

ÜSTEL FONKSİYONLAR

$$z = x + iy$$

$$\begin{aligned} f(z) &= e^z = e^{x+iy} = e^x e^{iy} \\ &= e^x (\cos y + i \sin y) \end{aligned}$$

$$f(z) = u(x, y) + i v(x, y)$$

$$u(x, y) = e^x \cos y$$

$$v(x, y) = e^x \sin y$$

$$e^0 = 1 \quad , \quad \frac{1}{e^z} = e^{-z}$$

$$\frac{e^{z_1}}{e^{z_2}} = e^{z_1 - z_2}$$

$$(e^z)^n = e^{zn} \quad n = 0, \pm 1, \pm 2, \dots$$

$$e^{2\pi i} = 1 \Rightarrow e^{z+2\pi i} = e^z$$

$$\frac{d}{dz} e^{iz} = i e^{iz}$$

$$|e^z| = |e^{x+iy}| = e^x$$

TRİGONOMETRİK FONKSİYONLAR

$$e^{\pm ix} = \cos x \pm i \sin x$$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$

$$\frac{d}{dz} \cos z = -\sin z$$

$$\frac{d}{dz} \sin z = \cos z$$

$$\sin(-z) = -\sin(z)$$

$$\cos(-z) = \cos(z)$$

$$\sin(z_1 + z_2) = \sin z_1 \cos z_2 + \cos z_1 \sin z_2$$

$$\cos(z_1 + z_2) = \cos z_1 \cos z_2 - \sin z_1 \sin z_2$$

$$\sin^2 z + \cos^2 z = 1$$

$$\sin 2z = 2 \sin z \cos z$$

$$\cos 2z = \cos^2 z - \sin^2 z$$

$$\sin\left(z + \frac{\pi}{2}\right) = \cos z$$

$$\sin\left(z - \frac{\pi}{2}\right) = -\cos z$$

$$|\sin z|^2 = \sin^2 x + \sinh^2 y$$

$$|\cosh z|^2 = \cos^2 x + \sinh^2 y$$

$\sin z = 0$ ancak ve ancak $z = n\pi$ $n = 0, \pm 1, \pm 2, \dots$

$\cos z = 0$ ancak ve ancak $z = \frac{\pi}{2} + n\pi$ $n = 0, \pm 1, \pm 2, \dots$

$$\tan z = \frac{\sin z}{\cos z}$$

$$\cot z = \frac{\cos z}{\sin z}$$

$$\sec z = \frac{1}{\cos z}$$

$$\operatorname{cosec} z = \frac{1}{\sin z}$$

- y reel sayı olduğu zaman hiperbolik fonksiyonlar tanımlanabilir.

$$\sinh y = \frac{e^y - e^{-y}}{2}$$

$$\cosh y = \frac{e^y + e^{-y}}{2}$$

$z_1 = x$ ve $z_2 = i y$ olmak üzere

$$\sin(z_1 + z_2) = \sin(x + i y) = \sin x \cosh y + i \cos x \sinh y$$

$$\cos(z_1 + z_2) = \cos(x + i y) = \cos x \cosh y - i \sin x \sinh y$$

$$\sin(i y) = i \sinh y$$

$$\cos(i y) = \cosh y$$

KAYNAKLAR

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