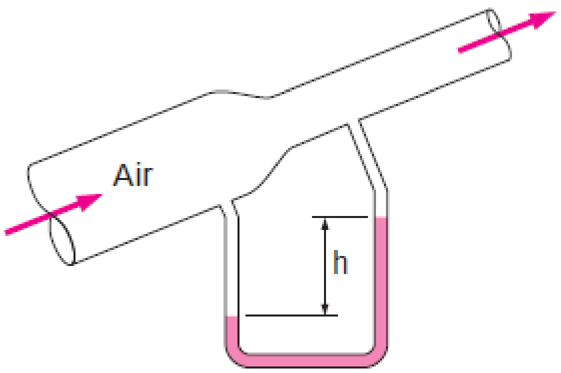
The maximum discharge rate occurs when the water height in the pool is a maximum, which is the case at the beginning and thus $z_1 = h$. Substituting, the maximum flow velocity and discharge rate become

$$V_{2,\max} = \sqrt{2gh} = \sqrt{2(9.81\text{ m/s}^2)(2\text{ m})} = 6.26\text{ m/s}$$

$$\dot{V}_{max} = A_{pipe} V_{2,max} = \frac{\pi D^2}{4} V_{2,max} = \frac{\pi (0.03\text{ m})^2}{4}(6.26\text{ m/s}) = 0.00443\text{ m}^3/\text{s} = \mathbf{4.43\ L/s}$$



Example: Air at 110 kPa and 50°C flows upward through a 6-cm-diameter inclined duct at a rate of 45 L/s. The duct diameter is then reduced to 4 cm through a reducer. The pressure change across the reducer is measured by a water manometer. The elevation difference between the two points on the pipe where the two arms of the manometer are attached is 0.20 m. Determine the differential height between the fluid levels of the two arms of the manometer. We take the density of water to be ρ = 1000 kg/m^3. The gas constant of air is R = 0.287 kPa·m3/kg·K.



Solution: We take points 1 and 2 at the lower and upper connection points, respectively, of the two arms of the manometer, and take the lower connection point as the reference level. Noting that the effect of elevation on the pressure change of a gas is negligible, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \rightarrow \quad P_1 - P_2 = \rho_{air} \frac{V_2^2 - V_1^2}{2}$$

$$\rho_{air} = \frac{P}{RT} = \frac{110\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(50+273\text{ K})} = 1.19\text{ kg/m}^3$$

$$V_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{\pi D_1^2/4} = \frac{0.045\text{ m}^3/\text{s}}{\pi(0.06\text{ m})^2/4} = 15.9\text{ m/s}$$

$$V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi D_2^2/4} = \frac{0.045\text{ m}^3/\text{s}}{\pi(0.04\text{ m})^2/4} = 35.8\text{ m/s}$$

Substituting,

$$P_1 - P_2 = (1.19\text{ kg/m}^3)\frac{(35.8\text{ m/s})^2 - (15.9\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = 612\text{ N/m}^2 = 612\text{ Pa}$$

The differential height of water in the manometer corresponding to this pressure change is determined from $\Delta P = \rho_{water} gh$

to be

$$h = \frac{P_1 - P_2}{\rho_{water} g} = \frac{612\text{ N/m}^2}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) = 0.0624\text{ m} = \mathbf{6.24\text{ cm}}$$

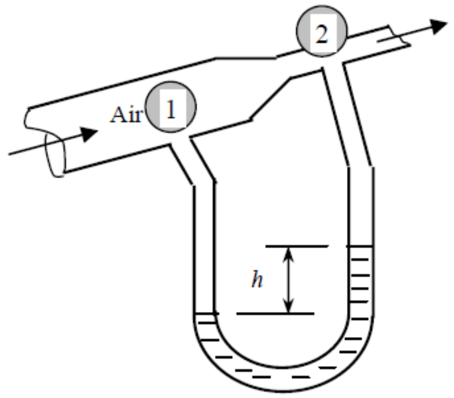
When the effect of air column on pressure change is considered, the pressure change becomes

$$P_1 - P_2 = \frac{\rho_{air}(V_2^2 - V_1^2)}{2} + \rho_{air} g(z_2 - z_1)$$

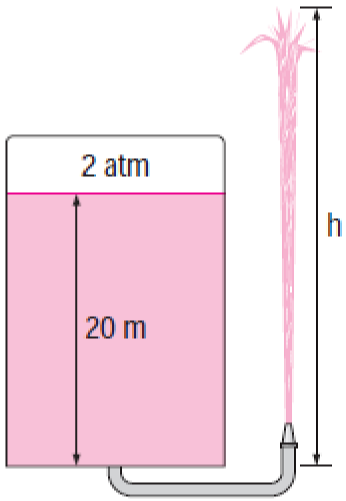
$$= (1.19\text{ kg/m}^3)\left[\frac{(35.8\text{ m/s})^2 - (15.9\text{ m/s})^2}{2} + (9.81\text{ m/s}^2)(0.2\text{ m})\right]\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right)$$

$$= (612+2)\text{ N/m}^2 = 614\text{ N/m}^2 = 614\text{ Pa}$$

This difference between the two results (612 and 614 Pa) is less than 1%. Therefore, the effect of air column on pressure change is, indeed, negligible as assumed. In other words, the pressure change of air in the duct is almost entirely due to velocity change, and the effect of elevation change is negligible. Also, if we were to account for the Δz of air flow, then it would be more proper to account for the Δz of air in the manometer by using $\rho_{water} - \rho_{air}$ instead of ρ_{water} when calculating *h*. The additional air column in the manometer tends to cancel out the change in pressure due to the elevation difference in the flow in this case.



Example: The water level in a tank is 20 m above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The tank cover is airtight, and the air pressure above the water surface is 2 atm gage. The system is at sea level. Determine the maximum height to which the water stream could rise. We take the density of water to be 1000 kg/m^3.



Solution: We take point 1 at the free surface of water in the tank, and point 2 at the top of the water trajectory. Also, we take the reference level at the bottom of the tank. At the top of the water trajectory $\mathbf{V}_2$ = 0, and atmospheric pressure pertains. Noting that z_1 = 20 m, P_1,gage = 2 atm, P_2 = Patm, and that the fluid velocity at the free surface of the tank is very low ($V_1 \cong 0$), the Bernoulli equation between these two points simplifies to

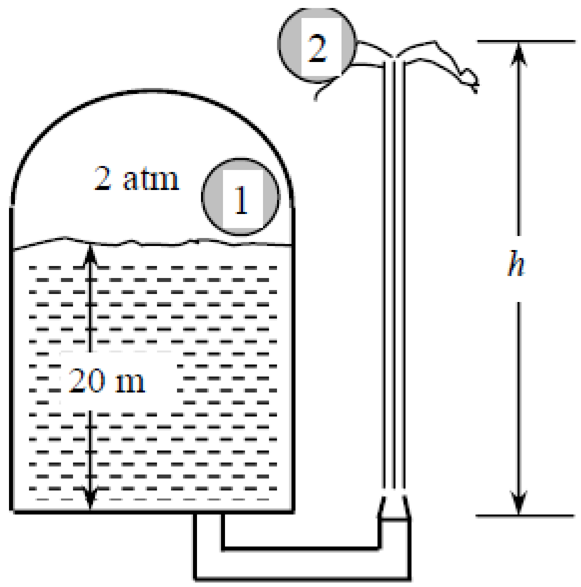
$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2$$

$$\frac{P_1}{\rho g}+z_1=\frac{P_{atm}}{\rho g}+z_2$$

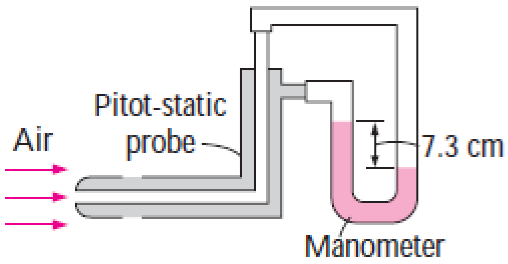
$$z_2 = \frac{P_1 - P_{atm}}{\rho g} + z_1 = \frac{P_{1,\text{gage}}}{\rho g} + z_1$$

Substituting,

$$z_2 = \frac{2\ \text{atm}}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{101{,}325\ \text{N/m}^2}{1\ \text{atm}}\right)\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) + 20 = \mathbf{40.7\ m}$$



Example: A Pitot-static probe connected to a water manometer is used to measure the velocity of air. If the deflection (the vertical distance between the fluid levels in the two arms) is 7.3 cm, determine the air velocity. Take the density of air to be 1.25 kg/m^3. We take the density of water to be ρ = 1000 kg/m^3. The density of air is given to be 1.25 kg/m^3.



Solution: We take point 1 on the side of the probe where the entrance is parallel to flow and is connected to the static arm of the Pitot-static probe, and point 2 at the tip of the probe where the entrance is normal to flow and is connected to the dynamic arm of the Pitot-static probe. Noting that point 2 is a stagnation point and thus $V_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

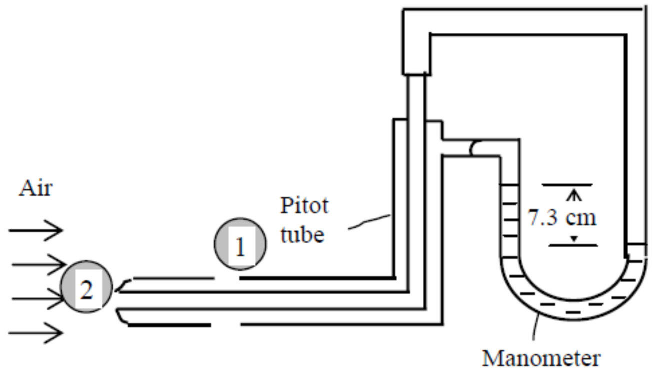
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} \quad V_1 = \sqrt{\frac{2(P_2 - P_1)}{\rho_{air}}} \quad (1)$$

The pressure rise at the tip of the Pitot-static probe is simply the pressure change indicated by the differential water column of the manometer,

$$P_2 - P_1 = \rho_{\text{water}} g h \quad (2)$$

Combining Eqs. (1) and (2) and substituting, the flow velocity is determined to be

$$V_1 = \sqrt{\frac{2\rho_{\text{water}} g h}{\rho_{\text{air}}}} = \sqrt{\frac{2(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.073\ \text{m})}{1.25\ \text{kg/m}^3}} = \mathbf{33.8\ m/s}$$



Example: In a hydroelectric power plant, water enters the türbine nozzles at 700 kPa absolute with a low velocity. If the nozzle outlets are exposed to atmospheric pressure of 100 kPa, determine the maximum velocity to which water can be accelerated by the nozzles before striking the turbine blades. We take the density of water to be $\rho = 1000$ kg/m^3.

Solution: We take points 1 and 2 at the inlet and exit of the nozzle, respectively. Noting that $V_1 \cong 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2$$

$$\frac{P_1}{\rho g}=\frac{P_{atm}}{\rho g}+\frac{V_2^2}{2g}$$

$$V_2=\sqrt{\frac{2(P_1-P_{atm})}{\rho}}$$

Substituting the given values, the nozzle exit velocity is determined to be

$$V_1=\sqrt{\frac{2(700-100)\text{ kPa}}{1000\text{ kg/m}^3}\left(\frac{1000\text{ N/m}^2}{1\text{ kPa}}\right)\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)}=\mathbf{34.6\ m/s}$$

This is the maximum nozzle exit velocity, and the actual velocity will be less because of friction between water and the walls of the nozzle.