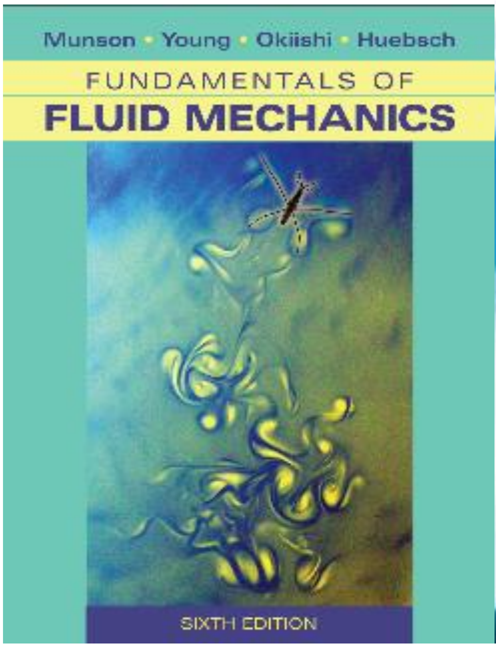
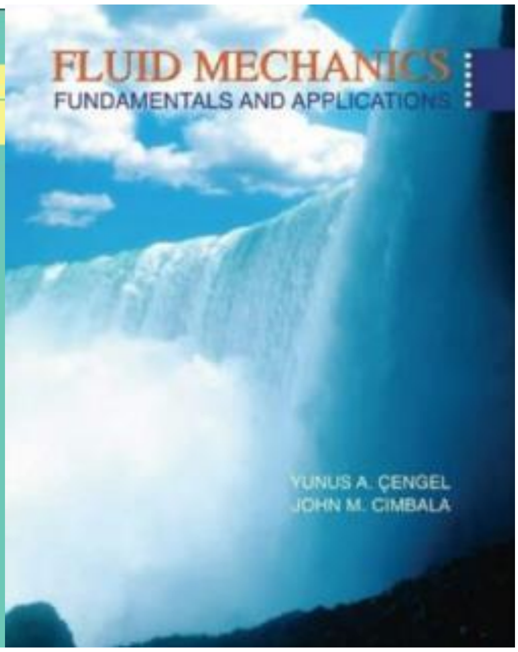
FLUID MECHANICS





PROF. DR. METİN GÜNER
COMPILER

ANKARA UNIVERSITY
FACULTY OF AGRICULTURE
DEPARTMENT OF AGRICULTURAL MACHINERY AND TECHNOLOGIES ENGINEERING

5. FLOW IN PIPES

EXAMPLES

Example: In cold climates, water pipes may freeze and burst if proper precautions are not taken. In such an occurrence, the exposed part of a pipe on the ground ruptures, and water shoots up to 34 m. Estimate the gage pressure of water in the pipe. State your assumptions and discuss if the actual pressure is more or less than the value you predicted. The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). The water pressure in the pipe at the burst section is equal to the water main pressure. Friction between the water and air is negligible. The irreversibilities that may occur at the burst section of the pipe due to abrupt expansion are negligible. We take the density of water to be 1000 kg/m^3.

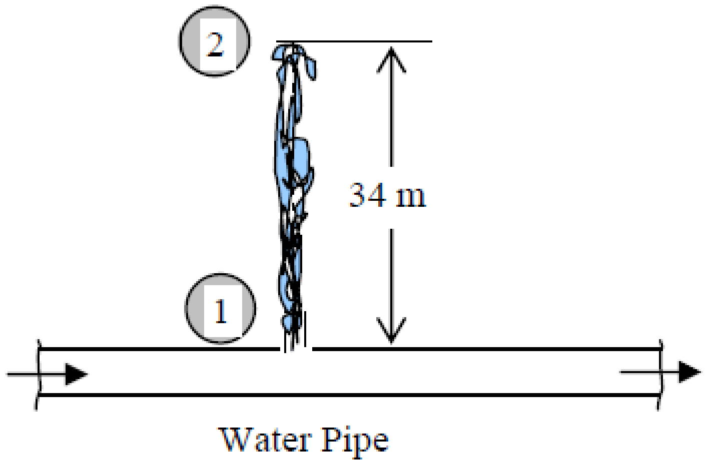
Solution: This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. The water height will be maximum under the stated assumptions. The velocity inside the hose is relatively low ($V_1 \cong 0$) and we take the burst section of the pipe as the reference level ($z_1 = 0$). At the top of the water trajectory $V_2 = 0$, and atmospheric pressure pertains. Then the Bernoulli equation simplifies to

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{P_1}{\rho g}=\frac{P_{atm}}{\rho g}+z_2 \quad\rightarrow\quad \frac{P_1-P_{atm}}{\rho g}=z_2 \quad\rightarrow\quad \frac{P_{1,gage}}{\rho g}=z_2$$

Solving for P1,gage and substituting,

$$P_{1,gage}=\rho g z_2=(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(34\ \text{m})\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right)\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)=\mathbf{334\ kPa}$$

Therefore, the pressure in the main must be at least 334 kPa above the atmospheric pressure. The result obtained by the Bernoulli equation represents a limit, since frictional losses are neglected, and should be interpreted accordingly. It tells us that the water pressure (gage) cannot possibly be less than 334 kPa (giving us a lower limit), and in all likelihood, the pressure will be much higher.



Example: A Pitot-static probe is used to measure the velocity of an aircraft flying at 3000 m. If the differential pressure reading is 3 kPa, determine the velocity of the aircraft. The air flow over the aircraft is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). Standard atmospheric conditions exist. The wind effects are negligible. The density of the atmosphere at an elevation of 3000 m is ρ= 0.909 kg/m^3.

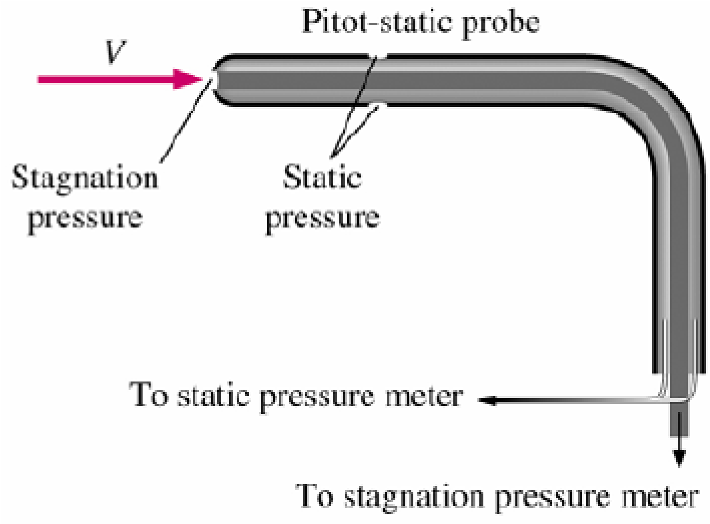
Solution: We take point 1 at the entrance of the tube whose opening is parallel to flow, and point 2 at the entrance of the tube whose entrance is normal to flow. Noting that point 2 is a stagnation point and thus $V_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{V_1^2}{2g}=\frac{P_2-P_1}{\rho g} \quad\rightarrow\quad \frac{V_1^2}{2}=\frac{P_{stag}-P_1}{\rho}$$

Solving for $V1$ and substituting,

$$V_1=\sqrt{\frac{2(P_{stag}-P_1)}{\rho}}=\sqrt{\frac{2(3000\ \text{N/m}^2)}{0.909\ \text{kg/m}^3}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)}=\mathbf{81.2\ m/s}=\mathbf{292\ km/h}$$

Note that the velocity of an aircraft can be determined by simply measuring the differential pressure on a Pitot-static probe.



Example: While traveling on a dirt road, the bottom of a car hits a sharp rock and a small hole develops at the bottom of its gas tank. If the height of the gasoline in the tank is 30 cm, determine the initial velocity of the gasoline at the hole. Discuss how the velocity will change with time and how the flow will be affected if the lid of the tank is closed tightly. The flow is steady, incompressible, and irrotational with negligible frictional effects (so that the Bernoulli equation is applicable). The air space in the tank is at atmospheric pressure. The splashing of the gasoline in the tank during travel is not considered.

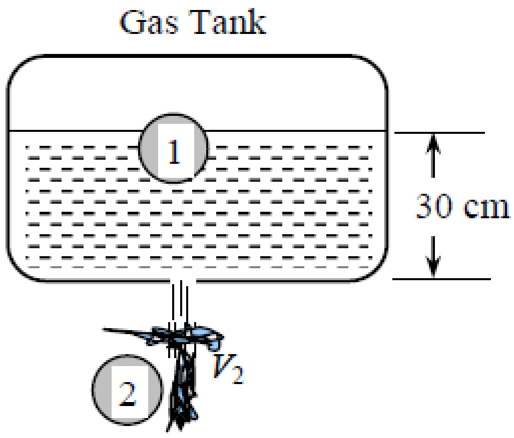
Solution: This problem involves the conversion of flow, kinetic, and potential energies to each other without involving any pumps, turbines, and wasteful components with large frictional losses, and thus it is suitable for the use of the Bernoulli equation. We take point 1 to be at the free surface of gasoline in the tank so that $P_1 = P_{atm}$ (open to the atmosphere) $V_1 \cong 0$ (the tank is large relative to the outlet), and $z_1 = 0.3$ m and $z_2 = 0$ (we take the reference level at the hole. Also, $P_2 = P_{atm}$ (gasoline discharges into the atmosphere). Then the Bernoulli equation simplifies to

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \rightarrow \quad z_1 = \frac{V_2^2}{2g}$$

Solving for V_2 and substituting,

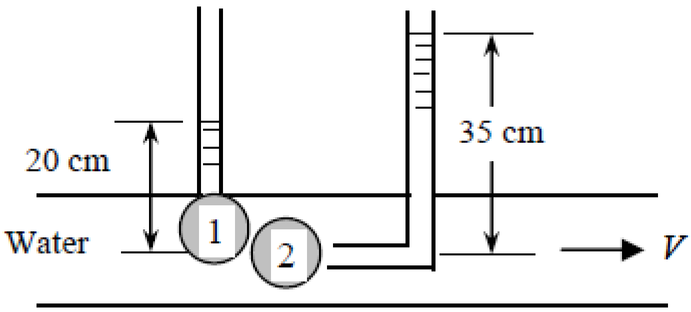
$$V_2 = \sqrt{2gz_1} = \sqrt{2(9.81\,\text{m/s}^2)(0.3\,\text{m})} = \mathbf{2.43\ m/s}$$

Therefore, the gasoline will initially leave the tank with a velocity of 2.43 m/s. The Bernoulli equation applies along a streamline, and streamlines generally do not make sharp turns. The velocity will be less than 2.43 m/s since the hole is probably sharp-edged and it will cause some head loss. As the gasoline level is reduced, the velocity will decrease since velocity is proportional to the square root of liquid height. If the lid is tightly closed and no air can replace the lost gasoline volume, the pressure above the gasoline level will be reduced, and the velocity will be decreased.



Example: A piezometer and a Pitot tube are tapped into a 3-cm diameter horizontal water pipe, and the height of the water columns are measured to be 20 cm in the piezometer and 35 cm in the Pitot tube (both measured from the top surface of the pipe). Determine the velocity at the center of the pipe. The flow is steady, incompressible, and irrotational with negligible frictional effects in the short distance between the two pressure measurement locations (so that the Bernoulli equation is applicable).

Solution: We take points 1 and 2 along the centerline of the pipe, with point 1 directly under the piezometer and point 2 at the entrance of the Pitot-static probe (the stagnation point). This is a steady flow with straight and parallel streamlines, and thus the static pressure at any point is equal to the hydrostatic pressure at that point. Noting that point 2 is a stagnation point and thus $V_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \rightarrow \quad \frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g}$$

Substituting the P_1 and P_2 expressions give

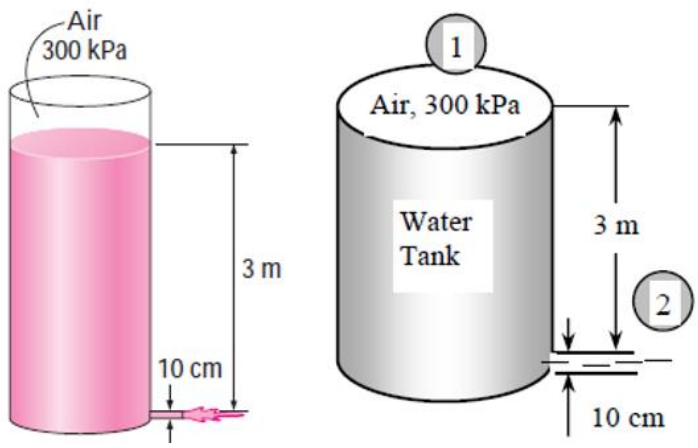
$$\frac{V_1^2}{2g}=\frac{P_2-P_1}{\rho g}=\frac{\rho g(h_{\text{pitot}}+R)-\rho g(h_{\text{piezo}}+R)}{\rho g}=\frac{\rho g(h_{\text{pitot}}-h_{\text{piezo}})}{\rho g}=h_{\text{pitot}}-h_{\text{piezo}}$$

Solving for V_1 and substituting,

$$V_1=\sqrt{2g(h_{\text{pitot}}-h_{\text{piezo}})}=\sqrt{2(9.81\,\text{m/s}^2)[(0.35-0.20)\,\text{m}]}=\mathbf{1.72\ m/s}$$

Note that to determine the flow velocity, all we need is to measure the height of the excess fluid column in the Pitot-static probe.

Example: A pressurized tank of water has a 10-cm-diameter orifice at the bottom, where water discharges to the atmosphere. The water level is 3 m above the outlet. The tank air pressure above the water level is 300 kPa (absolute) while the atmospheric pressure is 100 kPa. Neglecting frictional effects, determine the initial discharge rate of water from the tank. We take the density of water to be 1000 kg/m^3.



Solution: We take point 1 at the free surface of the tank, and point 2 at the exit of orifice, which is also taken to be the reference level ($z_2 = 0$). Noting that the fluid velocity at the free surface is very low ($V_1 \cong 0$) and water discharges into the atmosphere (and thus $P_2 = P$atm), the Bernoulli equation simplifies to

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad \frac{V_2^2}{2g}=\frac{P_1-P_2}{\rho g}+z_1$$

Solving for $V2$ and substituting, the discharge velocity is determined to

$$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho} + 2gz_1} = \sqrt{\frac{2(300-100)\text{ kPa}}{1000\text{ kg/m}^3}\left(\frac{1000\text{ N/m}^2}{1\text{ kPa}}\right)\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) + 2(9.81\text{ m/s}^2)(3\text{ m})}$$

$$= 21.4\text{ m/s}$$

Then the initial rate of discharge of water becomes

$$\dot{V} = A_{\text{orifice}} V_2 = \frac{\pi D^2}{4} V_2 = \frac{\pi(0.10\text{ m})^2}{4}(21.4\text{ m/s}) = \mathbf{0.168\ m^3/s}$$

Note that this is the maximum flow rate since the frictional effects are ignored. Also, the velocity and the flow rate will decrease as the water level in the tank decreases.

Example: The water in a 10-m-diameter, 2-m-high aboveground swimming pool is to be emptied by unplugging a 3-cmdiameter, 25-m-long horizontal pipe attached to the bottom of the pool. Determine the maximum discharge rate of water through the pipe. Also, explain why the actual flow rate will be less.

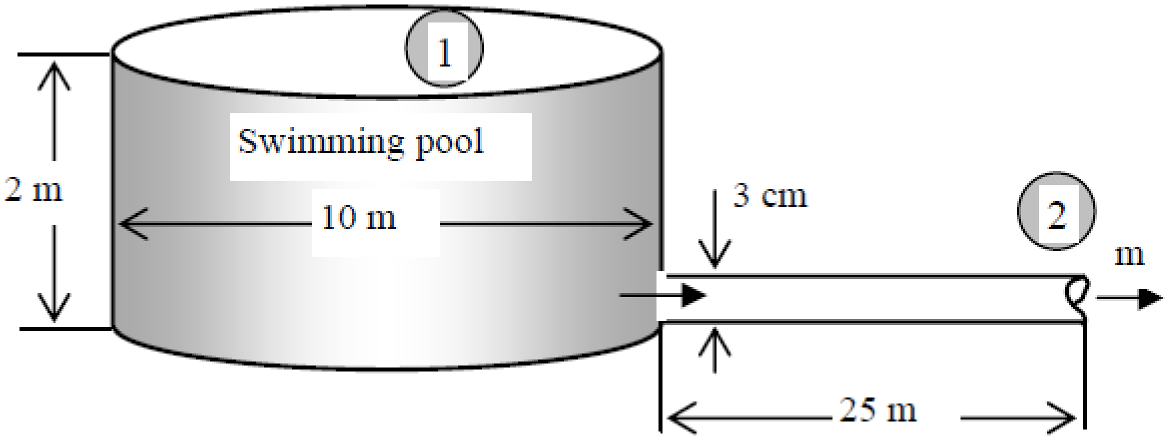
Solution: We take point 1 at the free surface of the pool, and point 2 at the exit of pipe. We take the reference level at the pipe exit ($z_2 = 0$). Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P$atm) and that the fluid velocity at the free surface is very low ($V_1 \cong 0$), the Bernoulli equation between these two points simplifies to

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \rightarrow \quad z_1 = \frac{V_2^2}{2g} \quad \rightarrow \quad V_2 = \sqrt{2gz_1}$$

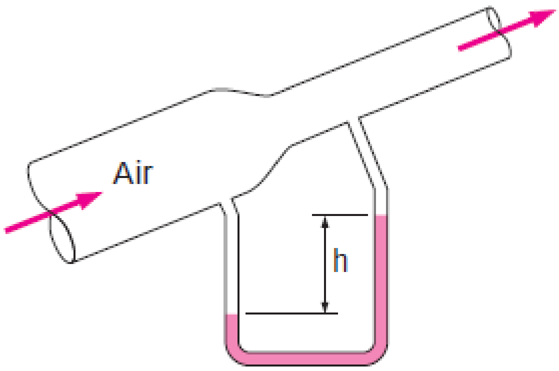
The maximum discharge rate occurs when the water height in the pool is a maximum, which is the case at the beginning and thus $z_1 = h$. Substituting, the maximum flow velocity and discharge rate become

$$V_{2,\max} = \sqrt{2gh} = \sqrt{2(9.81\text{ m/s}^2)(2\text{ m})} = 6.26\text{ m/s}$$

$$\dot{V}_{\max} = A_{\text{pipe}} V_{2,\max} = \frac{\pi D^2}{4} V_{2,\max} = \frac{\pi(0.03\text{ m})^2}{4}(6.26\text{ m/s}) = 0.00443\text{ m}^3/\text{s} = \mathbf{4.43\ L/s}$$



Example: Air at 110 kPa and 50°C flows upward through a 6-cm-diameter inclined duct at a rate of 45 L/s. The duct diameter is then reduced to 4 cm through a reducer. The pressure change across the reducer is measured by a water manometer. The elevation difference between the two points on the pipe where the two arms of the manometer are attached is 0.20 m. Determine the differential height between the fluid levels of the two arms of the manometer. We take the density of water to be ρ = 1000 kg/m^3. The gas constant of air is R = 0.287 kPa·m3/kg·K.



Solution: We take points 1 and 2 at the lower and upper connection points, respectively, of the two arms of the manometer, and take the lower connection point as the reference level. Noting that the effect of elevation on the pressure change of a gas is negligible, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad \rightarrow \quad P_1-P_2=\rho_{\text{air}}\frac{V_2^2-V_1^2}{2}$$

$$\rho_{air}=\frac{P}{RT}=\frac{110\text{ kPa}}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(50+273\text{ K})}=1.19\text{ kg/m}^3$$

$$V_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{\pi D_1^2/4} = \frac{0.045\ \text{m}^3/\text{s}}{\pi(0.06\ \text{m})^2/4} = 15.9\ \text{m/s}$$

$$V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\pi D_2^2/4} = \frac{0.045\ \text{m}^3/\text{s}}{\pi(0.04\ \text{m})^2/4} = 35.8\ \text{m/s}$$

Substituting,

$$P_1 - P_2 = (1.19\ \text{kg/m}^3)\frac{(35.8\ \text{m/s})^2 - (15.9\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 612\ \text{N/m}^2 = 612\ \text{Pa}$$

The differential height of water in the manometer corresponding to this pressure change is determined from $\Delta P = \rho_{water} g h$

to be

$$h = \frac{P_1 - P_2}{\rho_{water} g} = \frac{612\ \text{N/m}^2}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = 0.0624\ \text{m} = \mathbf{6.24\ cm}$$

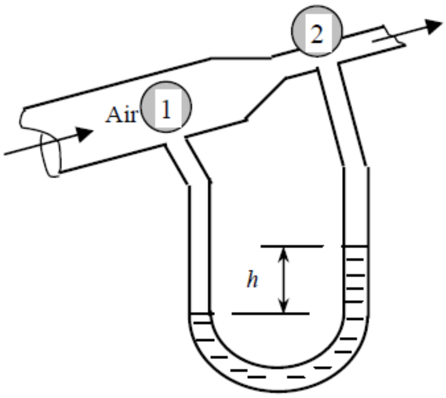
When the effect of air column on pressure change is considered, the pressure change becomes

$$P_1 - P_2 = \frac{\rho_{air}(V_2^2 - V_1^2)}{2} + \rho_{air} g (z_2 - z_1)$$

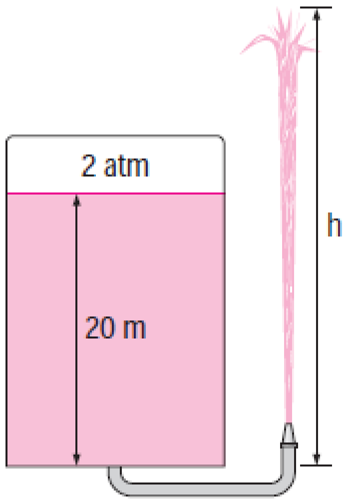
$$= (1.19\ \text{kg/m}^3)\left[\frac{(35.8\ \text{m/s})^2 - (15.9\ \text{m/s})^2}{2} + (9.81\ \text{m/s}^2)(0.2\ \text{m})\right]\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= (612 + 2)\ \text{N/m}^2 = 614\ \text{N/m}^2 = 614\ \text{Pa}$$

This difference between the two results (612 and 614 Pa) is less than 1%. Therefore, the effect of air column on pressure change is, indeed, negligible as assumed. In other words, the pressure change of air in the duct is almost entirely due to velocity change, and the effect of elevation change is negligible. Also, if we were to account for the Δz of air flow, then it would be more proper to account for the Δz of air in the manometer by using ρ_{water} - ρ_{air} instead of ρ_{water} when calculating *h*. The additional air column in the manometer tends to cancel out the change in pressure due to the elevation difference in the flow in this case.



Example: The water level in a tank is 20 m above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The tank cover is airtight, and the air pressure above the water surface is 2 atm gage. The system is at sea level. Determine the maximum height to which the water stream could rise. We take the density of water to be 1000 kg/m^3.



Solution: We take point 1 at the free surface of water in the tank, and point 2 at the top of the water trajectory. Also, we take the reference level at the bottom of the tank. At the top of the water trajectory $\mathbf{V}_2 = 0$, and atmospheric pressure pertains. Noting that $z_1 = 20$ m, $P_{1,gage} = 2$ atm, $P_2 = P_{atm}$, and that the fluid velocity at the free surface of the tank is very low ($V_1 \cong 0$), the Bernoulli equation between these two points simplifies to

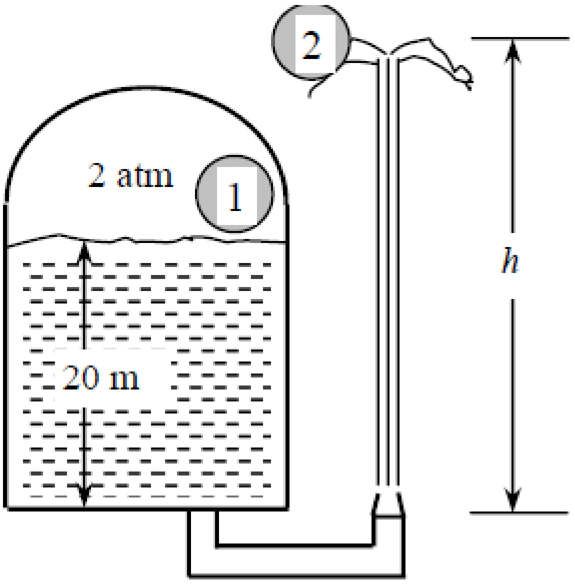
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{P_1}{\rho g} + z_1 = \frac{P_{atm}}{\rho g} + z_2$$

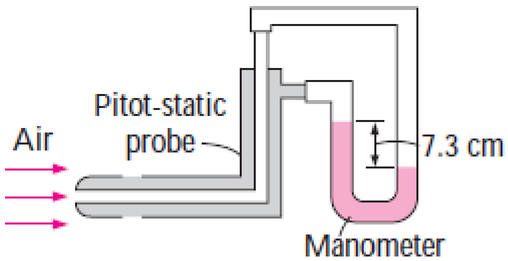
$$z_2 = \frac{P_1 - P_{atm}}{\rho g} + z_1 = \frac{P_{1,gage}}{\rho g} + z_1$$

Substituting,

$$z_2 = \frac{2\text{ atm}}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)}\left(\frac{101{,}325\text{ N/m}^2}{1\text{ atm}}\right)\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) + 20 = \mathbf{40.7\text{ m}}$$



Example: A Pitot-static probe connected to a water manometer is used to measure the velocity of air. If the deflection (the vertical distance between the fluid levels in the two arms) is 7.3 cm, determine the air velocity. Take the density of air to be 1.25 kg/m^3. We take the density of water to be ρ = 1000 kg/m^3. The density of air is given to be 1.25 kg/m^3.



Solution: We take point 1 on the side of the probe where the entrance is parallel to flow and is connected to the static arm of the Pitot-static probe, and point 2 at the tip of the probe where the entrance is normal to flow and is connected to the dynamic arm of the Pitot-static probe. Noting that point 2 is a stagnation point and thus $V_2 = 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

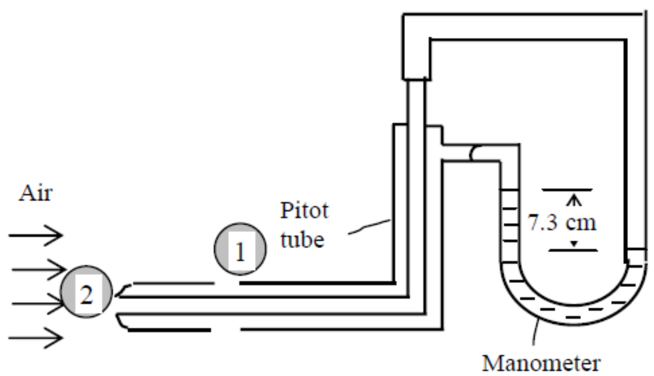
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} \qquad V_1 = \sqrt{\frac{2(P_2 - P_1)}{\rho_{air}}} \qquad (1)$$

The pressure rise at the tip of the Pitot-static probe is simply the pressure change indicated by the differential water column of the manometer,

$$P_2 - P_1 = \rho_{\text{water}} g h \qquad (2)$$

Combining Eqs. (1) and (2) and substituting, the flow velocity is determined to be

$$V_1 = \sqrt{\frac{2\rho_{\text{water}} g h}{\rho_{\text{air}}}} = \sqrt{\frac{2(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.073\ \text{m})}{1.25\ \text{kg/m}^3}} = \mathbf{33.8\ m/s}$$



Example: In a hydroelectric power plant, water enters the türbine nozzles at 700 kPa absolute with a low velocity. If the nozzle outlets are exposed to atmospheric pressure of 100 kPa, determine the maximum velocity to which water can be accelerated by the nozzles before striking the turbine blades. We take the density of water to be $\rho = 1000$ kg/m^3.

Solution: We take points 1 and 2 at the inlet and exit of the nozzle, respectively. Noting that $V_1 \cong 0$ and $z_1 = z_2$, the application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

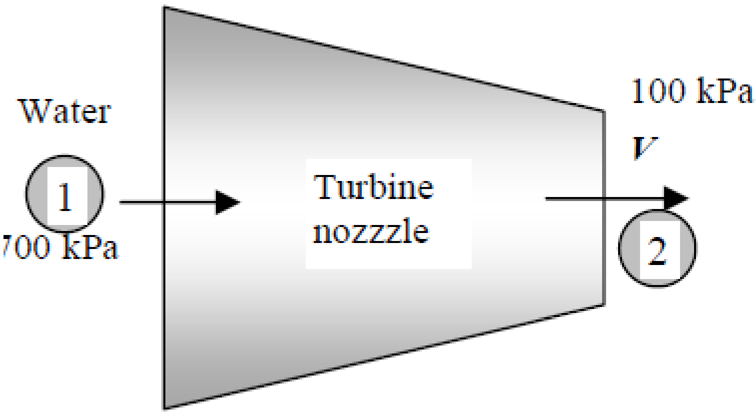
$$\frac{P_1}{\rho g} = \frac{P_{atm}}{\rho g} + \frac{V_2^2}{2g}$$

$$V_2 = \sqrt{\frac{2(P_1 - P_{atm})}{\rho}}$$

Substituting the given values, the nozzle exit velocity is determined to be

$$V_1 = \sqrt{\frac{2(700-100)\ \text{kPa}}{1000\ \text{kg/m}^3}\left(\frac{1000\ \text{N/m}^2}{1\ \text{kPa}}\right)\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)} = \mathbf{34.6\ m/s}$$

This is the maximum nozzle exit velocity, and the actual velocity will be less because of friction between water and the walls of the nozzle.



5. FLOW IN PIPES

Liquid or gas flow through *pipes* or *ducts* is commonly used in heating and cooling applications and fluid distribution networks. The fluid in such applications is usually forced to flow by a fan or pump through a flow section. We pay particular attention to *friction,* which is directly related to the *pressure drop* and *head loss* during flow through pipes and ducts. The pressure drop is then used to determine the pumping power requirement. A typical piping system involves pipes of different diameters connected to each other by various fittings or elbows to route the fluid, valves to control the flow rate, and pumps to pressurize the fluid.

The terms *pipe, duct*, and *conduit* are usually used interchangeably for flow sections. In general, flow sections of circular cross section are referred to as *pipes* (especially when the fluid is a liquid), and flow sections of noncircular cross section as *ducts* (especially when the fluid is a gas). Smalldiameter pipes are usually referred to as *tubes.* Given this uncertainty, we will use more descriptive phrases (such as *a circular pipe* or *a rectangular duct*) whenever necessary to avoid any misunderstandings.

You have probably noticed that most fluids, especially liquids, are transported in *circular pipes.* This is because pipes with a circular cross section can withstand large pressure differences between the inside and the outside without undergoing significant distortion. *Noncircular pipes* are usually used in applications such as the heating and cooling systems of buildings where the pressure difference is relatively small, the manufacturing and installation costs are lower, and the available space is limited for ductwork (Fig.5.1).

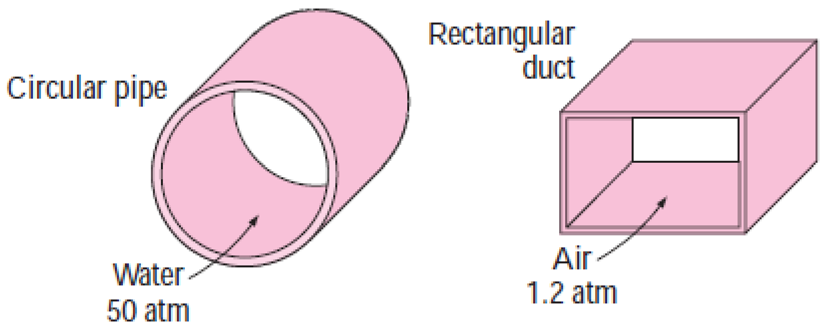


Figure 5.1. Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any significant distortion, but noncircular pipes cannot

The fluid velocity in a pipe changes from *zero* at the surface because of the no-slip condition to a maximum at the pipe center. In fluid flow, it is convenient to work with an *average* velocity V_{avg}, which remains constant in incompressible flow when the cross-sectional area of the pipe is constant (Fig.5.2). The average velocity in heating and cooling applications may change somewhat because of changes in density with temperature. But, in practice, we evaluate the fluid properties at some average temperature and treat them as constants. The convenience of working with constant properties usually more than justifies the slight loss in accuracy.

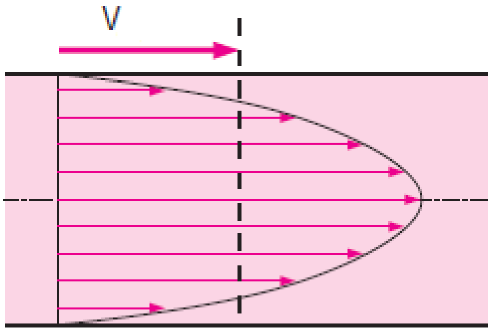


Figure 5.2. Average velocity *V* is defined as the average speed through a cross section. For fully developed laminar pipe flow, *V* is half of maximum velocity.

5.1 General Characteristics of Pipe Flow

Before we apply the various governing equations to pipe flow examples, we will discuss some of the basic concepts of pipe flow. With these ground rules established we can then proceed to formulate and solve various important flow problems.

Although not all conduits used to transport fluid from one location to another are round in cross section, most of the common ones are. These include typical water pipes, hydraulic hoses, and other conduits that are designed to withstand a considerable pressure difference across their walls without undue distortion of their shape. Typical conduits of noncircular cross section include heating and air conditioning ducts that are often of rectangular cross section. Normally the pressure difference between the inside and outside of these ducts is relatively small. Most of the basic principles involved are independent of the cross-sectional shape, although the details of the flow may be dependent on it. Unless otherwise specified, we will assume that the conduit is round, although we will show how to account for other shapes.

For all flows involved in this chapter, we assume that the pipe is completely filled with the fluid being transported as is shown in Fig.5.3*a*. Thus, we will not consider a concrete pipe through which rainwater flows without completely filling the pipe, as is shown in Fig.5.3*b*. The difference between open-channel flow and the pipe flow of this chapter is in the fundamental mechanism that drives the flow. For open-channel flow, gravity alone is the driving force—the water flows down a hill. For pipe flow, gravity may be important (the pipe need not be horizontal), but the main driving force is likely to be a pressure gradient along the pipe. If the pipe is not full, it is not possible to maintain this pressure difference, P_1-P_2.

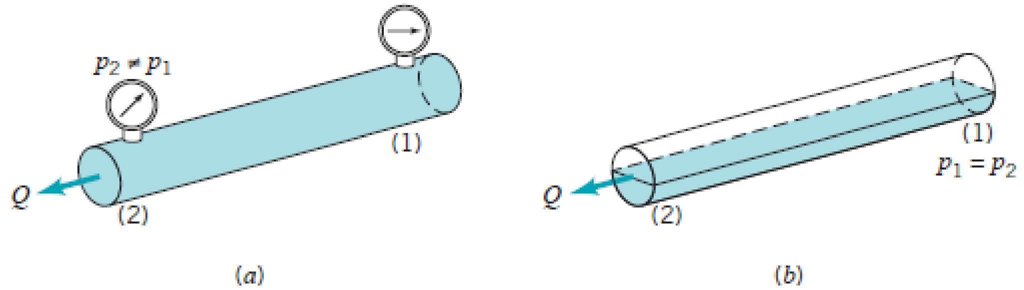


Figure 5.3. (*a*) Pipe flow. (*b*) Open-channel flow.

5.1.1. Laminar or Turbulent Flow

The flow of a fluid in a pipe may be laminar flow or it may be turbulent flow. Osborne Reynolds 11842–19122, a British scientist and mathematician, was the first to distinguish the difference between these two classifications of flow by using a simple apparatus as shown by the figure, which is a sketch of Reynolds' dye experiment. Reynolds injected dye into a pipe in which water flowed due to gravity. The entrance region of the pipe is depicted in Fig.5.4*a*. If water runs through a pipe of diameter *D* with an average velocity *V*, the following characteristics are observed by injecting neutrally buoyant dye as shown. For "small enough flowrates" the dye streak (a streakline) will remain as a well-defined line as it flows along, with only slight blurring due to molecular diffusion of the dye into the surrounding water. For a somewhat larger "intermediate flowrate" the dye streak fluctuates in time and space, and intermittent bursts of irregular behavior appear along the streak. On the other hand, for "large enough flowrates" the dye streak almost immediately becomes blurred and spreads across the entire pipe in a random fashion. These three characteristics, denoted as ***laminar***, ***transitional***, and ***turbulent*** flow, respectively, are illustrated in Fig. 5.4*b*.

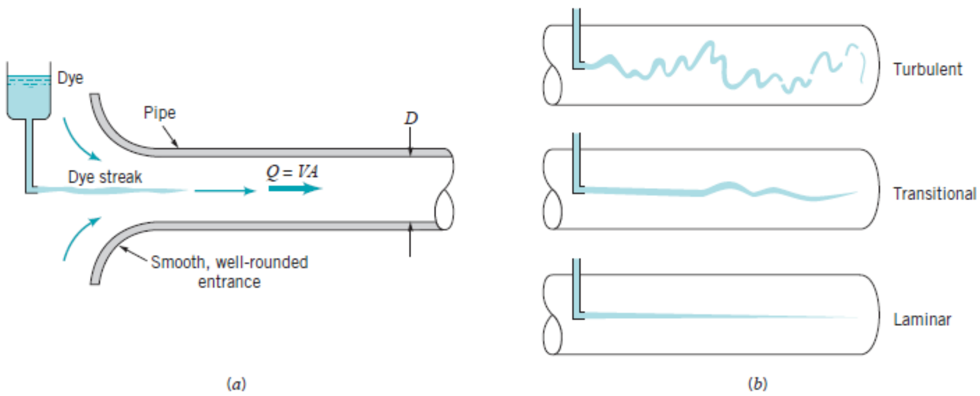


Figure 5.4. (*a*) Experiment to illustrate type of flow. (*b*) Typical dye streaks.

The curves shown in the below Fig.5.5 represent the *x* component of the velocity as a function of time at a point *A* in the flow. The random fluctuations of the turbulent flow (with the associated particle mixing) are what disperse the dye throughout the pipe and cause the blurred appearance illustrated in Fig. 5.5*b*. For laminar flow in a pipe there is only one component of velocity, $\boldsymbol{V} = u\hat{\boldsymbol{\imath}}$. For turbulent flow the predominant component of velocity is also along the pipe, but it is unsteady (random) and accompanied by random components normal to the pipe axis, $\boldsymbol{V} = u\hat{\boldsymbol{\imath}} + v\hat{\boldsymbol{\jmath}} + w\hat{\boldsymbol{k}}$. Such motion in a typical flow occurs too fast for our eyes to follow. Slow motion pictures of the flow can more clearly reveal the irregular, random, turbulent nature of the flow.

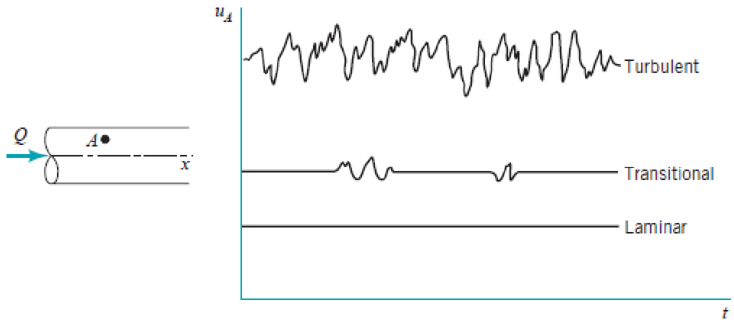


Figure 5.5. Time dependence of fluid velocity at a point.

The transition from laminar to turbulent flow depends on the ***geometry, surface roughness, flow velocity, surface temperature,*** **and *type of fluid,*** among other things. After exhaustive experiments in the 1880s, Osborne Reynolds discovered that the flow regime depends mainly on the ratio of *inertial forces* to *viscous forces* in the fluid. This ratio is called the ***Reynolds number*** and is expressed for internal flow in a circular pipe as

$$Re = \frac{Inertial\ forces}{Viscous\ forces} = \frac{VD}{\nu} = \frac{\rho VD}{\mu}$$

Where V is the average flow velocity (m/s), *D is the* characteristic length of the geometry (diameter in this case, in m), and $\nu = \mu/\rho$ is the kinematic viscosity of the fluid (m^2/s). Note that the Reynolds number is a *dimensionless* quantity. Also, kinematic viscosity has the unit m^2/s, and can be viewed as ***viscous diffusivity*** or ***diffusivity for momentum***.

The Reynolds number ranges for which laminar, transitional, or turbulent pipe flows are obtained cannot be precisely given. The actual transition from laminar to turbulent flow may take place at various Reynolds numbers, depending on how much the flow is disturbed by vibrations of the pipe, roughness of the entrance region, and the like. For general engineering purposes (i.e., without undue precautions to eliminate such disturbances), the following values are appropriate:

The flow in a round pipe is laminar if the Reynolds number is less than approximately 2100. The flow in a round pipe is turbulent if the Reynolds number is greater than approximately 4000. For Reynolds numbers between these two limits, (2100<Re<4000) the flow may switch between laminar and turbulent conditions in an apparently random fashion (transitional flow).

$Re \leq 2100 \rightarrow Laminar\ flow$
$2100 < Re < 4000 \rightarrow Transitional\ flow$
$Re \geq 4000 \rightarrow Turbulent\ flow$

5.1.2 Entrance Region and Fully Developed Flow

Any fluid flowing in a pipe had to enter the pipe at some location. The region of flow near where the fluid enters the pipe is termed the *entrance region* and is illustrated in Fig.5.6. It may be the first few feet of a pipe connected to a tank or the initial portion of a long run of a hot air duct coming from a furnace.

As is shown in Fig.5.6, the fluid typically enters the pipe with a nearly uniform velocity profile at section (1). As the fluid moves through the pipe, viscous effects cause it to stick to the pipe wall (the no-slip boundary condition). This is true whether the fluid is relatively inviscid air or a very viscous oil. Thus, a *boundary layer* in which viscous effects are important is produced along the pipe wall such that the initial velocity profile changes with distance along the pipe, *x*, until the fluid reaches the end of the entrance length, section (2), beyond which the velocity profile does not vary with *x*.

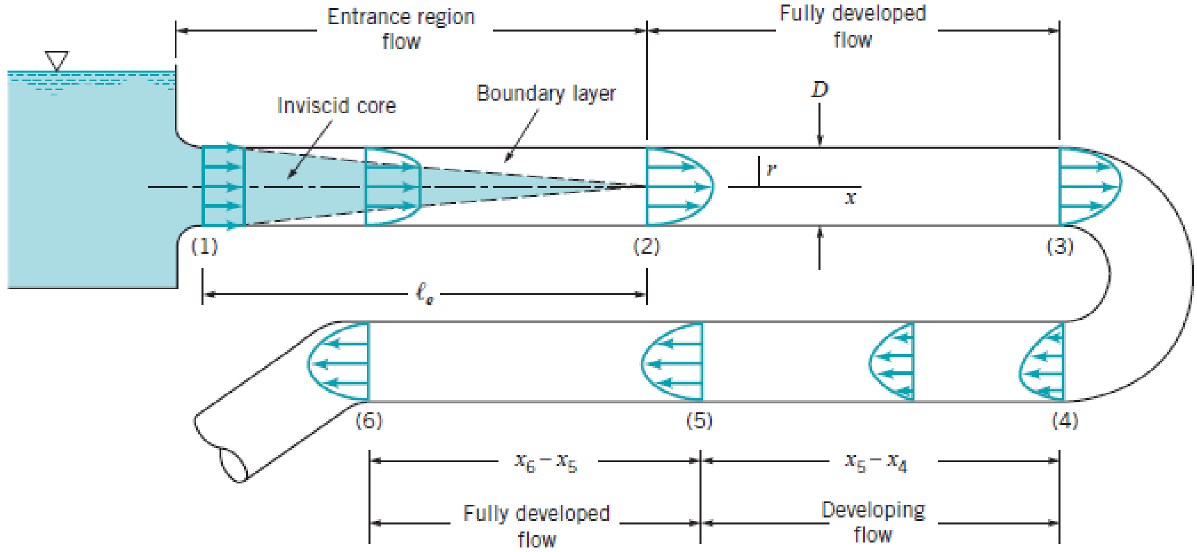


Figure 5.6. Entrance region, developing flow, and fully developed flow in a pipe system.

The boundary layer has grown in thickness to completely fill the pipe. Viscous effects are of considerable importance within the boundary layer. For fluid outside the boundary layer [within the *inviscid core* surrounding the centerline from (1) to (2)], viscous effects are negligible.

The shape of the velocity profile in the pipe depends on whether the flow is laminar or turbulent, as does the length of the entrance region, L_e. As with many other properties of pipe flow, the dimensionless ***entrance length,*** L_e/D correlates quite well with the Reynolds number. Typical entrance lengths are given by

$Le/D=0.06\ Re$ for laminar flow

$Le/D=4.4\ (Re)^{1/6}$ for turbulent flow

For very low Reynolds number flows the entrance length can be quite short ($Le=0.6D$ *if* $Re=10$) if whereas for large Reynolds number flows it may take a length equal to many pipe diameters before the end of the entrance region is reached ($Le=120D$ *for* $Re=2000$). For many practical engineering problems, $20D < Le < 30D$ for $10^4 < Re < 10^5$.

Calculation of the velocity profile and pressure distribution within the entrance region is quite complex. However, once the fluid reaches the end of the entrance region, section (2) of Fig.5.6, the flow is simpler to describe because the velocity is a function of only the distance from the pipe centerline, r, and independent of x. This is true until the character of the pipe changes in some way, such as a change in diameter, or the fluid flows through a bend, valve, or some other component at section (3). The flow between (2) and (3) is termed ***fully developed flow.*** Beyond the interruption of the fully developed flow [at section (4)], the flow gradually begins its return to its fully developed character [section (5)] and continues with this profile until the next pipe system component is reached [section (6)]. In many cases the pipe is long enough so that there is a considerable length of fully developed flow compared with the developing flow length $[(x_3 - x_2) \gg L_e \ and\ (x_6 - x_5) \gg (x_5 - x_4)]$. In other cases, the distances between one component (bend, tee, valve, etc.) of the pipe system and the next component is so short that fully developed flow is never achieved.

5.1.3. Pressure and Shear Stress

Fully developed steady flow in a constant diameter pipe may be driven by gravity and/or pressure forces. For horizontal pipe flow, gravity has no effect except for a hydrostatic pressure variation across the pipe, $\gamma D,$ that is usually negligible. It is the pressure difference, $\Delta P = P_1 - P_2$ between one section of the horizontal pipe and another which forces the fluid through the pipe. Viscous effects provide the restraining force that exactly balances the pressure force, thereby allowing the fluid to flow through the pipe with no acceleration. If viscous effects were absent in such flows, the pressure would be constant throughout the pipe, except for the hydrostatic variation.

In non-fully developed flow regions, such as the entrance region of a pipe, the fluid accelerates or decelerates as it flows (the velocity profile changes from a uniform profile at the entrance of the pipe to its fully developed profile at the end of the entrance region). Thus, in the entrance region there is a balance between pressure, viscous, and inertia (acceleration) forces. The result is a pressure distribution along the horizontal pipe as shown in Fig.5.7. The magnitude of the pressure gradient, $\frac{\partial P}{\partial x}$, is larger in the entrance region than in the fully developed region, where it is a constant, $\frac{\partial P}{\partial x} = -\frac{\Delta P}{L} < 0$.

The fact that there is a nonzero pressure gradient along the horizontal pipe is a result of viscous effects. If the viscosity were zero, the pressure would not vary with *x*. The need for the pressure drop can be viewed from two different standpoints. In terms of a force balance, the pressure force is needed to overcome the viscous forces generated. In terms of an energy balance, the work done by the pressure force is needed to overcome the viscous dissipation of energy throughout the fluid. If the pipe is not horizontal, the pressure gradient along it is due in part to the component of weight in that direction. This contribution due to the weight either enhances or retards the flow, depending on whether the flow is downhill or uphill.

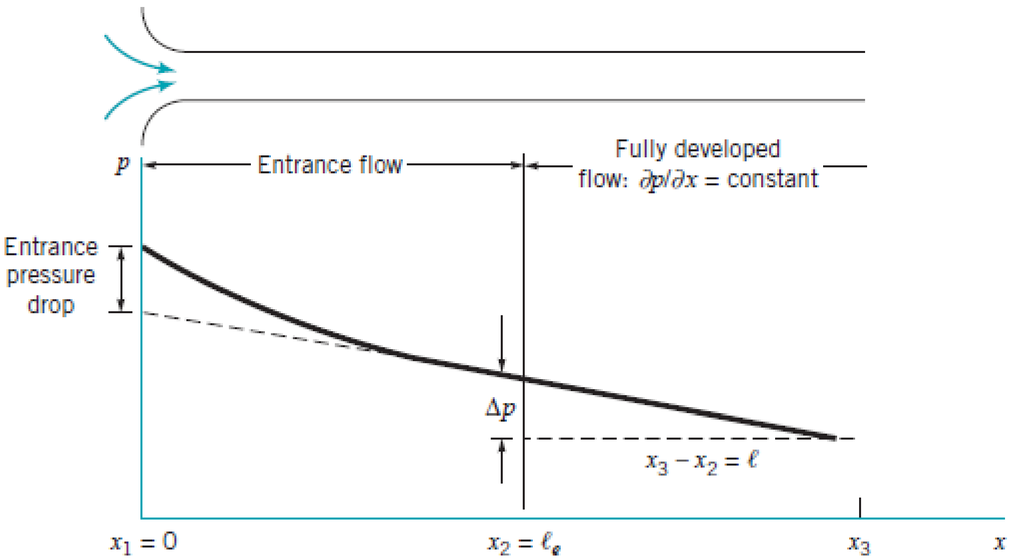


Figure 5.7. Pressure distribution along a horizontal pipe.

The nature of the pipe flow is strongly dependent on whether the flow is laminar or turbulent. This is a direct consequence of the differences in the nature of the shear stress in laminar and turbulent flows. The shear stress in laminar flow is a direct result of momentum transfer among the randomly moving molecules (a microscopic phenomenon). The shear stress in turbulent flow is largely a result of momentum transfer among the randomly moving, finite-sized fluid particles (a macroscopic phenomenon). The net result is that the physical properties of the shear stress are quite different for laminar flow than for turbulent flow.

5.2. Fully Developed Laminar Flow

We mentioned that flow in pipes is laminar for $Re \leq 2100$, and that the flow is fully developed if the pipe is sufficiently long (relative to the entry length) so that the entrance effects are negligible. In this section we consider the steady laminar flow of an incompressible fluid with constant properties in the fully developed region of a straight circular pipe. We obtain the momentum equation by applying a momentum balance to a differential volume element, and obtain the velocity profile by solving it. Then we use it to obtain a relation for the friction factor. An important aspect of the analysis here is that it is one of the few available for viscous flow.

In fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline and the velocity profile $u(r)$ remains unchanged in the flow direction. There is no motion in the radial direction, and thus the velocity component in the direction normal to flow is everywhere zero. There is no acceleration since the flow is steady and fully developed., denoted τ_w, the ***wall shear stress*** (Fig.5.8). Hence, the shear stress distribution throughout the pipe is a linear function of the radial coordinate. The following equations can be written for laminar flow. Although we are discussing laminar flow, a closer consideration of the assumptions involved in the derivation of below Eqs. r*eveals that these equations are valid for both laminar and turbulent flow.*

$$\frac{\Delta p}{\ell} = \frac{2\tau}{r} \qquad \tau = \frac{2\tau_w r}{D} \qquad \Delta p = \frac{4\ell\tau_w}{D}$$

Where: ΔP is the pressure difference (Pa), τ is the shear stress N/m^2, l is the length of pipe, r is the radial coordinate, τ_w is the *wall shear stress* (Pa) **(**At r = D/2 (the pipe wall) the shear stress is a maximum), D is the pipe diameter (m)

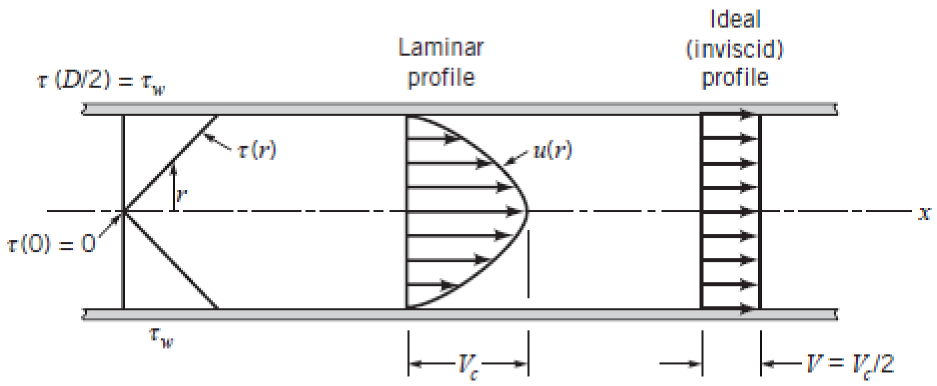


Figure 5.8. Shear stress distribution within the fluid in a pipe (laminar or turbulent flow) and typical velocity profiles

For laminar flow of a Newtonian fluid, the shear stress is simply proportional to the velocity gradient. In the notation associated with our pipe flow, this becomes

$$\tau = -\mu \frac{du}{dr}$$

The negative sign is included to give $\tau > 0$ with du/dr<0 (the velocity decreases from the pipe centerline to the pipe wall).

The velocity profile can be written as

$$u(r) = \left(\frac{\Delta p D^2}{16\mu\ell}\right)\left[1 - \left(\frac{2r}{D}\right)^2\right] = V_c\left[1 - \left(\frac{2r}{D}\right)^2\right]$$

$$u(r) = \mathbf{V_c}\left(1 - \frac{r^2}{R^2}\right)$$

Where; $V_c = \Delta P D^2/(16\mu l)$ is the centerline velocity. An alternative expression can be written by using the relationship between the wall shear stress and the pressure gradient to give

$$u(r) = \frac{\tau_w D}{4\mu}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Where R=D/2 is the pipe radius.

The flowrate is can be written as follow.

$$V = \frac{\pi R^2 V_c}{2\pi R^2} = \frac{V_c}{2} = \frac{\Delta p D^2}{32\mu\ell}$$

$$Q = \frac{\pi D^4 \Delta p}{128\mu\ell}$$

The *maximum velocity* in fully developed laminar flow in a circular pipe are $V_c = 2V$

The equations for nonhorizontal pipes (Fig.5.9)

$$\frac{\Delta p - \gamma\ell \sin\theta}{\ell} = \frac{2\tau}{r}$$

Thus, all of the results for the horizontal pipe are valid provided the pressure gradient is adjusted for the elevation term, that is, ΔP *is* replaced by $\Delta P - \gamma l sin\theta$ so that

$$V = \frac{(\Delta p - \gamma\ell \sin\theta)D^2}{32\mu\ell}$$

$$Q = \frac{\pi(\Delta p - \gamma\ell \sin\theta)D^4}{128\mu\ell}$$

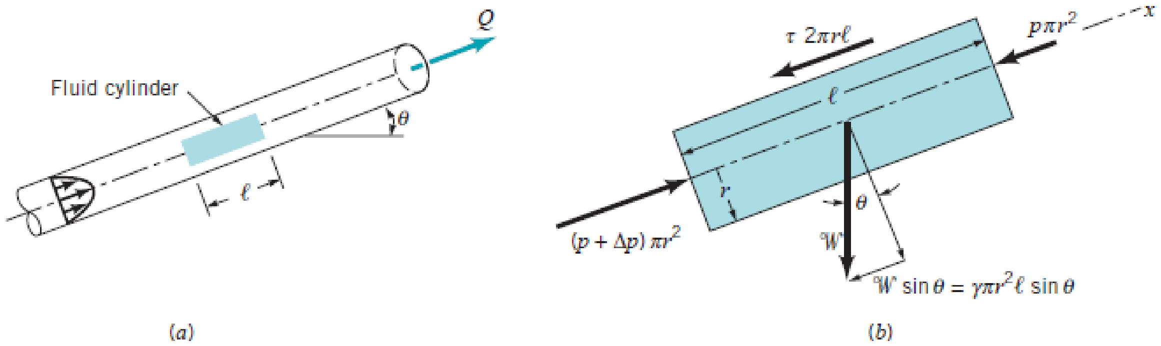


Figure 5.9. Free-body diagram of a fluid cylinder for flow in a nonhorizontal pipe.

It is seen that the driving force for pipe flow can be either a pressure drop in the flow direction, ΔP, or the component of weight in the flow direction, $-\gamma l sin\theta$. If the flow is downhill, gravity helps the flow (a smaller pressure drop is required; $sin\theta < 0$). If the flow is uphill, gravity works against the flow (a larger pressure drop is required; $sin\theta > 0$) . Note that $\gamma l sin\theta = \gamma\Delta z$ (where Δz *is* the change in elevation) is a hydrostatic type pressure term. If there is no flow V=0 and $\Delta P = \gamma l sin\theta = \gamma\Delta z$, as expected for fluid statics (Fig 5.9).

It is usually advantageous to describe a process in terms of dimensionless quantities. To this end we rewrite the pressure drop equation for laminar horizontal pipe flow, as $\Delta P = 32\mu l V/D^2$ and divide both sides by the dynamic pressure, $\rho V^2/2$ obtain the dimensionless form as

$$\Delta p = f\frac{\ell}{D}\frac{\rho V^2}{2} \qquad f = \Delta p(D/\ell)/(\rho V^2/2)$$

is termed the ***friction factor,*** or sometimes the *Darcy friction factor (*This parameter should not be confused with the less-used Fanning friction. which is defined to be f/4. In this text we will use only the Darcy fiction factor.) Thus the friction factor for laminar fully developed pipe flow is simply f= 64/Re. By substituting the pressure drop in terms of the wall shear stress, we obtain an alternate expression for the friction factor as a dimensionless wall shear stress

$$f = \frac{8\tau_w}{\rho V^2}$$

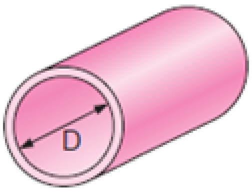
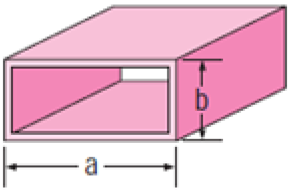
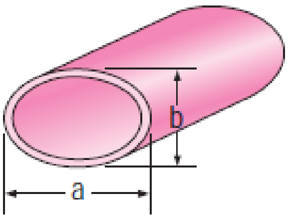
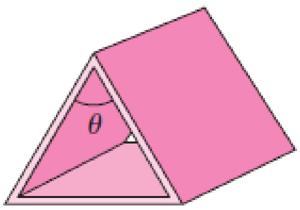
Knowledge of the friction factor will allow us to obtain a variety of information regarding pipe flow. For turbulent flow the dependence of the friction factor on the Reynolds number is much more complex than that given by f=64/Re for laminar flow.

5.2.1. Laminar Flow in Noncircular Pipes

The friction factor *f* relations are given in the below Table 5.1. for *fully developed laminar flow* in pipes of various cross sections. The Reynolds number for flow in these pipes is based on the hydraulic diameter $D_h=4Ac/p$, where *Ac* is the cross-sectional area of the pipe and *p* is its wetted perimeter.

Table 5.1. Friction factor for fully developed laminar flow

Friction factor for fully developed *laminar flow* in pipes of various cross sections ($D_h = 4A_c/p$ and $\text{Re} = V_{avg}\, D_h/\nu$)

Tube Geometry	a/b or $\theta°$	Friction Factor f
Circle	—	64.00/Re
		
Rectangle	a/b	
	1	56.92/Re
	2	62.20/Re
	3	68.36/Re
	4	72.92/Re
	6	78.80/Re
	8	82.32/Re
	∞	96.00/Re
Ellipse	a/b	
	1	64.00/Re
	2	67.28/Re
	4	72.96/Re
	8	76.60/Re
	16	78.16/Re
Isosceles triangle	θ	
	10°	50.80/Re
	30°	52.28/Re
	60°	53.32/Re
	90°	52.60/Re
	120°	50.96/Re

5.3. Fully Developed Turbulent Flow

Consider a long section of pipe that is initially filled with a fluid at rest. As the valve is opened to start the flow, the flow velocity and, hence, the Reynolds number increase from zero (no flow) to their maximum steady-state flow values, as is shown in Fig.5.10. Assume this transient process is slow enough so that unsteady effects are negligible (quasi-steady flow). For an initial time period the

Reynolds number is small enough for laminar flow to occur. At some time the Reynolds number reaches 2100, and the flow begins its transition to turbulent conditions. Intermittent spots or bursts of turbulence appear. As the Reynolds number is increased, the entire flow field becomes turbulent. The flow remains turbulent as long as the Reynolds number exceeds approximately 4000.

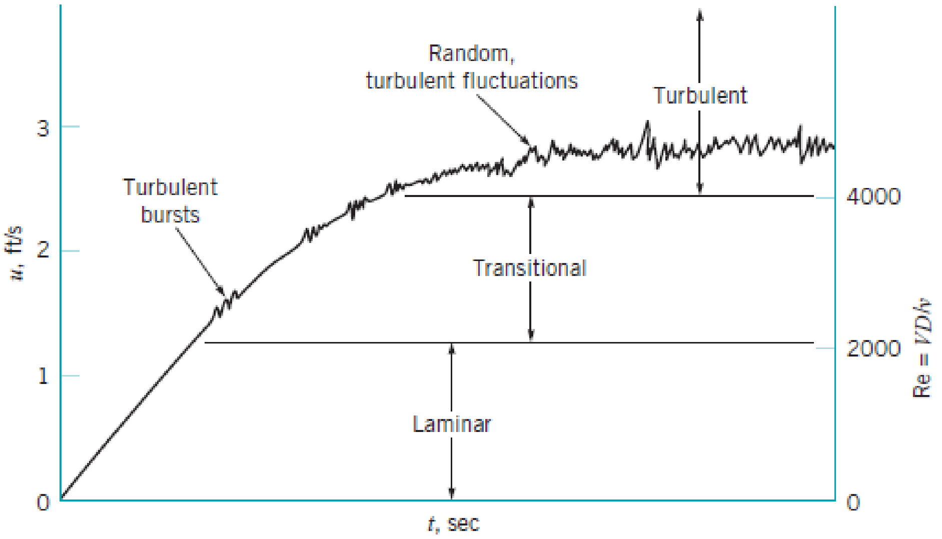


Figure 5.10. Transition from laminar to turbulent flow in a pipe.

A typical trace of the axial component of velocity measured at a given location in the flow, u=u(t), is shown in Fig.5.11. Its irregular, random nature is the distinguishing feature of turbulent flow. The character of many of the important properties of the flow (pressure drop, heat transfer, etc.) depends strongly on the existence and nature of the turbulent fluctuations or randomness indicated.

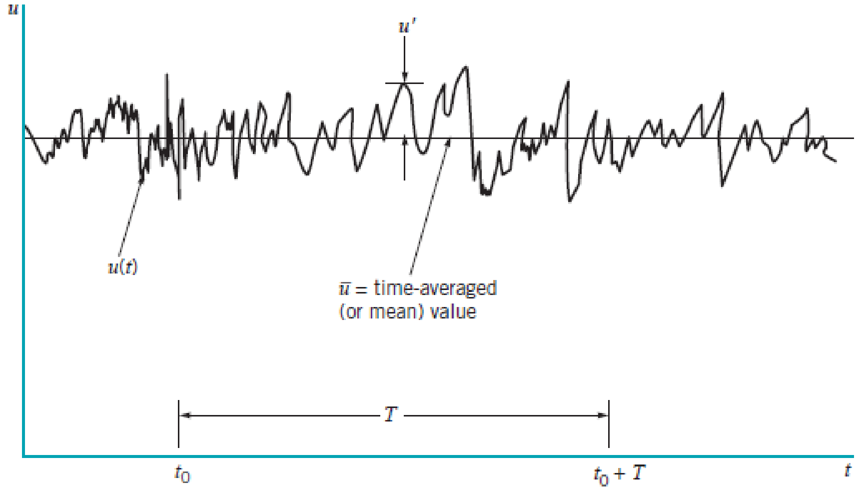


Figure 5.11. The time-averaged, $\overline{u}$,and fluctuating, u' , description of a parameter for turbulent flow

Turbulent flow shear stress is larger than laminar flow shear stress because of the irregular, random motion. The *total shear stress* in turbulent flow can be expressed as

$$\tau = \mu \frac{d\overline{u}}{dy} - \rho\overline{u'v'} = \tau_{\text{lam}} + \tau_{\text{turb}}$$

Note that if the flow is laminar $u' = v' = 0$, so that $\overline{u'v'} = 0$ and the above equation reduces o the customary random molecule-motion induced laminar shear stress, $\tau_{lam} = \mu d\bar{u}/\text{dy}$. For turbulent flow it is found that the turbulent shear stress, $\tau_{turb} = -\rho\overline{u'v'}$ is positive. Hence, the shear stress is greater in turbulent flow than in laminar flow.

Considerable information concerning turbulent velocity profiles has been obtained through the use of dimensional analysis, experimentation, numerical simulations, and semiempirical theoretical efforts. As is indicated in Fig.5.12, fully developed turbulent flow in a pipe can be broken into three regions which are characterized by their distances from the wall: the viscous sublayer very near the pipe wall, the overlap region, and the outer turbulent layer throughout the center portion of the flow.

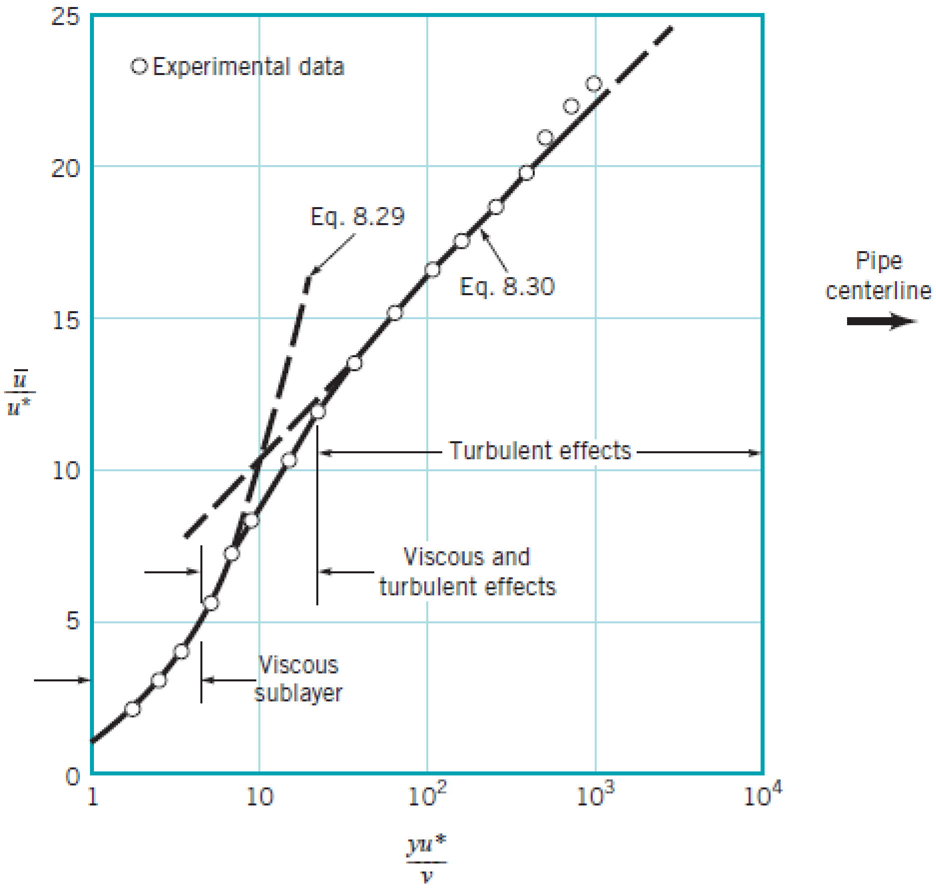


Figure 5.12. Typical structure of the turbulent velocity profile in a pipe

Within the viscous sublayer the viscous shear stress is dominant compared with the turbulent (or Reynolds) stress, and the random, eddying nature of the flow is essentially absent. In the outer turbulent layer the Reynolds stress is dominant, and there is considerable mixing and randomness to the flow. The character of the flow within these two regions is entirely different. For example, within the viscous sublayer the fluid viscosity is an important parameter; the density is unimportant. In the outer layer the opposite is true. By a careful use of dimensional analysis arguments for the flow in each layer and by a matching of the results in the common overlap layer, it has been possible to obtain the following conclusions about the turbulent velocity profile in a smooth pipe. In the viscous sublayer the velocity profile can be written in dimensionless form as

$$\frac{\bar{u}}{u^*} = \frac{yu^*}{\nu}$$

Where y=R-r is the distance measured from the wall, $\bar{u}$ is the time-averaged x component of velocity, and $u^* = (\frac{\tau_w}{\rho})^{1/2}$ is termed ***the friction velocity***. Note that u^* is not an actual velocity of the fluid-it is merely a quantity that has dimensions of velocity.

This equation is known as the ***law of the wall,*** and it is found to satisfactorily correlate with experimental data for smooth surfaces for $0 \leq yu^*/\nu \leq 5$. The viscous sublayer is usually quite thin. For viscous sublayer can be calculated by

$$0 \leq yu^*/\nu \leq 5$$

Thus, thickness of viscous sublayer: $y = \delta_{sublayer} = \frac{5\nu}{u^*}$ This is valid very near the smooth wall. We conclude that *the thickness of the viscous sublayer is proportional to the kinematic viscosity and inversely proportional to the average flow velocity*. In other words, the viscous sublayer is suppressed and it gets thinner as the velocity (and thus the Reynolds number) increases. Consequently, the velocity profile becomes nearly flat and thus the velocity distribution becomes more uniform at very high Reynolds numbers. The quantity ν/u^* has dimensions of length and is called the ***viscous length;*** it is used to nondimensionalize the distance y from the surface.

Dimensional analysis arguments indicate that in the overlap region the velocity should vary as the logarithm of y. Thus, the following expression has been proposed:

$$\frac{\bar{u}}{u^*} = 2.5 \ln\left(\frac{yu^*}{\nu}\right) + 5.0$$

Where the constants 2.5 and 5.0 have been determined experimentally. As is indicated in the above Fig.5.12, for regions not too close to the smooth wall, but not all the way out to the pipe center, The last equation gives a reasonable correlation with the experimental data. Note that the horizontal scale is a logarithmic scale. This tends to exaggerate the size of the viscous sublayer relative to the remainder of the flow. The viscous sublayer is usually quite thin. Similar results can be obtained for turbulent flow past rough walls (5.13).

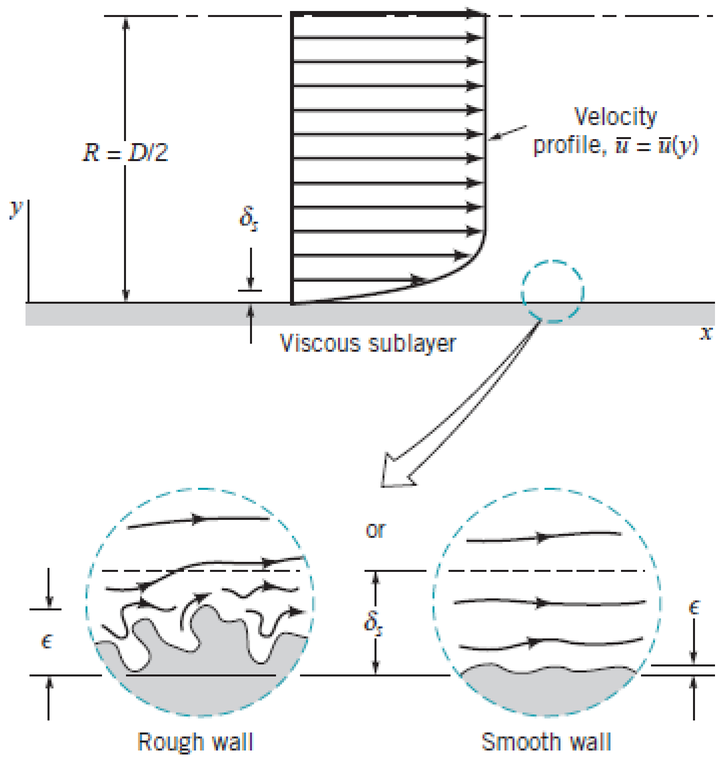


Figure 5.13. Flow in the viscous sublayer near rough and smooth walls.

A number of other correlations exist for the velocity profile in turbulent pipe flow. In the central region (the outer turbulent layer) the expression

$$\frac{V_c - \bar{u}}{u^*} = 2.5 \ln\left(\frac{R}{y}\right)$$

Where; V_c is the centerline velocity, is often suggested as a good correlation with experimental data. Another often-used (and relatively easy to use) correlation is the empirical *power-law velocity profile*

$$\frac{\bar{u}}{V_c} = \left(1 - \frac{r}{R}\right)^{1/n}$$

In this representation, the value of n is a function of the Reynolds number, as is indicated in below Fig.5.14.

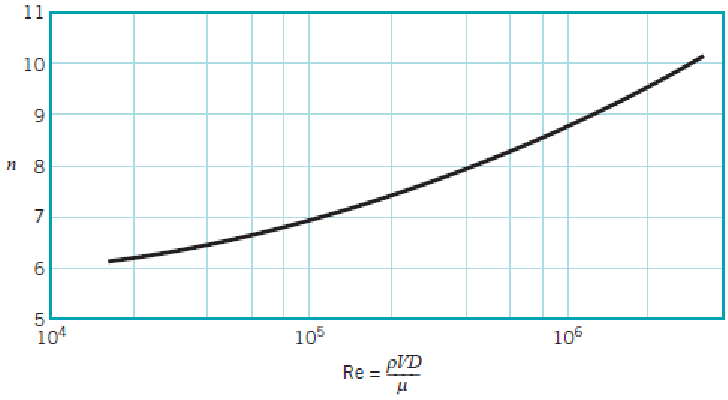


Figure 5.14. Exponent, *n*, for power-law velocity profiles.

The one-seventh power-law velocity profile (n=7) is often used as a reasonable approximation for many practical flows. Typical turbulent velocity profiles based on this power-law representation are shown in the below Fig.5.15.

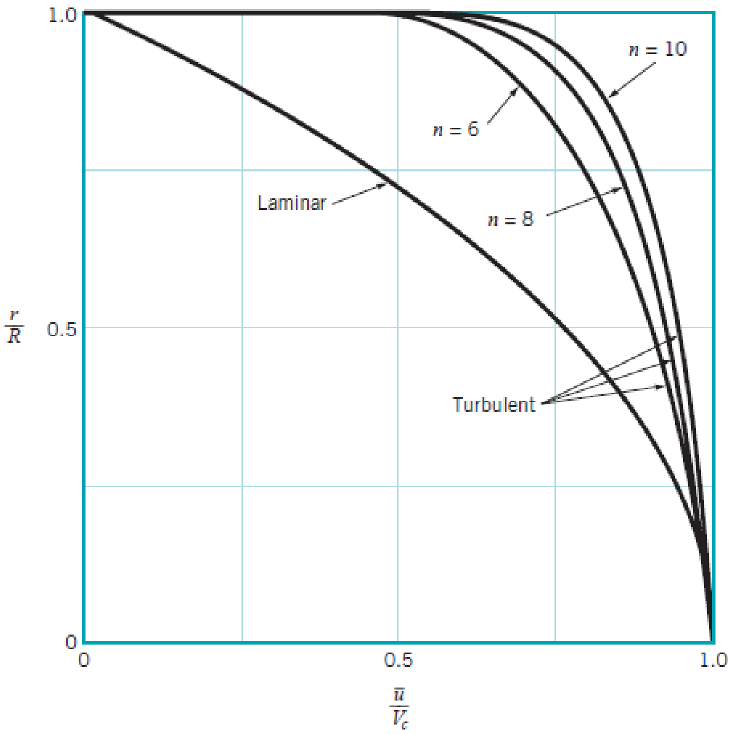


Figure 5.15. Typical laminar flow and turbulent flow velocity profiles.

Pressure Drop and Head Loss

The overall head loss for the pipe system consists of the head loss due to viscous effects in the straight pipes, termed the ***major loss*** and denoted, h_{Lmajor}, and the head loss in the various pipe components, termed the ***minor loss*** and denoted, h_{Lminor}. That is, $h_L = h_{Lmajor} + h_{Lminor}$

The head loss designations of "major" and "minor" do not necessarily reflect the relative importance of each type of loss. For a pipe system that contains many components and a relatively short length of pipe, the minor loss may actually be larger than the major loss.

$$\boldsymbol{h_{Lmajor} = f\frac{L}{D}\frac{V^2}{2g}}$$

This equation called the *Darcy–Weisbach equation*, is valid for any fully developed, steady, incompressible pipe flow—whether the pipe is horizontal or on a hill. The friction factor (f) in fully developed turbulent pipe flow depends on the ***Reynolds number*** ($Re = \rho VD/\mu$) and ***the relative roughness,*** ε/D, which is the ratio of the mean height of roughness of the pipe to the pipe diameter and are not present in the laminar formulation because the two parameters ρ and ε are not important in fully developed laminar pipe flow. Typical roughness values for various pipe surfaces are given in Table 5.2.

Table 5.2. Rougness for new pipes

Equivalent Roughness for New Pipes [From Moody (Ref. 7) and Colebrook (Ref. 8)]

	Equivalent Roughness, ε
Pipe	**Millimeters**
Riveted steel	0.9–9.0
Concrete	0.3–3.0
Wood stave	0.18–0.9
Cast iron	0.26
Galvanized iron	0.15
Commercial steel or wrought iron	0.045
Drawn tubing	0.0015
Plastic, glass	0.0 (smooth)

The below figure shows the functional dependence of *f* on Re and *ε/D* and is called the ***Moody chart*** in honor of L. F. Moody, who, along with C. F. Colebrook, correlated the original data of Nikuradse in terms of the relative roughness of commercially available pipe materials. It should be noted that the values of *ε/D* do not necessarily correspond to the actual values obtained by a microscopic determination of the average height of the roughness of the surface (Fig 16). They do, however, provide the correct correlation for $f = \emptyset(Re, \frac{\varepsilon}{D})$.

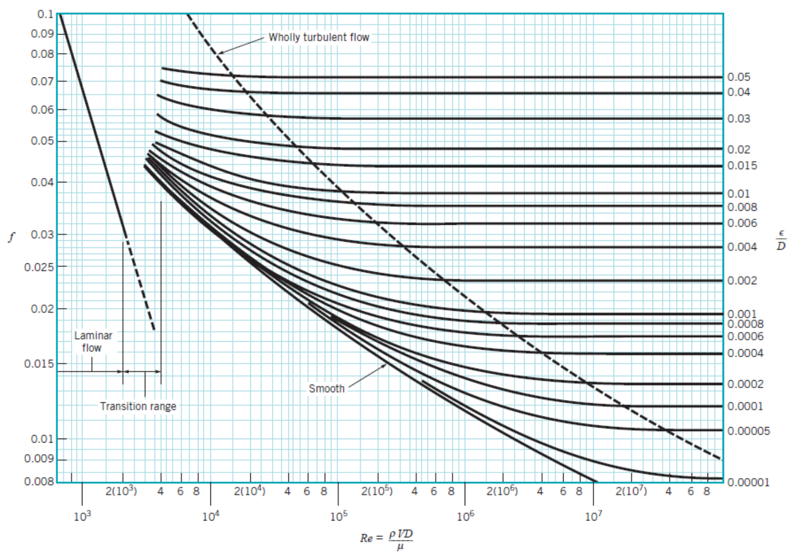


Figure 5.16. Friction factor as a function of Reynolds number and relative roughness for round pipes-the Moody chart.

Note that even for smooth pipes (ε = 0) the friction factor is not zero. That is, there is a head loss in any pipe, no matter how smooth the surface is made. This is a result of the no-slip boundary condition that requires any fluid to stick to any solid surface it flows over. There is always some microscopic surface roughness that produces the no-slip behavior (and thus) on the molecular level, even when the roughness is considerably less than the viscous sublayer thickness. Such pipes are called *hydraulically smooth*.

The Moody chart covers an extremely wide range in flow parameters. The Moody chart, on the other hand, is universally valid for all steady, fully developed, incompressible pipe flows. The following equation from Colebrook is valid for the entire nonlaminar range of the Moody chart

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

In fact, the Moody chart is a graphical representation of this equation, which is an empirical fit of the pipe flow pressure drop data. The above Equation is called the ***Colebrook formula.*** A difficulty with its use is that it is implicit in the dependence of *f*. That is, for given conditions (Re and ε/D), it is not possible to solve for *f* without some sort of iterative scheme. With the use of modern computers and calculators, such calculations are not difficult. A word of caution is in order concerning the use of the Moody chart or the equivalent Colebrook formula. Because of various inherent inaccuracies involved (uncertainty in the relative roughness, uncertainty in the experimental data used to produce the Moody chart, etc.), the use of several place accuracy in pipe flow problems is usually not justified. As a rule of thumb, a 10% accuracy is the best expected. It is possible to obtain an equation that adequately approximates the Colebrook_Moody chart relationship but does not require an iterative scheme. For example, an alternate form, which is easier to use, is given by by S. E. Haaland in 1983 as

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right]$$

Where one can solve for *f* explicitly. The results obtained from this relation are within 2 percent of those obtained from the Colebrook equation

To avoid tedious iterations in head loss, flow rate, and diameter calculations, Swamee and Jain proposed the following explicit relations in 1976 that are accurate to within 2 percent of the Moody chart

for $10^{-6} < \frac{\varepsilon}{D} < 10^{-2}$ *and* $3000 < Re < 3\times10^{8}$

$$h_L = 1.07\frac{Q^2 L}{gD^5}\left\{ln\left[\frac{\varepsilon}{3.7D} + 4.62\left(\frac{\nu D}{Q}\right)^{0.9}\right]\right\}^{-2}$$

for $Re > 2000$

$$Q = -0.965\left(\frac{gD^5 h_L}{L}\right)^{0.5} ln\left[\frac{\varepsilon}{3.7D} + \left(\frac{3.17\nu^2 L}{gD^3 h_L}\right)^2\right]$$

for $10^{-6} < \frac{\varepsilon}{D} < 10^{-2}$ and $5000 < Re < 3\times10^{8}$

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{LQ^2}{gh_L}\right)^{4.75} + \nu Q^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04}$$

If $Re \leq 10^5$ and the pipe is hydraulically smooth (ε=0), we can take $f=0.316/Re^{0.25}$.

Note that all quantities are dimensional and the units simplify to the desired unit (for example, to m or ft in the last relation) when consistent units are used. Noting that the Moody chart is accurate to within 15 percent of experimental data, we should have no reservation in using these approximate relations in the design of piping systems.

As discussed in the above section, the head loss in long, straight sections of pipe, the major losses, can be calculated by use of the friction factor obtained from either the Moody chart or the Colebrook equation. Most pipe systems, however, consist of considerably more than straight pipes. These additional components (valves, bends, tees, and the like) add to the overall head loss of the system. Such losses are generally termed *minor losses*, h_{Lminor}, with the corresponding head loss denoted In this section we indicate how to determine the various minor losses that commonly occur in pipe systems.

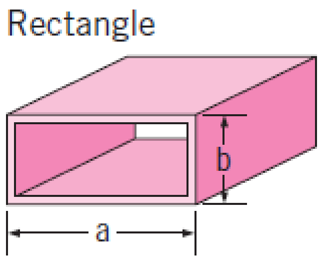
The friction loses for noncircular pipes can be calculated by *Darcy–Weisbach equation.* But The Reynolds number for flow in these pipes is based on the hydraulic diameter $D_h=4R=4Ac/p$, where ***R*** is the ***hydraulic radius (***Hydraulic radius is defined as the ratio of the channel's cross-sectional area of the flow to its ***wetted perimeter***)***.*** *Ac* is the cross-sectional area of the flow and *p* is its wetted perimeter (the length of the perimeter of the cross section in contact with the fluid).

$$R = \frac{A_c}{p} \qquad D_h = 4\frac{A_c}{p} \qquad Re = \frac{4RV}{\nu} = \frac{D_h V}{\nu}$$

The relative rougness is ε/4R.

$$h_{Lmajor} = f\frac{L}{D_h}\frac{V^2}{2g}$$

Example: The water is fully flowing through a duct that has a=25 cm and b=10 cm. Determine hydraulic radius of the duct. If water is filled up to half of the pipe. What is hydraulic radius?



Solution: If the duct is full of water.

$$R = \frac{A_c}{p} = \frac{0.25\times0.10}{2(0.25+0.10)} = 0.036\, m$$

If water is filled up to half of the pipe:

$$R = \frac{A_c}{p} = \frac{0.25\times0.10/2}{2(0.25+0.10/2)} = 0.018\, m$$

The head loss associated with flow through a valve is a common minor loss. The purpose of a valve is to provide a means to regulate the flowrate. This is accomplished by changing the geometry of the system (i.e., closing or opening the valve alters the flow pattern through the valve), which in turn alters the losses associated with the flow through the valve. The flow resistance or head loss through the valve may be a significant portion of the resistance in the system. In fact, with the valve closed, the resistance to the flow is infinite—the fluid cannot flow. Such minör losses may be very important indeed. With the valve wide open the extra resistance due to the presence of the valve may or may not be negligible.

The flow pattern through a typical component such as a valve is shown in Fig.5.17. It is not difficult to realize that a theoretical analysis to predict the details of such flows to obtain the head loss for these components is not, as yet, possible. Thus, the head loss information for essentially all components is given in dimensionless form and based on experimental data. The most common method used to determine these head losses or pressure drops is to specify the ***loss coefficient*****,** K_L**.**

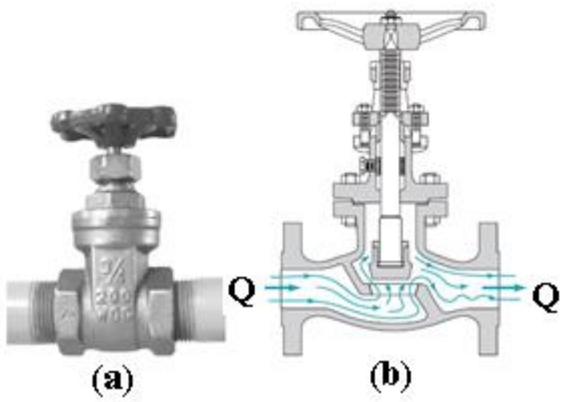


Figure 5.17. Flow through a valve. Losses due to pipe

Minor losses are usually expressed in terms of the **loss coefficient** K_L (Fig 5.18, 5.19, 5.20 and 5.21) (also called the **resistance coefficient**), defined as

$$h_{Lminor} = K_L \frac{V^2}{2g}$$

where h_L is the *additional* irreversible head loss in the piping system caused by insertion of the component, and is defined as $h_L = \Delta P_L / \rho g$. The pressure drop across a component that has a loss coefficient of is equal to the dynamic pressure, $\rho V^2/2$.

Minor losses are sometimes given in terms of an *equivalent length*, l_{eq}. In this terminology, the head loss through a component is given in terms of the equivalent length of pipe that would produce the same head loss as the component. That is,

$$l_{eq} = K_L \frac{D}{f}$$

where D and f are based on the pipe containing the component. The head loss of the pipe system is the same as that produced in a straight pipe whose length is equal to the pipes of the original system plus the sum of the additional equivalent lengths of all of the components of the system. Most pipe flow analyses, including those in this book, use the loss coefficient method rather than the equivalent length method to determine the minor losses.

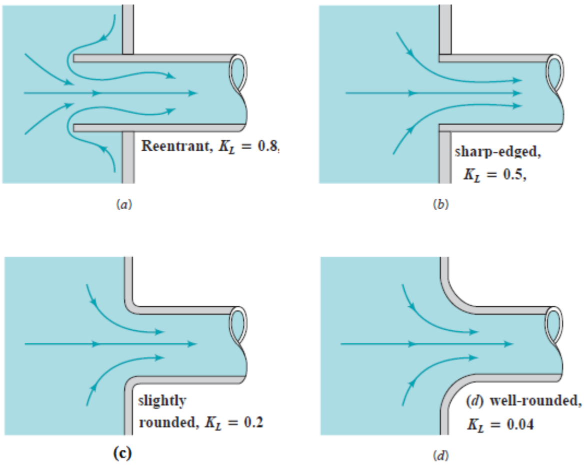


Figure 5.18. Entrance flow conditions and loss coefficient

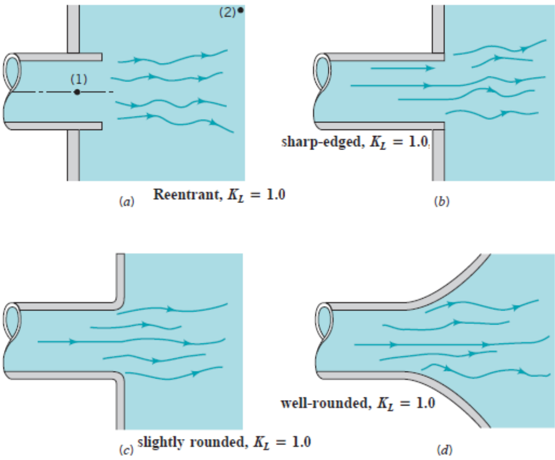


Figure 5.19. Exit flow conditions and loss coefficient.

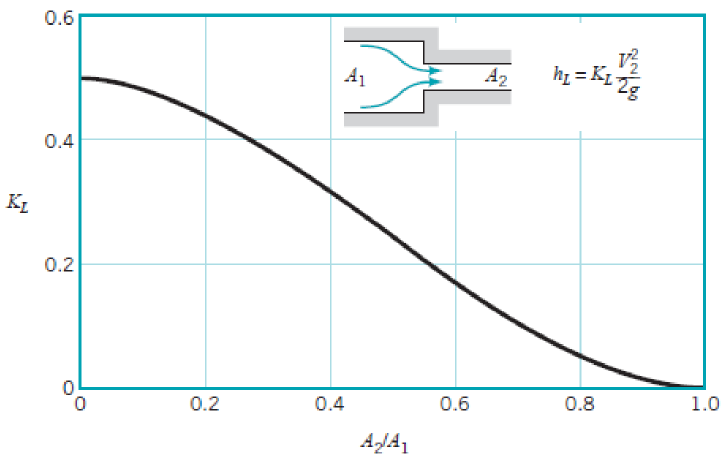


Figure 5.20. Loss coefficient for a sudden contraction

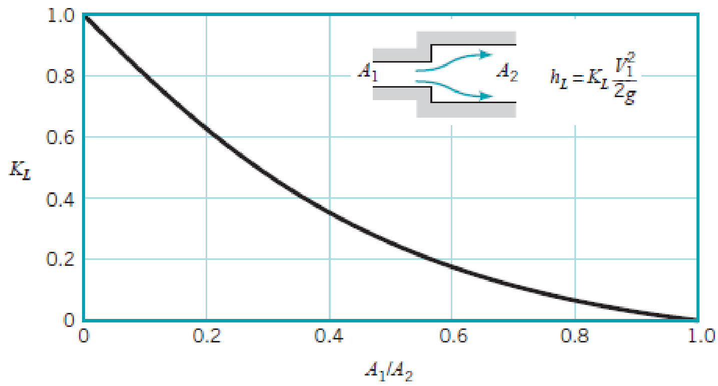


Figure 5.21. Loss coefficient for a sudden expansion

The total head loss in a piping system is determined from $h_L = h_{Lmajor} + h_{Lminor}$

$$h_L = f\frac{L}{D}\frac{V^2}{2g} + K_L\frac{V^2}{2g}$$

Once the useful pump head is known, the *mechanical power that needs to be delivered by the pump to the fluid* and the *electric power consumed by the motor of the pump* for a specified flow rate are determined from

$$\dot{W}_p = \frac{\rho g Q h_L}{\eta_p} \text{ for pump}$$

$$\dot{W}_e = \frac{\rho g Q h_L}{\eta_p \eta_m} \text{ for electrik motor}$$

Where; η_p is the efficiency of the pump. $\eta_m$is the efficiency of the motor.

5.4. Pipe Flowrate Measurement

It is often necessary to determine experimentally the flowrate in a pipe. In before chapter we introduced various types of flow-measuring devices (venturi meter, nozzle meter, orifice meter, etc.) and discussed their operation under the assumption that viscous effects were not important. In this section we will indicate how to account for the ever-present viscous effects in these flow meters. We will also indicate other types of commonly used flow meters. Orifice, nozzle and Venturi meters involve the concept “high velocity gives low pressure.”

5.4.1 Pipe Flowrate Meters

Three of the most common devices used to measure the instantaneous flowrate in pipes are *the orifice meter, the nozzle meter, and the venturi meter*. Each of these meters operates on the principle that a decrease in flow area in a pipe causes an increase in velocity that is accompanied by a decrease in pressure. Correlation of the pressure difference with the velocity provides a means of measuring the flowrate. In the absence of viscous effects and under the assumption of a horizontal pipe, application of the Bernoulli equation between points (1) and (2) shown in Fig.5.22 gave

$$Q_{\text{ideal}} = A_2 V_2 = A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho(1 - \beta^4)}}$$

Where; $\beta = \frac{D_2}{D_1}$. Based on the results of the previous sections of this chapter, we anticipate that there is a head loss between (1) and (2) so that the governing equations become

$$Q = A_1 V_1 = A_2 V_2$$

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

The ideal situation has h_L=0 and results in above equation. The difficulty in including the head loss is that there is no accurate expression for it. The net result is that empirical coefficients are used in the flowrate equations to account for the complex real-world effects brought on by the nonzero viscosity. The coefficients are discussed in this section.

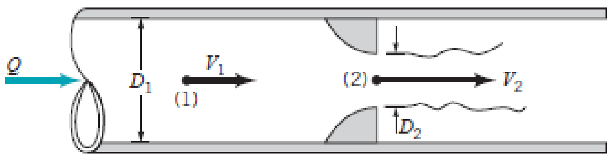


Figure 5.22. Typical pipe flow meter geometry.

A typical ***orifice meter*** is constructed by inserting between two flanges of a pipe a flat plate with a hole, as shown in the below Fig.5.23.

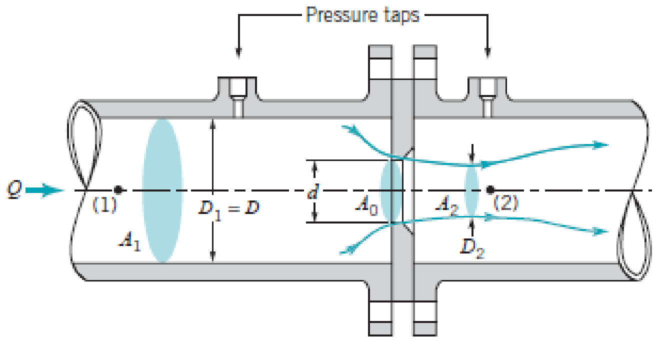


Figure 5.23. Typical orifice meter construction.

The pressure at point (2) within the vena contracta is less than that at point (1). Nonideal effects occur for two reasons. First, the vena contracta area, A_2 is less than the area of the hole, A_0, by an unknown amount. Thus, $A_2=C_cA_0$, where C_c is the contraction coefficient (Cc<1). Second, the swirling flow and turbulent motion near the orifice plate introduce a head loss that cannot be calculated theoretically. Thus, an *orifice discharge coefficient*, C_0, is used to take these effects into account. That is,

$$Q = C_o Q_{\text{ideal}} = C_o A_o \sqrt{\frac{2(p_1 - p_2)}{\rho(1 - \beta^4)}}$$

Where; $A_0 = \frac{\pi d^2}{4}$ is the area of the hole in the orifice plate. The value of C_0 is a function of $\beta = \frac{d}{D}$ and the Reynolds number $Re = \rho VD/\mu$, where $V = \frac{Q}{A_1}$. Typical values of C_0 are given in the below Fig. 5.24. The experimentally determined data for orifice discharge coefficient for $0.25 < \beta < 0.75$ *and* $10^4 < Re < 10^7$ is expressed as

$$\text{Orifice meters:} \quad \mathbf{C_0} = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{Re^{0.75}}$$

For flows with high Reynolds numbers (Re ≥ 30.000), the value of C_0 can be taken to be $\boldsymbol{C_0=0.61}$ for orifices.

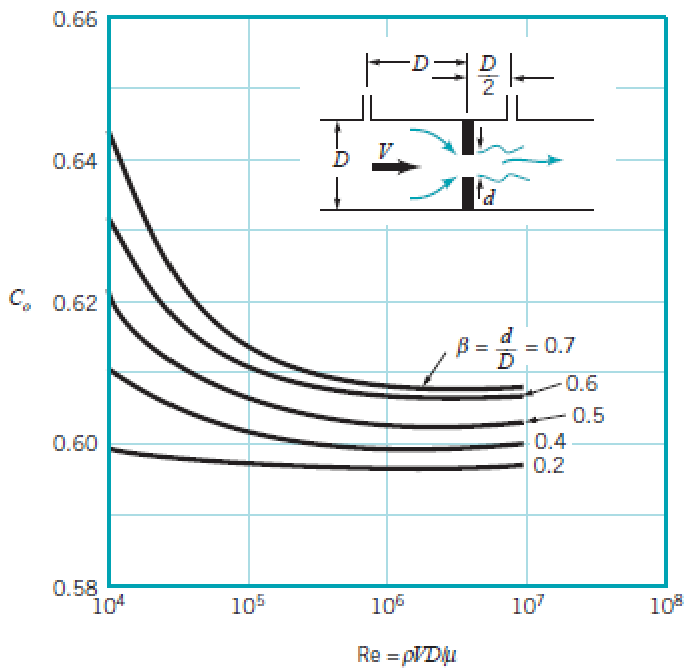


Figure 5.24. Orifice meter discharge coefficient

For a given value of C_0, the flowrate is proportional to the square root of the pressure diffrence. Note that the value of C_0 depends on the specific construction of the orifice meter (i.e., the placement of the pressure taps, whether the orifice plate edge is square or beveled, etc.). Very precise conditions governing the construction of standard orifice meters have been established to provide the greatest accuracy possible.

Another type of pipe flow meter that is based on the same principles used in the orifice meter is the ***nozzle meter*,** three variations of which are shown in the below Fig.5.25.

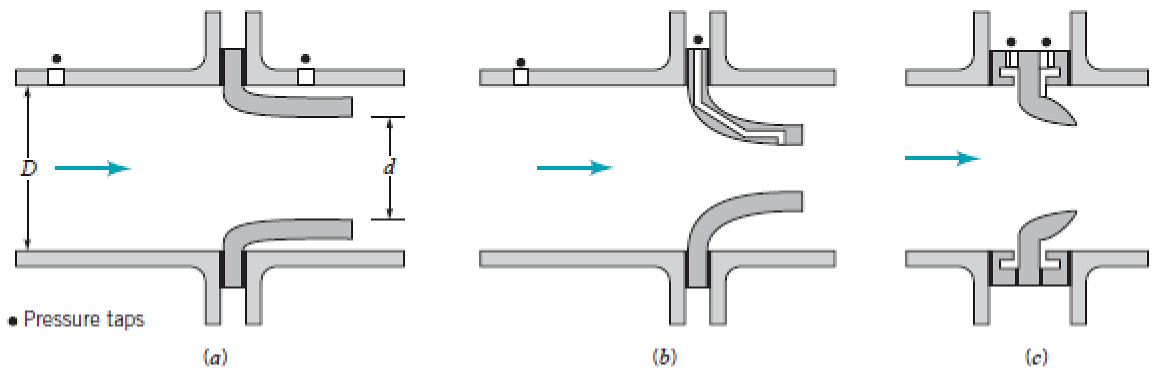


Figure 5.25. Typical nozzle meter construction.

This device uses a contoured nozzle (typically placed between flanges of pipe sections) rather than a simple (and less expensive) plate with a hole as in an orifice meter. The resulting flow pattern for the nozzle meter is closer to ideal than the orifice meter flow. There is only a slight vena contracta and the secondary flow separation is less severe, but there still are viscous effects. These are accounted for by use of the ***nozzle discharge coefficient***, C_n, where

$$Q = C_n Q_{\text{ideal}} = C_n A_n \sqrt{\frac{2(p_1 - p_2)}{\rho(1 - \beta^4)}}$$

With $A_n = \frac{\pi d^2}{4}$. As with the orifice meter, the value of C_n is a function of the diameter rato, with $A_n = \frac{\pi d^2}{4}$. And the Reynolds number, $Re = \frac{\rho V D}{\mu}$. Typical values obtained from experiments are shown in the below Fig.5.26. Again, precise values of C_n depend on the specific details of the nozzle design. Note that $C_n > C_0$ the nozzle meter is more efficient (less energy dissipated) than the orifice meter. C_n for $0.25 < \beta < 0.75$ *and* $10^4 < Re < 10^7$ can be calculated from the following equation.

$$\mathbf{C_n} = 0.9975 - \frac{6.53\beta^{0.5}}{\text{Re}^{0.5}}$$

For flows with high Reynolds numbers (Re≥ 30.000), the value of C_n can be taken to be ***C_n=0.96*** for flow nozzles.

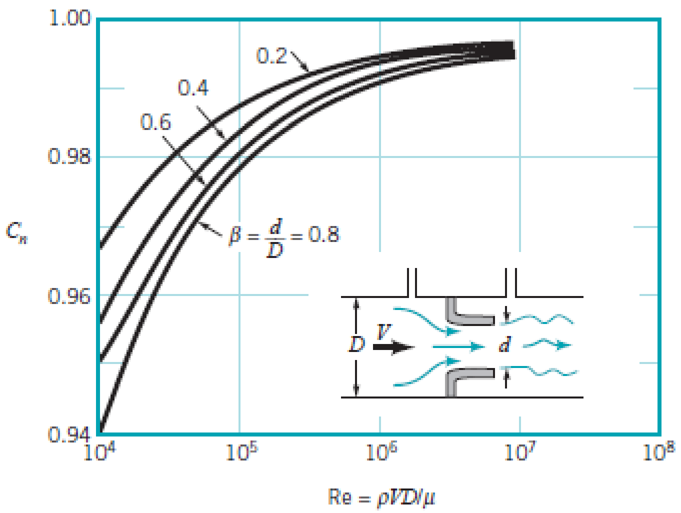


Figure 5.26. Nozzle meter discharge coefficient

The most precise and most expensive of the three obstruction-type flow meters is the ***Venturimeter*** shown in Fig.5.27.

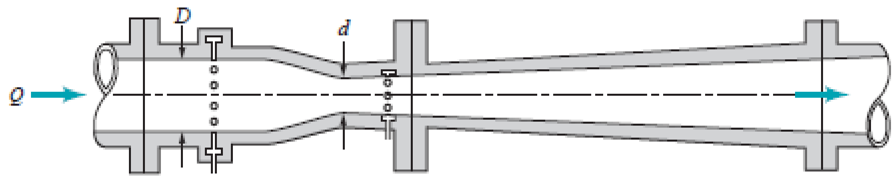


Figure 5.27. Typical venturi meter construction.

Although the operating principle for this device is the same as for the orifice or nozzle meters, the geometry of the venturi meter is designed to reduce head losses to a minimum. This is accomplished by providing a relatively streamlined contraction (which eliminates separation ahead of the throat) and a very gradual expansion downstream of the throat (which eliminates separation in this decelerating portion of the device). Most of the head loss that occurs in a well-designed venturi meter is due to friction losses along the walls rather than losses associated with separated flows and the inefficient mixing motion that accompanies such flow. Thus, the flowrate through a Venturi meter is given by

$$Q = C_v Q_{\text{ideal}} = C A_T \sqrt{\frac{2(p_1 - p_2)}{\rho(1 - \beta^4)}}$$

Where; $A_T = \frac{\pi d^2}{4}$ is the throat area. The range of values of Cv, the Venturi discharge coefficient, is given in the following Fig. 5.28.

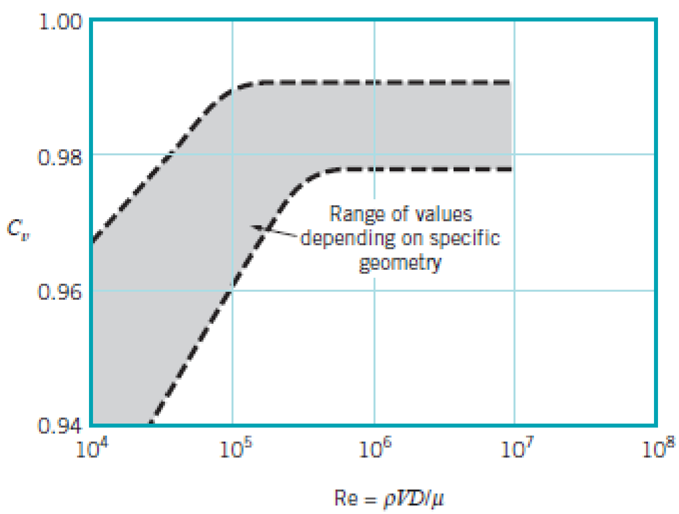


Figure 5.28. Venturi meter discharge coefficient.

The throat-to-pipe diameter ratio $\left(\beta = \frac{d}{D}\right)$, the Reynolds number, and the shape of the converging and diverging sections of the meter are among the parameters that affect the value of C_v.

Owing to its streamlined design, the discharge coefficients of Venturi meters are very high, ranging between 0.95 and 0.99 (the higher values are for the higher Reynolds numbers) for most flows. In the absence of specific data, we can take ***Cv*=0.98** for Venturi meters. Also, the Reynolds number depends on the flow velocity, which is not known a priori. Therefore, the solution is iterative in nature when curve-fit correlations are used for *Cv*.

5.5. Chapter Summary and Study Guide

This chapter discussed the flow of a viscous fluid in a pipe. General characteristics of laminar, turbulent, fully developed, and entrance flows are considered. Poiseuille's equation is obtained to describe the relationship among the various parameters for fully developed laminar flow.

Some of the important equations in this chapter are given below.

Entrance length	$\frac{\ell_e}{D} = 0.06\ \text{Re}$ for laminar flow $\frac{\ell_e}{D} = 4.4\ (\text{Re})^{1/6}$ for turbulent flow
Pressure drop for fully developed laminar pipe flow	$\Delta p = \frac{4\ell\tau_w}{D}$
Velocity profile for fully developed laminar pipe flow	$u(r) = \left(\frac{\Delta p D^2}{16\mu\ell}\right)\left[1 - \left(\frac{2r}{D}\right)^2\right] = V_c\left[1 - \left(\frac{2r}{D}\right)^2\right]$
Volume flowrate for fully developed laminar pipe flow	$Q = \frac{\pi D^4\ \Delta p}{128\mu\ell}$
Friction factor for fully developed laminar pipe flow	$f = \frac{64}{\text{Re}}$
Pressure drop for a horizontal pipe	$\Delta p = f\frac{\ell}{D}\frac{\rho V^2}{2}$
Head loss due to major losses	$h_{L\ \text{major}} = f\frac{\ell}{D}\frac{V^2}{2g}$
Colebrook formula	$\frac{1}{\sqrt{f}} = -2.0\ \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$

Explicit alternative to Colebrook formula	$\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right]$
Head loss due to minor losses	$h_{L\,\text{minor}} = K_L \frac{V^2}{2g}$
Volume flowrate for orifice, nozzle, or Venturi meter	$Q = C_i A_i \sqrt{\frac{2(p_1 - p_2)}{\rho(1 - \beta^4)}}$

The design and analysis of piping systems involve the determination of the head loss, flow rate, or the pipe diameter. Tedious iterations in these calculations can be avoided by the approximate Swamee–Jain formulas expressed as

for $10^{-6} < \frac{\varepsilon}{D} < 10^{-2}$ *and* $3000 < Re < 3\times10^{8}$

$$h_L = 1.07\frac{Q^2 L}{gD^5}\left\{ln\left[\frac{\varepsilon}{3.7D} + 4.62\left(\frac{\nu D}{Q}\right)^{0.9}\right]\right\}^{-2}$$

for $Re > 2000$

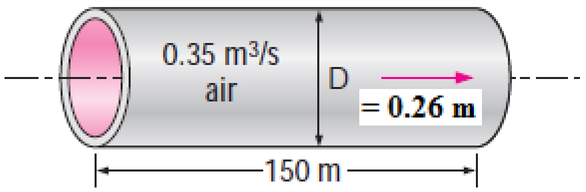
$$Q = -0.965\left(\frac{gD^5 h_L}{L}\right)^{0.5} ln\left[\frac{\varepsilon}{3.7D} + \left(\frac{3.17\nu^2 L}{gD^3 h_L}\right)^2\right]$$

for $10^{-6} < \frac{\varepsilon}{D} < 10^{-2}$ and $5000 < Re < 3\times10^{8}$

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{LQ^2}{gh_L}\right)^{4.75} + \nu Q^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04}$$

EXAMPLES

Example: Heated air at 1 atm and 35°C is to be transported in a 150-m-long circular plastic duct at a rate of 0.35 m^3/s. If the head loss in the pipe is not to exceed 20 m, determine the major loss and pressure drop of the duct. The the friction factor, *f*, is 0.018.The density, dynamic viscosity, and kinematic viscosity of air at 35°C are ρ=1.145 kg/m^3, μ=1.895×10^5 kg/m.s, and ν=1.655×10^{-5} m^2/s.



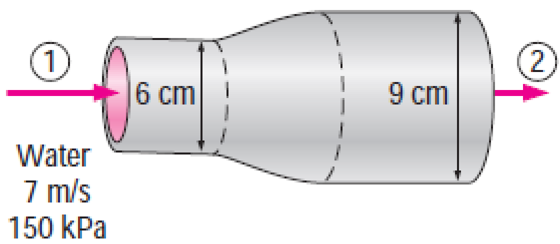
Solution:

$$V = \frac{4Q}{\pi D^2} = \frac{4 \times 0.35}{\pi \times 0.26^2} = 6.6\, m/s$$

$$h_L = f\frac{l}{D}\frac{V^2}{2g} = 0.018\frac{150 \times 6.6^2}{0.26 \times 2 \times 9.81} = 23.06\, m$$

$$\Delta P = h_L\gamma = 23.06 \times 9810 = 226218.6\, Pa$$

Example: A 6-cm-diameter horizontal water pipe expands gradually to a 9-cm-diameter pipe (Fig). The walls of the expansion section are angled 30° from the horizontal. The average velocity and pressure of water before the expansion section are 7 m/s and 150 kPa, respectively. Determine the head loss in the expansion section and the pressure in the larger-diameter pipe. We take the density of water to be ρ=1000 kg/m^3. The loss coefficient for gradual expansion of θ=60° total included angle is K_L= 0.07.



Solution: $V_2 = V_1(\frac{D_1}{D_2})^2 = 7(\frac{6}{9})^2 = 3.11\, m/s$

$$h_L = K_L\frac{V^2}{2g} = 0.07\frac{V_1^2}{2g} = 0.07\frac{7^2}{2 \times 9.81} = 0.175\, m$$

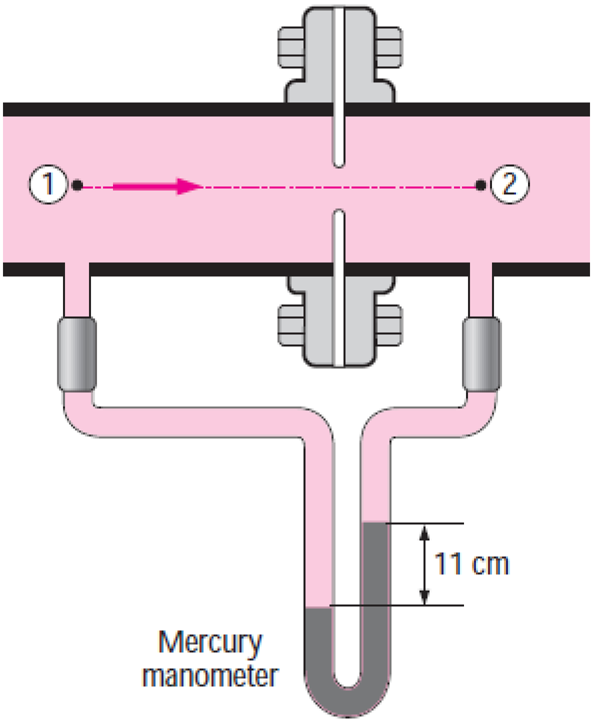
$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

$$\frac{150000}{1000} + \frac{7^2}{2} + 0 = \frac{P_2}{1000} + \frac{3.11}{2} + 0$$

$$P_2 = 172945\ Pa$$

Therefore, despite the head (and pressure) loss, the pressure *increases* from 150 to 169 kPa after the expansion. This is due to the conversion of dynamic pressure to static pressure when the average flow velocity is decreased in the larger pipe.

Example: The flow rate of methanol at 20°C (ρ=788.4 kg/m^3 and μ= 5.857×10^{-4} kg/m·s) through a 4-cm-diameter pipe is to be measured with a 3-cm-diameter orifice meter equipped with a mercury manometer across the orifice place, as shown in Fig.. If the differential height of the manometer is read to be 11 cm, determine the flow rate of methanol through the pipe and the average flow velocity. We take the density of mercury to be 13.600 kg/m^3. The flow is steady and incompressible. The discharge coefficient of the orifice meter is C_0=0.61.



Solution: The diameter ratio and the throat area of the orifice are

$$\beta = \frac{d}{D} = \frac{3}{4} = 0.75$$

$$A_0 = \frac{\pi d^2}{4} = \frac{\pi \times 0.03^2}{4} = 7.069 \times 10^{-4}\ m^2$$

The pressure drop across the orifice plate can be expressed as

$$\Delta P = P_1 - P_2 = (\rho_{Hg} - \rho_{met})gh$$

Then the flow rate relation for obstruction meters becomes

$$A_0 C_0 \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}} = A_0 C_0 \sqrt{\frac{2(\rho_{Hg} - \rho_{met})gh}{\rho_{met}(1-\beta^4)}} = A_0 C_0 \sqrt{\frac{2(\rho_{Hg}/\rho_{met} - 1)gh}{1-\beta^4}}$$

Substituting, the flow rate is determined to be

$$Q = (7.069 \times 10^{-4}\ m^2)(0.61)\sqrt{\frac{2(13{,}600/788.4 - 1)(9.81\ m/s^2)(0.11\ m)}{1 - 0.75^4}}$$

$$= 3.09 \times 10^{-3}\ m^3/s$$

Which is equivalent to 3.09 L/s. The average flow velocity in the pipe is determined by dividing the flow rate by the cross-sectional area of the pipe

$$V = \frac{Q}{A_c} = \frac{Q}{\pi D^2/4} = \frac{3.09 \times 10^{-3}\ m^3/s}{\pi(0.04\ m)^2/4} = 2.46\ m/s$$

The Reynolds number of flow through the pipe is

$$Re = \frac{\rho V D}{\mu} = \frac{(788.4\ kg/m^3)(2.46\ m/s)(0.04\ m)}{5.857 \times 10^{-4}\ kg/m \cdot s} = 1.32 \times 10^5$$

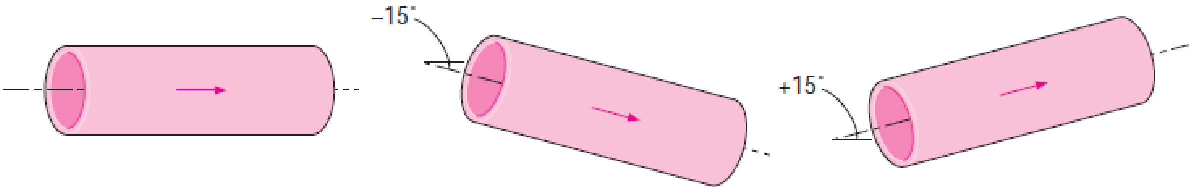
Substituting $\beta = 0.75$ and $Re = 1.32 \times 10^5$ into the orifice discharge coefficient relation

$$C_0 = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{Re^{0.75}}$$

gives $C_0 = 0.601$. Which is very close to the assumed value of 0.61.

Example: Oil at 20°C (ρ=888 kg/m^3 and m μ =0.800 kg/m·s) is flowing steadily through a 5-cm-diameter 40-m-long pipe (Fig.). The pressure at the pipe inlet and outlet are measured to be 745 and 97 kPa, respectively. Determine the flow rate of oil through the pipe assuming the pipe is (*a*) horizontal, (*b*) inclined 15°

upward, (*c*) inclined 15° downward. Also verify that the flow through the pipe is laminar. The density and dynamic viscosity of oil are given to be ρ= 888 kg/m^3 and μ = 0.800 kg/m·s, respectively.



Solution: The pressure drop across the pipe and the pipe cross-sectional area are

$$\Delta P = P_1 - P_2 = 745 - 97 = 648 \text{ kPa}$$

$$A_c = \pi D^2/4 = \pi(0.05 \text{ m})^2/4 = 0.001963 \text{ m}^2$$

(*a*) The flow rate for all three cases can be determined by

$$\mathbf{Q} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

where θ is the angle the pipe makes with the horizontal. For the horizontal case, θ=0 and thus sinθ=0 Therefore,

$$\mathbf{Q} = \frac{\Delta P\,\pi D^4}{128\mu L} = \frac{(648 \text{ kPa})\pi(0.05 \text{ m})^4}{128(0.800 \text{ kg/m}\cdot\text{s})(40 \text{ m})}\left(\frac{1000 \text{ N/m}^2}{1 \text{ kPa}}\right)\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right)$$

$$= 0.00311 \text{ m}^3/\text{s}$$

(*b*) For uphill flow with an inclination of 15°, we have θ=15°, and

$$\mathbf{Q} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

$$= \frac{[648{,}000 \text{ Pa} - (888 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(40 \text{ m})\sin 15°]\pi(0.05 \text{ m})^4}{128(0.800 \text{ kg/m}\cdot\text{s})(40 \text{ m})}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ Pa}\cdot\text{m}^2}\right)$$

$$= 0.00267 \text{ m}^3/\text{s}$$

(*c*) For downhill flow with an inclination of 15°, we have θ=-15°, and

$$Q = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

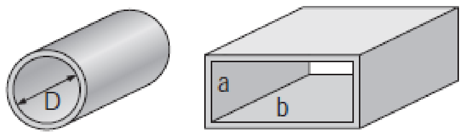
$$Q = \frac{[648{,}000\ \text{Pa} - (888\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(40\ \text{m})\sin(-15^\circ)]\pi(0.05\ \text{m})^4}{128(0.800\ \text{kg/m}\cdot\text{s})(40\ \text{m})}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{Pa}\cdot\text{m}^2}\right)$$

$$Q = 0.00354\ \text{m}^3/\text{s}$$

Example: Why are liquids usually transported in circular pipes?

Solution: Liquids are usually transported in circular pipes because pipes with a circular cross section can withstand large pressure differences between the inside and the outside without undergoing any significant distortion. Piping for gases at low pressure are often non-circular (e.g., air conditioning and heating ducts in buildings).

Example: What is the physical significance of the Reynolds number? How is it defined for (*a*) flow in a circular pipe of inner diameter *D* and (*b*) flow in a rectangular duct of cross section $a \times b$?

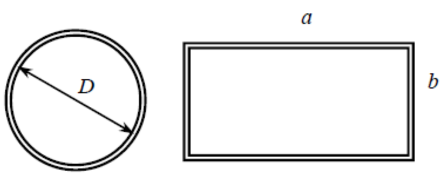


Solution: *Reynolds number* is the **ratio of the inertial forces to viscous forces**, and it serves as a criterion for determining the flow regime. At *large* Reynolds numbers, for example, the flow is turbulent since the inertia forces are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid. It is defined as follows:

a) For flow in a circular tube of inner diameter D: $Re = \frac{VD}{\nu}$

b) For flow in a rectangular duct of cross-section $a \times b$: $Re = \frac{VD_h}{\nu}$

Where $D_h = \frac{4A_c}{p} = \frac{4ab}{2(a+b)} = \frac{2ab}{(a+b)}$ is the hydraulic diameter.



Example: Consider a person walking first in air and then in water at the same speed. For which motion will the Reynolds number be higher?

Solution: Reynolds number is inversely proportional to kinematic viscosity, which is much smaller for water than for air (at 25°C, ν_{air} = 1.562×10-5 m^2/s and $\nu_{water} = \mu/\rho = 0.891\times10^{-3}/997 = 8.9\times10^{-7}$ m^2/s). Therefore, noting that Re = *VD*/ν, the Reynolds number is higher for motion in water for the same diameter and speed.

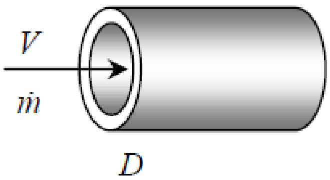
Example: Show that the Reynolds number for flow in a circular pipe of diameter *D* can be expressed as $Re = \frac{4\dot{m}}{\pi D\mu}$ .Where $\dot{m}$ is the mass flow rate.

Solution: Reynolds number for flow in a circular tube of diameter D is expressed as

$Re = \frac{VD}{\nu}$ Where $V = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho(\frac{\pi D^2}{4})} = \frac{4\dot{m}}{\rho\pi D^2}$ $and\ \nu = \frac{\mu}{\rho}$

Substituting, $Re = \frac{VD}{\nu} = \frac{4\dot{m}D}{\rho\pi D^2(\frac{\mu}{\rho})} = \frac{4\dot{m}}{\pi D\mu}$. Thus, $Re = \frac{4\dot{m}}{\pi D\mu}$

This result holds only for circular pipes.



Example: Which fluid at room temperature requires a larger pump to flow at a specified velocity in a given pipe: water or engine oil? Why?

Solution: Engine oil requires a larger pump because of its much larger viscosity. The density of oil is actually 10 to 15% smaller than that of water, and this makes the pumping requirement smaller for oil than water. However, the viscosity of oil is orders of magnitude larger than that of water, and is therefore the dominant factor in this comparison.

Example: What is hydraulic diameter? How is it defined? What is it equal to for a circular pipe of diameter *D*?

Solution: For flow through non-circular tubes, the Reynolds number and the friction factor are based on the *hydraulic diameter Dh* defined as $D_h = \frac{4A_c}{p}$ where *Ac* is the cross-sectional area of the tube and *p* is its perimeter. The hydraulic diameter is defined such that it **reduces to ordinary diameter *D* for circular**

tubes since $D_h = \frac{4A_c}{p} = \frac{4\pi D^2/4}{\pi D} = D$. Hydraulic diameter is a useful tool for dealing with non-circular pipes (e.g., air conditioning and heating ducts in buildings).

Example: Consider laminar flow in a circular pipe. Will the wall shear stress τ_w be higher near the inlet of the pipe or near the exit? Why? What would your response be if the flow were turbulent?

Solution: The wall shear stress τ_w is the highest at the tube inlet where the thickness of the boundary layer is nearly zero, and decreases gradually to the fully developed value. The same is true for turbulent flow.

Example: How does the wall shear stress vary along the flow direction in the fully developed region in (*a*) laminar flow and (*b*) turbulent flow?

Solution: The wall shear stress τw remains constant along the flow direction in the fully developed region in both laminar and turbulent flow. However, in the entrance region, τw starts out large, and decreases until the flow becomes fully developed.

Example: Someone claims that the shear stress at the center of a circular pipe during fully developed laminar flow is zero. Do you agree with this claim? Explain.

Solution: The shear stress at the center of a circular tube during fully developed laminar flow is zero since the shear stress is proportional to the velocity gradient, which is zero at the tube center. This result is due to the axisymmetry of the velocity profile.

Example: Consider fully developed laminar flow in a circular pipe. If the diameter of the pipe is reduced by half while the flow rate and the pipe length are held constant, the head loss will (*a*) double, (*b*) triple, (*c*) quadruple, (*d*) increase by a factor of 8, or (*e*) increase by a factor of 16.

Solution: In fully developed laminar flow in a circular pipe, the head loss is given by

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = \frac{64}{\text{Re}}\frac{L}{D}\frac{V^2}{2g} = \frac{64}{VD/\nu}\frac{L}{D}\frac{V^2}{2g} = \frac{64\nu}{D}\frac{L}{D}\frac{V}{2g}$$

The average velocity can be expressed in terms of the flow rate as $V = \frac{Q}{A_c} = \frac{Q}{\pi D^2/4}$ Substituting,

$$h_L = \frac{64\nu}{D^2}\frac{L}{2g}\left(\frac{Q}{\pi D^2/4}\right) = \frac{64\nu}{D^2}\frac{4L\,Q}{2g\pi D^2} = \frac{128\nu L\,Q}{g\pi D^4}$$

Therefore, at constant flow rate and pipe length, the head loss is inversely proportional to the 4th power of diameter, and thus reducing the pipe diameter by half increases the head loss by a factor of 16. This is a very significant increase in head loss, and shows why larger diameter tubes lead to much smaller pumping power requirements.

Example: What is turbulent viscosity? What is it caused by?

Solution: Turbulent viscosity μ_t is caused by turbulent eddies, and it accounts for momentum transport by turbulent eddies. It is expressed as $\tau_t = -\rho\overline{u'v'} = \mu_t \frac{\partial \bar{u}}{\partial y}$ where $\bar{u}$ is the mean value of velocity in the flow direction and $u'$and $u'$are the fluctuating components of velocity.

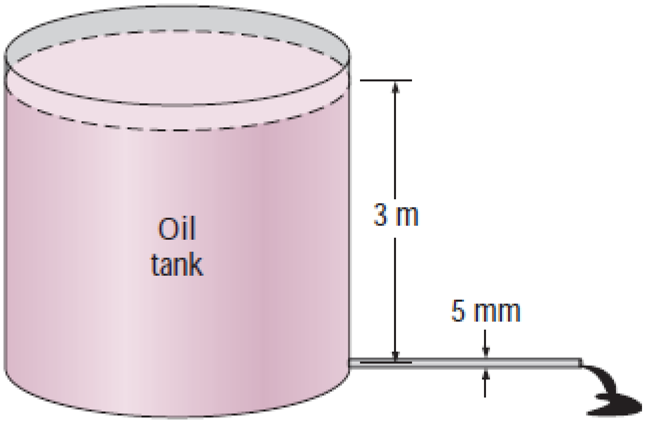
Example: How is head loss related to pressure loss? For a given fluid, explain how you would convert head loss to pressure loss.

Solution: The head loss is related to pressure loss by $h_L = \Delta P_L/\rho g$. For a given fluid, the head loss can be converted to pressure loss by multiplying the head loss by the acceleration of gravity and the density of the fluid. Thus, for constant density, head loss and pressure drop are linearly proportional to each other.

Example: Explain why the friction factor is independent of the Reynolds number at very large Reynolds numbers.

Solution: At very large Reynolds numbers, the flow is fully rough and the friction factor is independent of the Reynolds number. This is because the thickness of viscous sublayer decreases with increasing Reynolds number, and it be comes so thin that the surface roughness protrudes into the flow. The viscous effects in this case are produced in the main flow primarily by the protruding roughness elements, and the contribution of the viscous sublayer is negligible.

Example: Oil with a density of 850 kg/m^3 and kinematic viscosity of 0.00062 m^2/s is being discharged by a 5-mm-diameter, 40-m-long horizontal pipe from a storage tank open to the atmosphere. The height of the liquid level above the center of the pipe is 3 m. Disregarding the minor losses, determine the flow rate of oil through the pipe.



Solution: The dynamic viscosity is calculated to be

$$\mu = \rho \nu = (850\ \text{kg/m}^3)(0.00062\ \text{m}^2/s) = 0.527\ \text{kg/m}\cdot\text{s}$$

The pressure at the bottom of the tank is

$$P_{1,\text{gage}} = \rho g h$$

$$= (850\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(3\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= 25.02\ \text{kN/m}^2$$

Disregarding inlet and outlet losses, the pressure drop across the pipe is

$$\Delta P = P_1 - P_2 = P_1 - P_{\text{atm}} = P_{1,\text{gage}} = 25.02\ \text{kN/m}^2 = 25.02\ \text{kPa}$$

The flow rate through a horizontal pipe in laminar flow is determined from

$$\mathbf{Q}_{\text{horiz}} = \frac{\Delta P \pi D^4}{128 \mu L} = \frac{(25.02\ \text{kN/m}^2)\pi(0.005\ \text{m})^4}{128(0.527\ \text{kg/m}\cdot\text{s})(40\ \text{m})}\left(\frac{1000\ \text{kg}\cdot\text{m/s}^2}{1\ \text{kN}}\right)$$

$$= 1.821\times10^{-8}\ \text{m}^3/\text{s} \cong \mathbf{1.82\times10^{-8}\ m^3/s}$$

The average fluid velocity and the Reynolds number in this case are

$$V = \frac{\mathbf{Q}}{A_c} = \frac{\mathbf{Q}}{\pi D^2/4} = \frac{1.821\times10^{-8}\ \text{m}^3/\text{s}}{\pi(0.005\ \text{m})^2/4} = 9.27\times10^{-4}\ \text{m/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(850\ \text{kg/m}^3)(9.27\times10^{-4}\ \text{m/s})(0.005\ \text{m})}{0.527\ \text{kg/m}\cdot\text{s}} = 0.0075$$

which is less than 2100. Therefore, the flow is *laminar* and the analysis above is valid. The flow rate will be even less when the inlet and outlet losses are considered, especially when the inlet is not well-rounded.

Example: Water at 10°C (ρ= 999.7 kg/m^3 and μ=1.307 ×10^{-3} kg/m · s) is flowing steadily in a 0.20-cm-diameter, 15-m-long pipe at an average velocity of 1.2 m/s. Determine (*a*) the pressure drop, (*b*) the head loss, and (*c*) the pumping power requirement to overcome this pressure drop.

Solution: (*a*) First we need to determine the flow regime. The Reynolds number of the flow is

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(999.7\ \text{kg/m}^3)(1.2\ \text{m/s})(2\times10^{-3}\ \text{m})}{1.307\times10^{-3}\ \text{kg/m}\cdot\text{s}} = 1836$$

which is less than 2100. Therefore, the flow is laminar. Then the friction factor and the pressure drop become

$$f = \frac{64}{\text{Re}} = \frac{64}{1836} = 0.0349$$

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2}$$

$$= 0.0349\frac{15\ \text{m}}{0.002\ \text{m}}\frac{(999.7\ \text{kg/m}^3)(1.2\ \text{m/s})^2}{2}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right) = \mathbf{188\ kPa}$$

(*b*) The head loss in the pipe is determined from

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V^2}{2g} = 0.0349\frac{15\ \text{m}}{0.002\ \text{m}}\frac{(1.2\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = \mathbf{19.2\ m}$$

(*c*) The volume flow rate and the pumping power requirements are

$$\mathbf{Q} = VA_c = V(\pi D^2/4) = (1.2\ \text{m/s})[\pi(0.002\ \text{m})^2/4] = 3.77\times10^{-6}\ \text{m}^3/s$$

$$\dot{W}_{\text{pump}} = \mathbf{Q}\,\Delta P = (3.77\times10^{-6}\ \text{m}^3/s)(188\ \text{kPa})\left(\frac{1000\ \text{W}}{1\ \text{kPa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{0.71\ W}$$

Therefore, power input in the amount of 0.71 W is needed to overcome the frictional losses in the flow due to viscosity. If the flow were instead *turbulent*, the pumping power would be much greater since the head loss in the pipe would be much greater.

Example: In fully developed laminar flow in a circular pipe, the velocity at *R*/2 (midway between the wall surface and the centerline) is measured to be 6 m/s. Determine the velocity at the center of the pipe.

Solution: The velocity profile in fully developed laminar flow in a circular pipe is given by

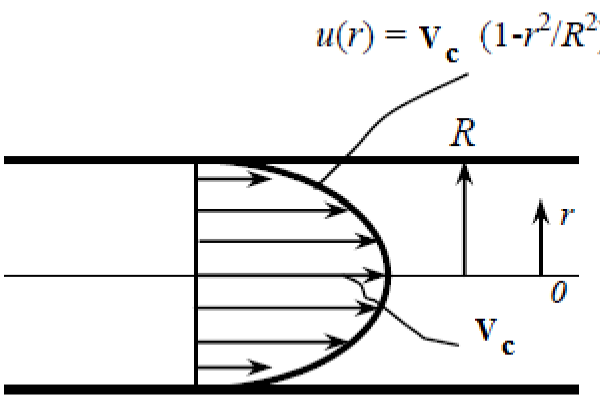
The velocity profile can be written as

$$u\,(r) = V_c\left[1 - (\frac{2r}{D})^2\right] = 6\left[1 - (\frac{2\times\frac{R}{2}}{2R})^2\right]$$

$$u\,(R/2) = V_c\left[1 - (\frac{2\times\frac{R}{2}}{2R})^2\right]$$

$$6 = V_c[1 - 0.25] \rightarrow V_c = 8\,m/s$$

V_c is the maximum velocity which occurs at pipe center, $r = 0$. At $r = R/2$,



Example: The velocity profile in fully developed laminar flow in a circular pipe of inner radius *R*= 2 cm, in m/s, is given by $u(r) = 4(1 - \frac{r^2}{R^2})$. Determine the average and maximum velocities in the pipe and the volume flow rate.

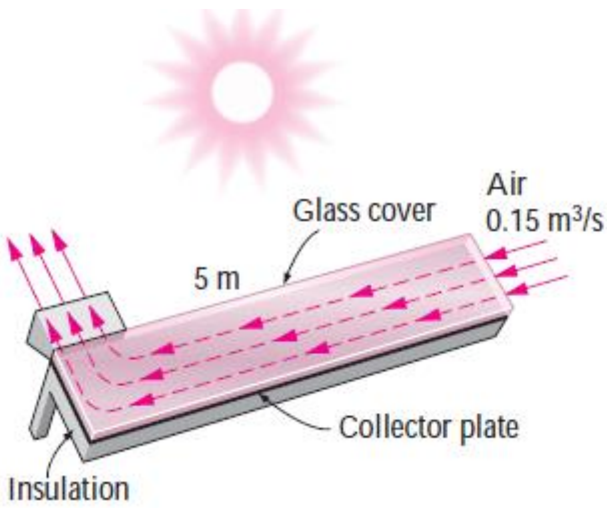
Solution: The velocity profile in fully developed laminar flow in a circular pipe is given by $u(r) = V_c(1 - \frac{r^2}{R^2})$.

The velocity profile in this case is given by $u(r) = 4\left(1 - \frac{r^2}{R^2}\right)$. Comparing the two relations above gives the maximum velocity to be V_c=4.00 m/s. Then the average velocity and volume flow rate become

$$V = \frac{V_c}{2} = \frac{4}{2} = 2\ m/s$$

$$Q = AV = \pi R^2 V = \pi \times 0.02^2 \times 2 = 0.00251\ m^3/s$$

Example: Consider an air solar collector that is 1 m wide and 5 m long and has a constant spacing of 3 cm between the glass cover and the collector plate (ε=0). Air flows at an average temperature of 45°C at a rate of 0.15 m^3/s through the 1-m-wide edge of the collector along the 5-m-long passageway. Disregarding the entrance and roughness effects, determine the pressure drop in the collector. The properties of air at 1 atm and 45° are ρ = 1.109 kg/m3, μ = 1.941×10^{-5} kg/m·s, and ν = 1.750×10-5 m^2/s.



Solution: Mass flow rate, cross-sectional area, hydraulic diameter, average velocity, and the Reynolds number are

$$\dot{m} = \rho \mathbf{Q} = (1.11\ \text{kg/m}^3)(0.15\ \text{m}^3/\text{s}) = 0.1665\ \text{kg/s}$$

$$A_c = a \times b = (1\ \text{m})(0.03\ \text{m}) = 0.03\ \text{m}^2$$

$$D_h = \frac{4A_c}{p} = \frac{4(0.03\ \text{m}^2)}{2(1+0.03)\ \text{m}} = 0.05825\ \text{m}$$

$$V = \frac{Q}{A_c} = \frac{0.15\,m^3/s}{0.03\,m^2} = 5\,m/s$$

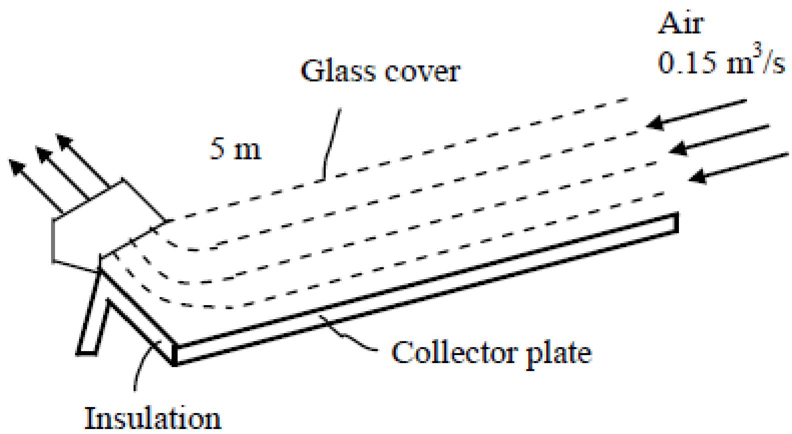
$$Re = \frac{VD_h}{\upsilon} = \frac{(5\,m/s)(0.05825\,m)}{1.750\times10^{-5}\,m^2/s} = 1.664\times10^4$$

Since Re is greater than 4000, the flow is turbulent. The friction factor corresponding to this Reynolds number for a smooth flow section ($\varepsilon/D = 0$) can be obtained from the Moody chart. But we use the Haaland equation,

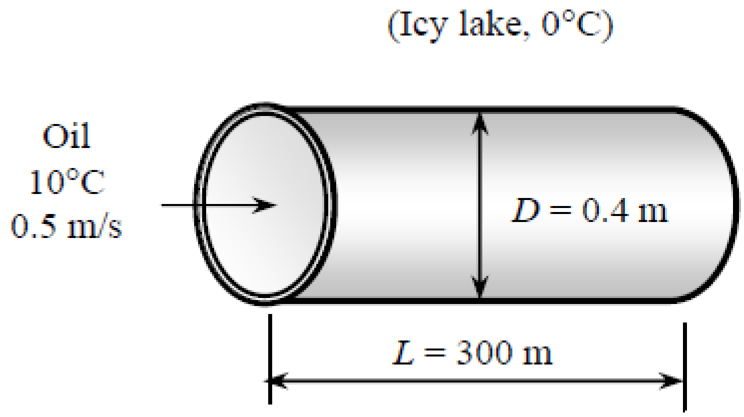
$$\frac{1}{\sqrt{f}} = -1.8\,log\left[\frac{6.9}{Re} + (\frac{\varepsilon}{3.7D})^{1.11}\right]$$

$$\frac{1}{\sqrt{f}} = -1.8\,log\left[\frac{6.9}{16640} + (0)^{1.11}\right] \rightarrow f = 0.027$$

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.027\frac{5}{0.05825}\frac{1.11\times5^2}{2} = 132.1\,Pa$$



Example: Consider the flow of oil with ρ=894 kg/m^3 and μ=2.33 kg/m · s in a 40-cm-diameter pipeline at an average velocity of 0.5 m/s. A 300-m-long section of the pipeline passes through the icy waters of a lake. Disregarding the entrance effects, determine the pumping power required to overcome the pressure losses and to maintain the flow of oil in the pipe.



Solution: The volume flow rate and the Reynolds number in this case are

$$\mathbf{Q} = VA_c = V\frac{\pi D^2}{4} = (0.5\ \mathrm{m/s})\frac{\pi(0.4\ \mathrm{m})^2}{4} = 0.0628\ \mathrm{m^3/s}$$

$$\mathrm{Re} = \frac{\rho VD}{\mu} = \frac{(894\ \mathrm{kg/m^3})(0.5\ \mathrm{m/s})(0.4\ \mathrm{m})}{2.33\ \mathrm{kg/m\cdot s}} = 76.7$$

which is less than 2100. Therefore, the flow is laminar, and the friction factor is

$$f = \frac{64}{\mathrm{Re}} = \frac{64}{76.7} = 0.834$$

Then the pressure drop in the pipe and the required pumping power become

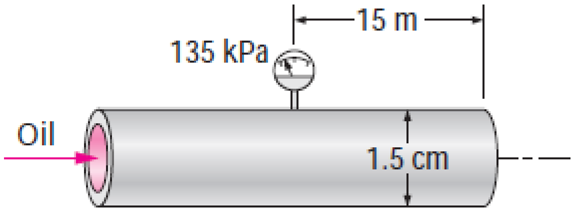
$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2}$$

$$= 0.834\frac{300\ \mathrm{m}}{0.4\ \mathrm{m}}\frac{(894\ \mathrm{kg/m^3})(0.5\ \mathrm{m/s})^2}{2}\left(\frac{1\ \mathrm{kN}}{1000\ \mathrm{kg\cdot m/s}}\right)\left(\frac{1\ \mathrm{kPa}}{1\ \mathrm{kN/m^2}}\right) = 69.9\ \mathrm{kPa}$$

$$\dot{W}_{\mathrm{pump}} = \mathbf{Q}\Delta P = (0.0628\ \mathrm{m^3/s})(69.9\ \mathrm{kPa})\left(\frac{1\ \mathrm{kW}}{1\ \mathrm{kPa\cdot m^3/s}}\right) = \mathbf{4.39\ kW}$$

The power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be much more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

Example: Oil with ρ=876 kg/m^3 and μ=0.24 kg/m · s is flowing through a 1.5-cm-diameter pipe that discharges into the atmosphere at 88 kPa. The absolute pressure 15 m before the exit is measured to be 135 kPa. Determine the flow rate of oil through the pipe if the pipe is (*a*) horizontal, (*b*) inclined 8° upward from the horizontal, and (*c*) inclined 8° downward from the horizontal.



Solution: The pressure drop across the pipe and the cross-sectional area are

$$\Delta P = P_1 - P_2 = 135 - 88 = 47 \text{ kPa}$$

$$A_c = \pi D^2 / 4 = \pi (0.015 \text{ m})^2 / 4 = 1.767 \times 10^{-4} \text{ m}^2$$

(*a*) The flow rate for all three cases can be determined from,

$$\mathbf{Q} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128 \mu L}$$

where θ is the angle the pipe makes with the horizontal. For the horizontal case, θ = 0 and thus sin θ = 0. Therefore,

$$\mathbf{Q}_{\text{horiz}} = \frac{\Delta P \pi D^4}{128 \mu L}$$

$$= \frac{(47 \text{ kPa})\pi (0.015 \text{ m})^4}{128(0.24 \text{ kg/m}\cdot\text{s})(15 \text{ m})}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right)\left(\frac{1000 \text{ N/m}^2}{1 \text{ kPa}}\right) = \mathbf{1.62 \times 10^{-5} \text{ m}^3/\text{s}}$$

(*b*) For uphill flow with an inclination of 8°, we have θ = +8°, and

$$\mathbf{Q}_{\text{uphill}} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128 \mu L}$$

$$= \frac{[(47{,}000 \text{ Pa} - (876 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(15 \text{ m}) \sin 8°]\pi (0.015 \text{ m})^4}{128(0.24 \text{ kg/m}\cdot\text{s})(15 \text{ m})}\left(\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ Pa}\cdot\text{m}^2}\right)$$

$$= \mathbf{1.00 \times 10^{-5} \text{ m}^3/\text{s}}$$

(*c*) For downhill flow with an inclination of 8°, we have θ = -8°, and

$$\dot{Q}_{downhill} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

$$= \frac{[(47{,}000\ \text{Pa} - (876\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(15\ \text{m})\sin(-8°)]\pi(0.015\ \text{m})^4}{128(0.24\ \text{kg/m}\cdot\text{s})(15\ \text{m})}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{Pa}\cdot\text{m}^2}\right)$$

$$= \mathbf{2.24\times10^{-5}\ m^3/s}$$

The flow rate is the highest for downhill flow case, as expected. The average fluid velocity and the Reynolds number in this case are

$$V = \frac{\dot{Q}}{A_c} = \frac{2.24\times10^{-5}\ \text{m}^3/\text{s}}{1.767\times10^{-4}\ \text{m}^2} = 0.127\ \text{m/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(876\ \text{kg/m}^3)(0.127\ \text{m/s})(0.015\ \text{m})}{0.24\ \text{kg/m}\cdot\text{s}} = 7.0$$

which is less than 2100. Therefore, the flow is **laminar** for all three cases, and the analysis above is valid. Note that the flow is driven by the combined effect of pressure difference and gravity. As can be seen from the calculated rates above, gravity opposes uphill flow, but helps downhill flow. Gravity has no effect on the flow rate in the horizontal case.

Example: In an air heating system, heated air at 40°C and 105 kPa absolute is distributed through a 0.2 m × 0.3 m rectangular duct made of commercial steel (*f=0.01833*) at a rate of 0.5 m^3/s. Determine the pressure drop and head loss through a 40-m-long section of the duct. The dynamic viscosity of air at 40 °C is $\mu = 1.918\times10^{-5}$ kg/m·s, and it is independent of pressure. The density of air listed in that table is for 1 atm

Solution: The density at 105 kPa and 315 K can be determined from the ideal gas relation to be

$$\rho = \frac{P}{RT} = \frac{105\ \text{kPa}}{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg.K})(40+273\ \text{K})} = 1.169\ \text{kg/m}^3$$

The hydraulic diameter, average velocity, and Reynolds number are

$$D_h = \frac{4A_c}{p} = \frac{4ab}{2(a+b)} = \frac{4(0.3\ \text{m})(0.20\ \text{m})}{2(0.3+0.20)\ \text{m}} = 0.24\ \text{m}$$

$$V = \frac{Q}{A_c} = \frac{Q}{a \times b} = \frac{0.5\,\text{m}^3/\text{s}}{(0.3\,\text{m})(0.2\,\text{m})} = 8.333\,\text{m/s}$$

$$\text{Re} = \frac{\rho V D_h}{\mu} = \frac{(1.169\,\text{kg/m}^3)(8.333\,\text{m/s})(0.24\,\text{m})}{1.918 \times 10^{-5}\,\text{kg/m}\cdot\text{s}} = 121\,900$$

which is greater than 4000. Therefore, the flow is turbulent. The pressure drop in the duct and the head loss become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2}$$

$$= 0.01833\frac{40\,\text{m}}{0.24\,\text{m}}\frac{(1.169\,\text{kg/m}^3)(8.333\,\text{m/s})^2}{2}$$

$$= 124\,\text{N/m}^2 = \mathbf{124\ Pa}$$

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V^2}{2g}$$

$$= 0.01833\frac{40\,\text{m}}{0.24\,\text{m}}\frac{(8.333\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = \mathbf{10.8\ m}$$

The required pumping power in this case is

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (0.5\,\text{m}^3/\text{s})(124\,\text{Pa})\left(\frac{1\ \text{W}}{1\ \text{Pa}\cdot\text{m}^3/\text{s}}\right) = 62\,\text{W}$$

Therefore, 62 W of mechanical power needs to be imparted to the fluid. The shaft power will be more than this due to fan inefficiency; the electrical power input will be even more due to motor inefficiency.

Example: What is minor loss in pipe flow? How is the minör loss coefficient K_L defined?

Solution: The head losses associated with the flow of a fluid through fittings, valves, bends, elbows, tees, inlets, exits, enlargements, contractions, etc. are called *minor losses*, and are expressed in terms of the *minor loss coefficient* as

$$K_L = \frac{h_L}{V^2/2g}$$

Example: Define equivalent length for minor loss in pipe flow. How is it related to the minor loss coefficient?

Solution: Equivalent length is the length of a straight pipe which would give the same head loss as the minor loss component. It is related to the minor loss coefficient by

$$L_e = \frac{D}{f} K_L$$

Example: What are the primary considerations when selecting a flowmeter to measure the flow rate of a fluid?

Solution: The primary considerations when selecting a *flowmeter* are **cost, size, pressure drop, capacity, accuracy, and reliability**.

Example: Explain how flow rate is measured with a Pitot-static tube, and discuss its advantages and disadvantages with respect to cost, pressure drop, reliability, and accuracy.

Solution: *Pitot-static tube* measures the difference between the stagnation and static pressure, which is the *dynamic pressure*, which is related to flow velocity by $V = \sqrt{\frac{2(P_1 - P_2)}{\rho}}$. Once the average flow velocity is determined, the flow rate is calculated from Q=AV. The Pitot tube is inexpensive, highly reliable since it has no moving parts, it has very small pressure drop, and its accuracy (which is about 3%) is acceptable for most engineering applications.

Example: Explain how flow rate is measured with obstruction type flowmeters. Compare orifice meters, flow nozzles, and Venturi meters with respect to cost, size, head loss, and accuracy.

Solution: An *obstruction flowmeter* measures the flow rate through a pipe by constricting the flow, and measuring the decrease in pressure due to the increase in velocity at (or downstream of) the constriction site. The flow rate for obstruction flowmeters is expressed as

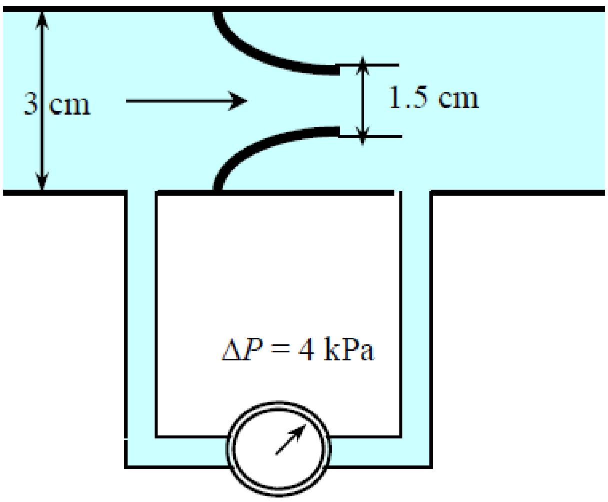
$$\boldsymbol{Q = A_0 A_0 \sqrt{\frac{2(P_1 - P_2)}{[\rho(1 - \beta^4)]}}}$$

Where $A_0 = \frac{\pi d^2}{4}$ is the cross-sectional area of the obstruction and β = *d* /*D* is the ratio of obstruction diameter to the pipe diameter. Of the three types of obstruction flow meters, the orifice meter is the cheapest, smallest, and least accurate, and it causes the greatest head loss. The Venturi meter is the most expensive, the largest, the most accurate, and it causes the smallest head loss. The nozzle meter is between the orifice and Venturi meters in all aspects. As diameter ratio β decreases, the pressure drop across the flowmeter increases, leading to a larger minör head loss associated with the flowmeter, but increasing the sensitivity of the measurement.

Example: How do positive displacement flowmeters operate? Why are they commonly used to meter gasoline, water, and natural gas?

Solution: A positive displacement flowmeter operates by trapping a certain amount of incoming fluid, displacing it to the discharge side of the meter, and counting the number of such discharge-recharge cycles to determine the total amount of fluid displaced. Positive displacement flowmeters are commonly used to meter gasoline, water, and natural gas because they are simple, reliable, inexpensive, and highly accurate even when the flow is unsteady.

Example: The flow rate of ammonia at 10°C (ρ=624.6 kg/m^3 and μ=1.697× 10^{-4} kg/m · s) through a 3-cm-diameter pipe is to be measured with a 1.5-cm-diameter flow nozzle equipped with a differential pressure gage. If the gage reads a pressure differential of 4 kPa, determine the flow rate of ammonia through the pipe, and the average flow velocity.



Solution: The diameter ratio and the throat area of the meter are

$$\beta = d / D = 1.5 / 3 = 0.50$$

$$A_0 = \pi d^2 / 4 = \pi (0.015 \text{ m})^2 / 4 = 1.767 \times 10^{-4} \text{ m}^2$$

Noting that ΔP = 4 kPa = 4000 N/m^2, the flow rate becomes

$$\mathbf{Q} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

$$= (1.767\times10^{-4}\ \text{m}^2)(0.96)\sqrt{\frac{2\times4000\ \text{N/m}^2}{(624.6\ \text{kg/m}^3)((1-0.50^4)}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)}$$

$$= \mathbf{0.627\times10^{-3}\ m^3/s}$$

which is equivalent to 0.627 L/s. The average flow velocity in the pipe is determined by dividing the flow rate by the crosssectional area of the pipe,

$$V = \frac{\mathbf{Q}}{A_c} = \frac{\mathbf{Q}}{\pi D^2/4} = \frac{0.627\times10^{-3}\ \text{m}^3/\text{s}}{\pi(0.03\ \text{m})^2/4} = \mathbf{0.887\ m/s}$$

The Reynolds number of flow through the pipe is

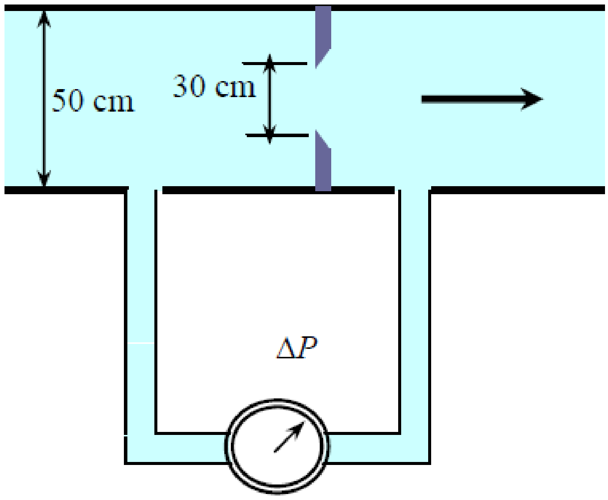
$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(624.6\ \text{kg/m}^3)(0.887\ \text{m/s})(0.03\ \text{m})}{1.697\times10^{-4}\ \text{kg/m}\cdot\text{s}} = 9.79\times10^4$$

Substituting the β and Re values into the orifice discharge coefficient relation gives

$$C_{\mathbf{n}} = 0.9975 - \frac{6.53\beta^{0.5}}{\text{Re}^{0.5}} = 0.9975 - \frac{6.53(0.50)^{0.5}}{(9.79\times10^4)^{0.5}} = 0.983$$

which is about 2% different than the assumed value of 0.96. Using this refined value of *Cn* , the flow rate becomes 0.642 L/s, which differs from our original result by only 2.4%. If the problem is solved using an equation solver such as EES, then the problem can be formulated using the curve-fit formula for *Cd* (which depends on Reynolds number), and all equations can be solved simultaneously by letting the equation solver perform the iterations as necessary.

Example: The flow rate of water at 20°C (ρ=998 kg/m^3 and μ=1.002 . 10^{-3} kg/m · s) through a 50-cm-diameter pipe is measured with an orifice meter with a 30-cm-diameter opening to be 250 L/s. Determine the pressure difference indicated by the orifice meter and the head loss. The discharge coefficient of the orifice meter is C_0 = 0.61.



Solution: The diameter ratio and the throat area of the orifice are

$$\beta = d / D = 30 / 50 = 0.60$$

$$A_0 = \pi d^2 / 4 = \pi (0.30 \text{ m})^2 / 4 = 0.07069 \text{ m}^2$$

For a pressure drop of $\Delta P = P1 - P2$ across the orifice plate, the flow rate is expressed as

$$\mathbf{Q} = A_o C_\mathbf{0} \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

Substituting,

$$0.25 \text{ m}^3/s = (0.07069 \text{ m}^2)(0.61) \sqrt{\frac{2\Delta P}{(998 \text{ kg/m}^3)((1-0.60^4)}}$$

which gives the pressure drop across the orifice plate to be

$$\Delta P = 14{,}600 \text{ kg} \cdot \text{m/s}^2 = \mathbf{14.6\ kPa}$$

It corresponds to a water column height of

$$h_w = \frac{\Delta P}{\rho_w g} = \frac{14{,}600 \text{ kg} \cdot \text{m/s}^2}{(998 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} = 1.49 \text{ m}$$

The percent pressure (or head) loss for orifice meters is given in Fig. for β = 0.6 to be 64%. Therefore,

$$h_L = (\text{Permanent loss fraction})(\text{Total head loss}) = 0.64(1.49 \text{ m}) = \mathbf{0.95\ m\ H_2O}$$

The head loss between the two measurement sections can also be estimated from the energy equation. Since $z_1 = z_2$, the head form of the energy equation simplifies to

$$h_L \approx \frac{P_1 - P_2}{\rho_f g} - \frac{V_2^2 - V_1^2}{2g}$$

$$= h_w - \frac{\left[(D/d)^4 - 1\right] V_1^2}{2g}$$

$$1.49 \text{ m} - \frac{\left[(50/30)^4 - 1\right](1.27 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = \mathbf{0.940\ m\ H_2O}$$

$$\text{where} \quad V_1 = \frac{\mathbf{Q}}{A_c} = \frac{\mathbf{Q}}{\pi D^2/4} = \frac{0.250 \text{ m}^3/\text{s}}{\pi(0.50 \text{ m})^2/4} = 1.27 \text{ m/s}$$

This head loss, though a reasonable estimate, is lower than the exact one calculated above because it does not take into account irreversible losses downstream of the pressure taps, where the flow is still "recovering," and is not yet fully developed. The Reynolds number of flow through the pipe is

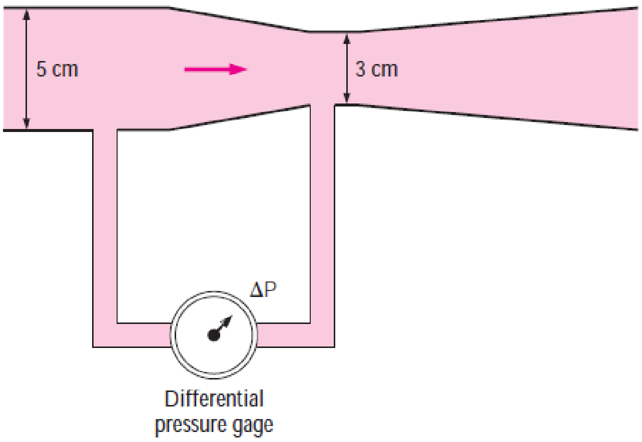
$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(998 \text{ kg/m}^3)(1.27 \text{ m/s})(0.50 \text{ m})}{1.002 \times 10^{-3} \text{ kg/m}\cdot\text{s}} = 6.32 \times 10^5$$

Substituting β and Re values into the orifice discharge coefficient relation

$$C_0 = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{\text{Re}^{0.75}}$$

gives C_0 = 0.605, which is very close to the assumed value of 0.61.

Example: A Venturi meter equipped with a differential pressure gage is used to measure the flow rate of water at 15°C (ρ=999.1 kg/m^3) through a 5-cm-diameter horizontal pipe. The diameter of the Venturi neck is 3 cm, and the measured pressure drop is 5 kPa. Taking the discharge coefficient to be 0.98, determine the volume flow rate of water and the average velocity through the pipe.



Solution: The diameter ratio and the throat area of the meter are

$$\beta = d / D = 3/5 = 0.60$$

$$A_0 = \pi d^2 / 4 = \pi (0.03\text{ m})^2 / 4 = 7.069 \times 10^{-4}\text{ m}^2$$

Noting that ΔP = 5 kPa = 5000 N/m^2, the flow rate becomes

$$\mathbf{Q} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

$$= (7.069 \times 10^{-4}\text{ m}^2)(0.98)\sqrt{\frac{2 \times 5000\text{ N/m}^2}{(999.1\text{ kg/m}^3)((1-0.60^4)}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)}$$

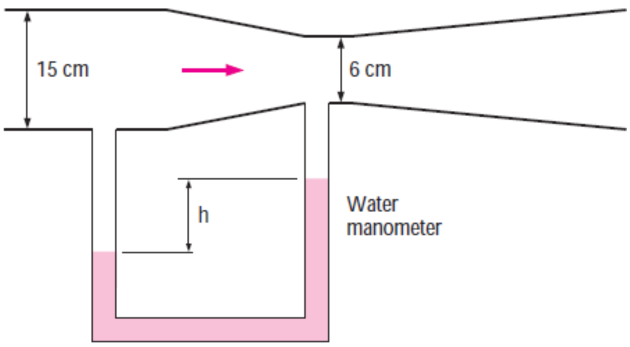
$$= \mathbf{0.00235\ m^3/s}$$

which is equivalent to 2.35 L/s. The average flow velocity in the pipe is determined by dividing the flow rate by the crosssectional area of the pipe,

$$V = \frac{\mathbf{Q}}{A_c} = \frac{\mathbf{Q}}{\pi D^2/4} = \frac{0.00235\text{ m}^3/\text{s}}{\pi(0.05\text{ m})^2/4} = \mathbf{1.20\ m/s}$$

Note that the flow rate is proportional to the square root of pressure difference across the Venturi meter.

Example: The mass flow rate of air at 20°C (ρ=1.204 kg/m^3) through a 15-cm-diameter duct is measured with a Venturi meter equipped with a water manometer. The Venturi neck has a diameter of 6 cm, and the manometer has a maximum differential height of 40 cm. Taking the discharge coefficient to be 0.98, determine the maximum mass flow rate of air this Venturi meter can measure. We take the density of water to be ρw = 1000 kg/m^3. The discharge coefficient of Venturi meter is given to be C_V = 0.98.



Solution: The diameter ratio and the throat area of the meter are

$$\beta = d / D = 6/15 = 0.40$$

$$A_0 = \pi d^2 / 4 = \pi (0.06\ \text{m})^2 / 4 = 0.002827\ \text{m}^2$$

The pressure drop across the Venturi meter can be expressed as

$$\Delta P = P_1 - P_2 = (\rho_w - \rho_f) g h$$

Then the flow rate relation for obstruction meters becomes

$$\mathbf{Q} = A_o \mathbf{C_v} \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

$$= A_o \mathbf{C_v} \sqrt{\frac{2(\rho_w - \rho_f) g h}{\rho_f (1-\beta^4)}} = A_o \mathbf{C_v} \sqrt{\frac{2(\rho_w / \rho_{air} - 1) g h}{1-\beta^4}}$$

Substituting and using h = 0.40 m, the maximum volume flow rate is determined to be

$$\mathbf{Q} = (0.002827\ \text{m}^2)(0.98) \sqrt{\frac{2(1000/1.204 - 1)(9.81\ \text{m/s}^2)(0.40\ \text{m})}{1 - 0.40^4}}$$

$$= 0.2265\ \text{m}^3 / s$$

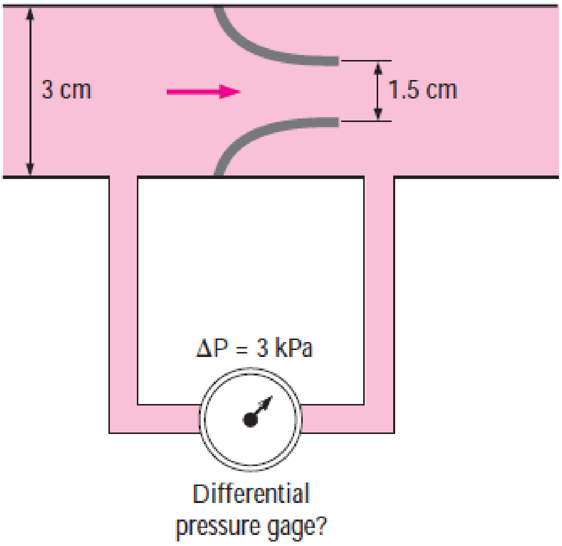
Then the maximum mass flow rate this Venturi meter can measure is

$$\dot{m} = \rho \mathbf{Q} = (1.204\ \text{kg/m}^3)(0.2265\ \text{m}^3/\text{s}) = \mathbf{0.273\ kg/s}$$

Also, the average flow velocity in the duct is

$$V = \frac{\mathbf{Q}}{A_c} = \frac{\mathbf{Q}}{\pi D^2 / 4} = \frac{0.2265\ \text{m}^3/\text{s}}{\pi (0.15\ \text{m})^2 / 4} = 12.8\ \text{m/s}$$

Example: A flow nozzle equipped with a differential pressure gage is used to measure the flow rate of water at 10°C (ρ=999.7 kg/m^3 and μ=1.307×10^{-3} kg/m·s) through a 3- cm-diameter horizontal pipe. The nozzle exit diameter is 1.5 cm, and the measured pressure drop is 3 kPa. Determine the volume flow rate of water, the average velocity through the pipe, and the head loss.



Solution: The diameter ratio and the throat area of the meter are

$$\beta = d / D = 1.5 / 3 = 0.50$$

$$A_0 = \pi d^2 / 4 = \pi (0.015\ \text{m})^2 / 4 = 1.767 \times 10^{-4}\ \text{m}^2$$

Noting that ΔP = 4 kPa = 4000 N/m^2, the flow rate becomes

$$\mathbf{Q} = A_o \mathbf{C_n} \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}}$$

$$= (1.767 \times 10^{-4}\ \text{m}^2)(0.96) \sqrt{\frac{2 \times 3000\ \text{N/m}^2}{(999.7\ \text{kg/m}^3)((1 - 0.50^4)} \left(\frac{1\ \text{kg} \cdot \text{m/s}^2}{1\ \text{N}} \right)}$$

$$= \mathbf{0.429 \times 10^{-3}\ m^3/s}$$

which is equivalent to 0.429 L/s. The average flow velocity in the pipe is determined by dividing the flow rate by the crosssectional area of the pipe,

$$V = \frac{\mathbf{Q}}{A_c} = \frac{\mathbf{Q}}{\pi D^2 / 4} = \frac{0.429 \times 10^{-3}\ \text{m}^3/\text{s}}{\pi (0.03\ \text{m})^2 / 4} = \mathbf{0.607\ m/s}$$

The water column height corresponding to a pressure drop of 3 kPa is

$$h_w = \frac{\Delta P}{\rho_w g} = \frac{3000\ \text{kg} \cdot \text{m/s}^2}{(999.7\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} = 0.306\ \text{m}$$

The percent pressure (or head) loss for nozzle meters is given in Fig. 8-59 for β = 0.5 to be 62%. Therefore,

$$h_L = (\text{Permanent loss fraction})(\text{Total head loss}) = 0.62(0.306\ \text{m}) = \mathbf{0.19\ m\ H_2O}$$

The head loss between the two measurement sections can be determined from the energy equation, which simplifies to (for $z_1 = z_2$)

$$h_L = \frac{P_1 - P_2}{\rho_f g} - \frac{V_2^2 - V_1^2}{2g} = h_w - \frac{[(D/d)^4 - 1]V_1^2}{2g} = 0.306\ \text{m} - \frac{[(3/1.5)^4 - 1](0.607\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 0.024\ \text{m H2O}$$

The Reynolds number of flow through the pipe is

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(999.7\ \text{kg/m}^3)(0.607\ \text{m/s})(0.03\ \text{m})}{1.307\times10^{-3}\ \text{kg/m}\cdot\text{s}} = 1.39\times10^4$$

Substituting the β and Re values into the orifice discharge coefficient relation gives

$$\mathbf{C_n} = 0.9975 - \frac{6.53\beta^{0.5}}{\text{Re}^{0.5}} = 0.9975 - \frac{6.53(0.50)^{0.5}}{(1.39\times10^4)^{0.5}} = 0.958$$

which is practically identical to the assumed value of 0.96.

Example: Milk with ρ=1010 kg/m^3 and μ=0.002 kg/m·s is flowing through a 3-cm-diameter and 100 m long horizontal pipe. The flow rate of milk is 0.0025 m^3/s and pipe is assumed smoth i.e. ε=0. Determine *a)* type of flow, *b)* the friction factor, *c*) the wall shear stress (τ_w), d) the friction velocity (u*), e) average velocity at y=R-r = 8 mm, f) the approximate thickness of the viscous sublayer, g) the pressure drop and head loses.

Solution: **a)** $V = \frac{4Q}{\pi D^2} = \frac{4\times0.0025}{\pi 0.03^2} = 3.54\ m/s$

$Re = \frac{\rho VD}{\mu} = \frac{1010\times3.54\times0.03}{0.002}$=53631 The flow type is turbulent because of Re=53631 >4000.

b) We can determine the friction factor by following equation due to Re=53631< 100000 and the pipe is assumed smooth (ε=0).

$f = \frac{0.316}{Re^{0.25}} = \frac{0.316}{53631^{0.25}} = 0.021$ or Haaland equation can be used.

$$\frac{1}{\sqrt{f}} = -1.8log\left[\left(\frac{\varepsilon}{3.7D}\right)^2 + \frac{6.9}{Re}\right] \quad \frac{1}{\sqrt{f}} = -1.8log\left[0 + \frac{6.9}{53631}\right] \rightarrow f = 0.0204$$

c) $\tau_w = f\rho\frac{V^2}{8} = 0.021\times1010\frac{3.54^2}{8} = 33.22\ Pa$

d) The friction velocity (u*)

$u^* = V(\frac{f}{8})^{1/2} = 3.54(\frac{0.021}{8})^{\frac{1}{2}} = 0.1814\, m/s$ or

$u^* = (\frac{\tau_w}{\rho})^{1/2} = (\frac{33.22}{1010})^{\frac{1}{2}} = 0.1814\, m/s$

e) The average velocity at y=R-r = 8 mm,

$$\bar{u} = u^* \times 2.5 \times ln\left\{\frac{yu^*}{\nu}\right\} + 5 \times u^*$$

$$\bar{u} = 0.1814 \times 2.5 \times ln\left\{\frac{0.008 \times 0.1814}{0.002/1010}\right\} + 5 \times 0.1814 = 3.9\, m/s$$

f) The approximate thickness of the viscous sublayer,

$$\delta_s = 5\frac{\nu}{u^*} = 5\frac{0.002/1010}{0.1814} = 0.00005458\, mm$$

g) The pressure drop and head loses

$\frac{\Delta P}{L} = \frac{4\tau_w}{D} \rightarrow \Delta P = L\frac{4\tau_w}{D} = 100\frac{4 \times 33.22}{0.03} = 442933\, Pa$ and

head loss $h_L = \frac{\Delta P}{\gamma} = \frac{442933}{1010 \times 9.81} = 44.70\, m$ or

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = 0.021\frac{100}{0.03}\frac{3.54^2}{2 \times 9.81} = 44.71\, m$$

$$\Delta P = h_L \gamma = 44.71 \times 1010 \times 9.81 = 442991\, Pa$$

NOTES:

***10^5 is read as “ten to the fifth power”

*****10^3 could be called "10 to the third power", "10 to the power 3" or simply "10 cubed"

*** 10^4 could be called "10 to the fourth power", "10 to the power 4" or "10 to the 4"

***10^{-3} is *read* that as ten to the minus third power.

Yüz Beş: One Hundred Five

Yüz On: One Hundred Ten

Yüz Yirmi: One Hundred Twenty

Yüz Yirmi Beş: One Hundred Twenty Five

İki Yüz Yirmi Beş: Two Hundred Twenty Five

Beş Yüz Elli: Five Hundred Fifty

101325: A Hundred and One Thousand Three Hundred and Twenty Five