INTEREST AND INVESTMENT COSTS

DEFINITIONS OF INTEREST

According to the classical definition, interest is the money returned to the owners of capital for use of their capital.

Modern economists seldom adhere to the classical definition. Instead, they prefer to substitute the term ***return on capital*** or ***return on investment*** for the classical ***interest.***

Engineers define interest as the ***compensation paid for the use of borrowed capital.***

TYPES OF INTEREST

Simple Interest

For example, if \$100 were the compensation demanded for giving someone the use of \$1000 for a period of one year, the principal would be \$1000, and the rate of interest would be lOO/lOOO = 0.1 or 10 percent/year.

The simplest form of interest requires compensation payment at a constant interest rate based only on the original principal. Thus, if \$1000 were loaned for a total time of 4 years at a constant interest rate of 10 percent/year, the simple interest earned would be

$$\$1000 \times 0.1 \times 4 = \$400$$

If ***P*** represents the principal, n the number of time units or interest periods, and ***i*** the interest rate based on the length of one interest period, the amount of simple interest Z during n interest periods is

$$Z=Pin$$

The principal must be repaid eventually; therefore, the entire amount S of principal plus simple interest due after n interest periods is

$$\boldsymbol{S = P + Z = P(1 + in)}$$

Compound Interest

Interest, like all negotiable capital, has a time value. If the interest were paid at the end of each time unit, the receiver could put this money to use for earning additional returns. Compound ***interest*** takes this factor into account by stipulating that interest is due regularly at the end of each interest period.

The compound amount due after any discrete number of interest perio can be determined as follows:

Period	Principal at start of period	Interest earned during period (i = interest rate based on length of one period)	Compound amount S at end of period
1	P	Pi	$P + Pi = P(1 + i)$
2	$P(1 + i)$	$P(1 + i)(i)$	$P(1 + i) + P(1 + i)(i) = P(1 + i)^2$
3	$P(1 + i)^2$	$P(1 + i)^*(i)$	$P(1 + i)^2 + P(1 + i)^*(i) = P(1 + i)^3$
n	$P(1 + i)^{n-1}$	$P(1 + i)^{n-1}(i)$	$P(1 + i)^n$

Therefore, the total amount of principal plus compounded interest due after interest periods and designated as S is†

$$S = P(1 + i)^n \quad ($$

NOMINAL AND EFFECTIVE INTEREST RATES

Then the interest rate based on the length of one interest period is *r/m*, and the amount S after 1 year is

$$S_{\text{after 1 year}} = P\left(1 + \frac{r}{m}\right)^m \tag{6}$$

Designating the effective interest rate as i_{eff}, the amount S after 1 year can be expressed in an alternate form as

$$S_{\text{after 1 year}} = P(1 + i_{\text{eff}}) \tag{7}$$

By equating Eqs. (6) and **(7)**, the following equation can be obtained for the effective interest rate in terms of the nominal interest rate and the number of periods per year:

$$\text{Effective annual interest rate} = i_{\text{eff}} = \left(1 + \frac{r}{m}\right)^m - 1 \tag{8}$$

Similarly, by definition,

$$\textbf{Nominal annual interest rate} = m\left(\frac{r}{m}\right) = r \tag{9}$$

The Basic Equations for Continuous Interest Compounding

Equations (6), (7), and (8) represent the basic expressions from which continuous-interest relationships can be developed. The symbol r represents the nominal interest rate with m interest periods per year. If the interest is compounded continuously, m approaches infinity, and Eq. (6) can be written as

$$S_{\text{after } n \text{ years}} = P \lim_{m \to \infty} \left(1 + \frac{r}{m}\right)^{mn} = P \lim_{m \to \infty} \left(1 + \frac{r}{m}\right)^{(m/r)(rn)} \tag{10}$$

The fundamental definition for the base of the natural system of logarithms ($e = 2.71828$) is†

$$\lim_{m \to \infty} \left(1 + \frac{r}{m}\right)^{m/r} = e = 2.71828\ldots \tag{11}$$

Thus, with continuous interest compounding at a nominal annual interest rate of r, the *amount* S an initial *principal* P will compound to in n years is†‡

$$S = Pe^{rn} \tag{12}$$

Similarly, from Eq. (8), the effective annual interest rate i_{eff}, which is the conventional interest rate that most executives comprehend, is expressed in terms of the nominal interest rate r compounded continuously as

$$i_{\text{eff}} = e^{r} - 1 \tag{13}$$

$$r = \ln(i_{\text{eff}} + 1) \tag{14}$$

Therefore,

$$e^{rn} = (1 + i_{\text{eff}})^{n} \tag{15}$$

and

$$S = Pe^{rn} = P(1 + i_{\text{eff}})^{n} \tag{16}$$