CAUCHY-RIEMANN DENKLEMLERİ

$$f(z)=u(x,y)+\boldsymbol{i}\,v(x,y)$$

$u(x,y)$ ve $v(x,y)$ bileşen fonksiyonlarının 1. mertebeden kısmi türevi elde edilecektir.

$$f'(z_0)=\lim_{z\to z_0}\frac{f(z)-f(z_0)}{z-z_0}$$

$$z=x+iy \quad , \quad z_0=x_0+iy_0 \;,\; \Delta z=z-z_0$$

$$\mathrm{Re}\left[f'(z_0)\right]=\lim_{(\Delta x,\Delta y)\to(0,0)}\mathrm{Re}\left\{\frac{f(z_0+\Delta z)-f(z_0)}{\Delta z}\right\}$$

$$\mathrm{Im}\left[f'(z_0)\right]=\lim_{(\Delta x,\Delta y)\to(0,0)}\mathrm{Im}\left\{\frac{f(z_0+\Delta z)-f(z_0)}{\Delta z}\right\}$$

$$\frac{f(z_0+\Delta z)-f(z_0)}{\Delta z}$$

$$=\frac{u(x_0+\Delta x,\, y_0+\Delta y)-u(x_0,y_0)}{\Delta x+\boldsymbol{i}\,\Delta y}$$

$$+\boldsymbol{i}\ \frac{\left[v(x_0+\Delta x, y_0+\Delta y)-v(x_0,y_0)\right]}{\Delta x+\boldsymbol{i}\,\Delta y}$$

x'e göre birinci mertebeden türevler

$\Delta y=0 \quad , \ \Delta x \to 0$:

$$\mathrm{Re}\left[f'(z_0)\right]=\lim_{\Delta x\to 0}\ \frac{u(x_0+\Delta x\,,y_0)-u(x_0\,,y_0)}{\Delta x}$$

$$=u_x(x_0\,,y_0)$$

$$\mathrm{Im}\left[f'(z0)\right]=\lim_{\Delta x\to 0}\ \frac{v(x_0+\Delta x\,,y_0)-v(x_0,y_0)}{\Delta x}$$

$$=v_x(x_0,y_0)$$

y'e göre birinci mertebeden türevler:

$$\Delta x = 0 \quad , \quad \Delta y \to 0$$

$$f'(z_0) = \lim_{\Delta y \to 0} \frac{\left[u(x_0, y_0 + \Delta y) - u(x_0, y_0)\right]}{\boldsymbol{i}\,\Delta y}$$

$$+ \lim_{\Delta y \to 0} \frac{\boldsymbol{i}\left[v(x_0, y_0 + \Delta y) - v(x_0, y_0)\right]}{\boldsymbol{i}\,\Delta y}$$

$$\mathrm{Re}\left[f'(z_0)\right] = \lim_{\Delta y \to 0} \frac{v(x_0\,, y_0 + \Delta y) - v(x_0\,, y_0)}{\Delta x}$$

$$= v_y(x_0\,, y_0)$$

$$\mathrm{Im}\left[f'(z_0)\right] = -\lim_{\Delta x \to 0} \frac{u(x_0\,, y_0 + \Delta y) - u(x_0, y_0)}{\Delta y}$$

$$= -u_y(x_0, y_0)$$

$$f'(z_0) = v_y(x_0, y_0) - \boldsymbol{i}\, u_y(x_0, y_0)$$

ve

$$f'(z_0) = u_x(x_0, y_0) + \boldsymbol{i}\, v_x(x_0, y_0)$$

$$u_x(x_0, y_0) = v_y(x_0, y_0)$$

$$u_y(x_0, y_0) = -v_x(x_0, y_0)$$

Teorem : $f(z)=u(x,y)+\boldsymbol{i}\,v(x,y)$ fonksiyonu olsun $z_0=x_0+\boldsymbol{i}\,y_0$ noktasında $f'(z_0)=u_x+\boldsymbol{i}\,v_x$ varsa, o zaman u(x,y) ve v(x,y)'nin, x_0 ve y_0 noktasında 1. mertebeden türevleri olmalıdır ve bu türevler **Cauchy-Riemann** denklemlerini sağlamalıdır.

KUTUPSAL KOORDİNATLARDA CAUCUHY-RIEMANN DENKLEMLERİ

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$z = x + iy \qquad z = re^{i\theta} \qquad z \neq 0$$

$$w = f(z) = u(r,\theta) + \boldsymbol{i}\, v(r,\theta)$$

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\,\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\,\frac{\partial y}{\partial r}$$

$$\frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x}\,\frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y}\,\frac{\partial y}{\partial \theta}$$

Benzer formüller $v(r,\theta)$ için de yazılabilir.

$$u_r = u_x \cos\theta + u_y \sin\theta$$

$$u_\theta = -u_x r \sin\theta + u_y r \cos\theta$$

$$v_r = v_x \cos\theta + v_y \sin\theta$$

$$v_\theta = -v_x r \sin\theta + v_y r \cos\theta$$

Cauchy-Riemann denklemleri kullanılırsa

$$u_x = v_y \qquad u_y = -v_x$$

$$u_r = \mathbf{v_y} \cos\theta - \mathbf{v_x} \sin\theta$$

$$u_\theta = -\mathbf{v_y} r \sin\theta - \mathbf{v_x} r \cos\theta$$

$$v_r = v_x \cos\theta + v_y \sin\theta$$

$$v_\theta = -v_x r \sin\theta + v_y r \cos\theta$$

kutupsal koordinatlarda **Cauchy-Riemann denklemleri** elde edilir.

$$v_\theta = r\, u_r \qquad u_\theta = -r\, v_r$$

KAYNAKLAR

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