



GGY 112

İSTATİSTİK

Doç. Dr. Furkan BAŞER
Ankara Üniversitesi Uygulamalı Bilimler Fakültesi



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9. HAFTA

REGRESYON



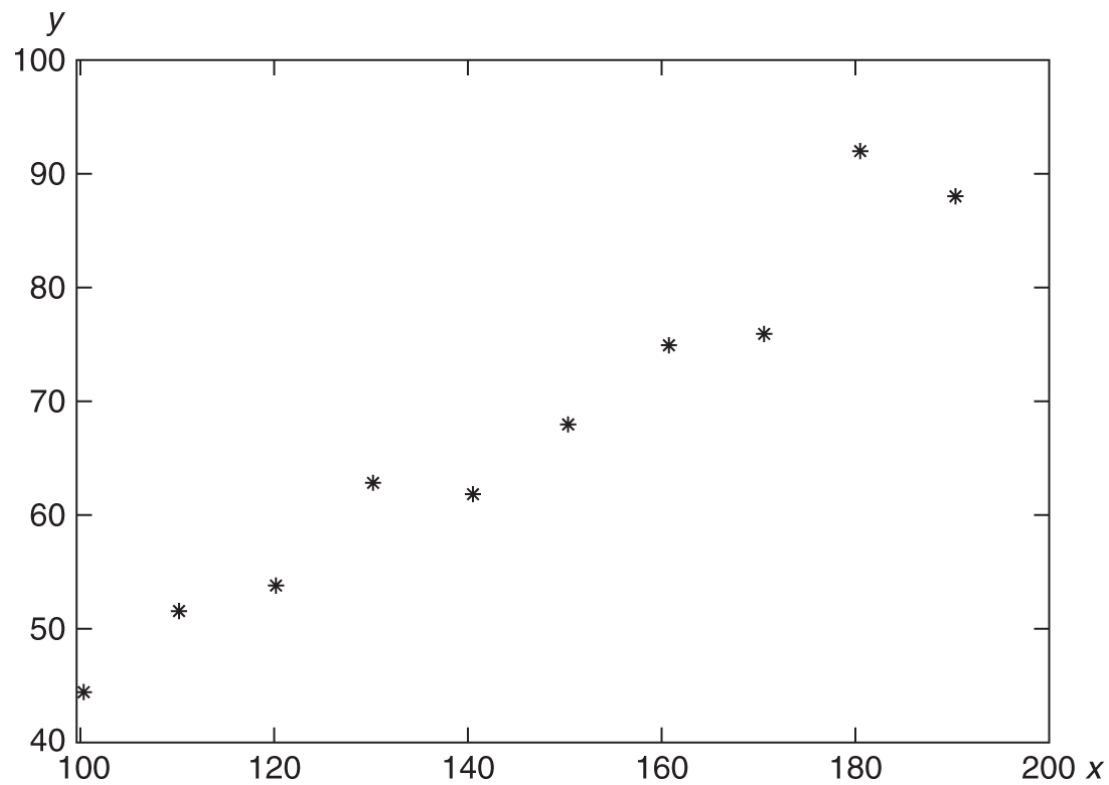
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Serpme diyagramı



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REGRESYON PARAMETRELERİNİN EN KÜÇÜK KARELER TAHMİN EDİCİLERİ



$$SS = \sum_{i=1}^n (Y_i - A - Bx_i)^2$$



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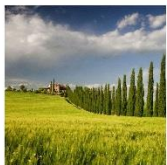


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TAHMİN EDİCİLERİN DAĞILIMI

$$Y_i \sim \mathcal{N}(\alpha + \beta x_i, \sigma^2)$$



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TAHMİN EDİCİLERİN DAĞILIMI

Simple Linear Regression

Sample size = 15

$x =$

$y =$

Data Points

49,12	↑
52,14	
38,9	
55,16	
32,8	
57,18	
54,14	
44,12	↓

Şeklin devamı bir sonraki slaytta yer almaktadır!



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TAHMİN EDİCİLERİN DAĞILIMI

The least squares estimators are as follows:

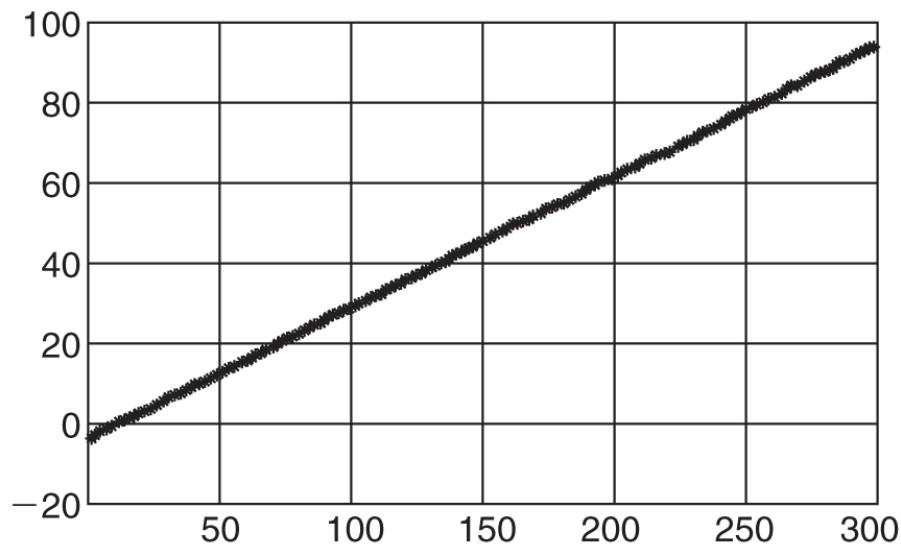
$$a = -2.51$$

$$\text{Average } x \text{ value} = 46.13$$

$$b = 0.32$$

$$\text{Sum of squares of the } x \text{ values} = 33212.0$$

The estimated regression line is $Y = -2.51 + 0.32x$



$$S(x, Y) = 416.2$$

$$S(x, x) = 1287.73$$

$$S(Y, Y) = 147.6$$

$$SS_R = 13.08$$



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TAHMİN EDİCİLERİN DAĞILIMI

ÖRNEK 9.3a Aşağıda x ve Y ile ilgili veriler bulunmaktadır. x , belirli bir ürünün ıslak karışımının nemliliğidir. Y , bitmiş ürünün yoğunluğudur.

x_i	y_i
5	7.4
6	9.3
7	10.6
10	15.4
12	18.1
15	22.2
18	24.1
20	24.8

Bu verileri doğrusal bir eğriye uygulayın ve SS_R değerini bulun.



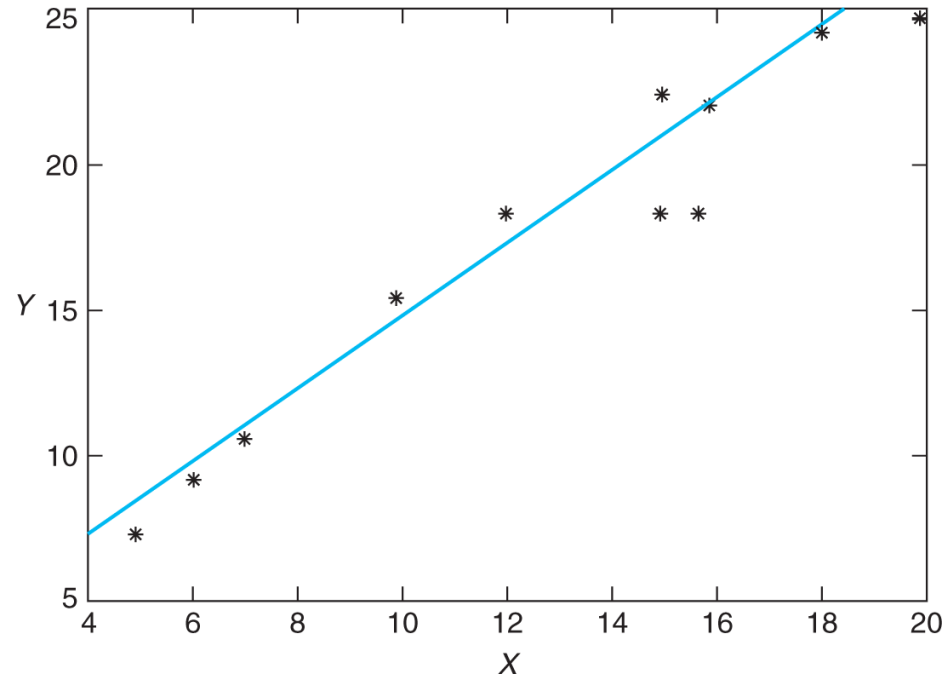
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REGRESYON PARAMETRELERİ ÜZERİNE İSTATİSTİKSEL KANITLAMALAR



Örnek 9.3a



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REGRESYON PARAMETRELERİ ÜZERİNE İSTATİSTİKSEL KANITLAMALAR



Simple Linear Regression

Sample size = 8

$x =$

$y =$

Add This Point To List

Remove Selected Point From List

Data Points

5, 7.4	↑
6, 9.3	
7, 10.6	
10, 15.4	
12, 18.1	
15, 22.2	
18, 24.1	
20, 24.8	↓

Start

Quit

Clear List

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REGRESYON PARAMETRELERİ ÜZERİNE İSTATİSTİKSEL KANITLAMALAR

The least squares estimators are as follows:

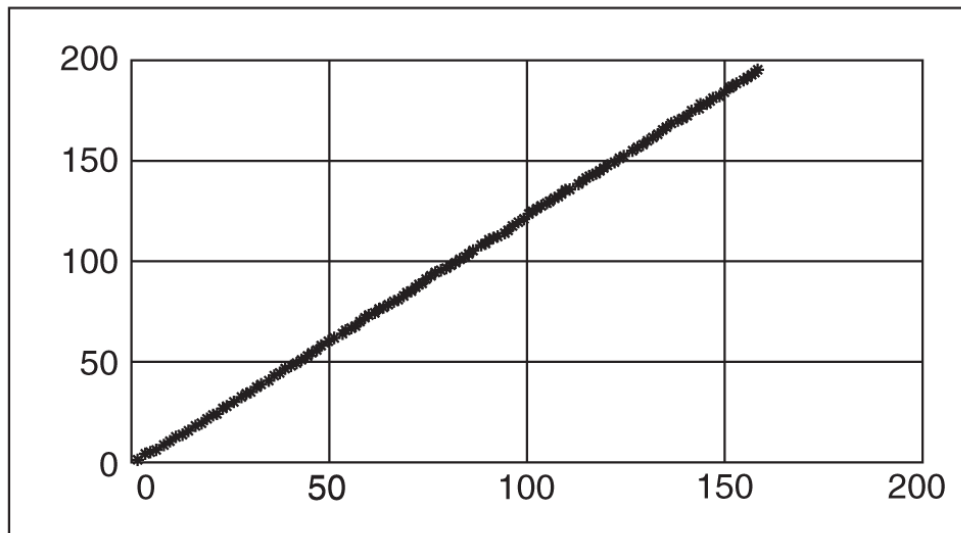
$$a = 2.46$$

$$\text{Average } x \text{ value} = 11.63$$

$$b = 1.21$$

$$\text{Sum of squares of the } x \text{ values} = 1303.0$$

The estimated regression line is $Y = 2.46 + 1.21x$



$$S(x, Y) = 267.66$$

$$S(x, x) = 221.88$$

$$S(Y, Y) = 332.37$$

$$SS_R = 9.47$$



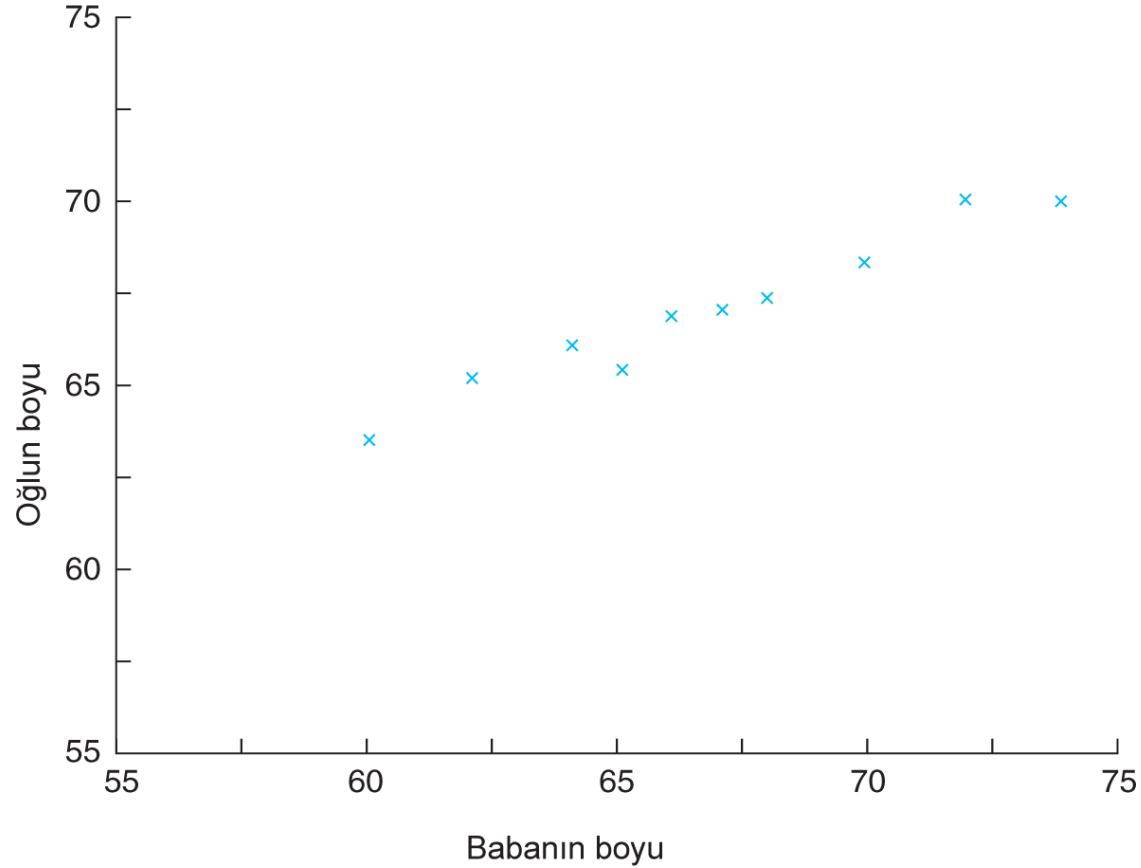
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REGRESYON PARAMETRELERİ ÜZERİNE İSTATİSTİKSEL KANITLAMALAR



Babanın boyuna karşı oğlun boyunun serpme diyagramı



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REGRESYON PARAMETRELERİ ÜZERİNE İSTATİSTİKSEL KANITLAMALAR



Motorlu Taşıt Ölümleri, Kuzeybatı ABD, 1988 ve 1989

İlçe	Ölümler (1988)	Ölümler (1989)
1	121	104
2	96	91
3	85	101
4	113	110
5	102	117
6	118	108
7	90	96
8	84	102
9	107	114
10	112	96
11	95	88
12	101	106



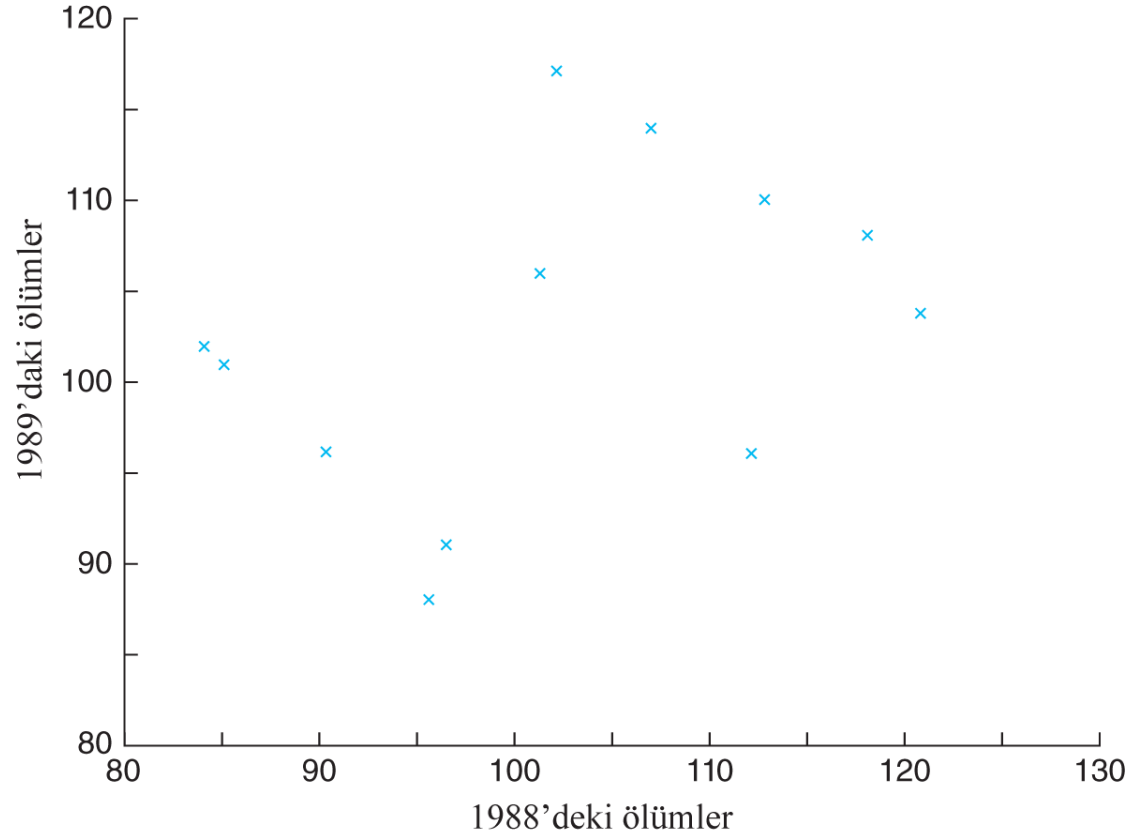
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REGRESYON PARAMETRELERİ ÜZERİNE İSTATİSTİKSEL KANITLAMALAR



1988 ölümlerine karşı 1989 ölümlerinin serpmeye diyagramı



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DAĞILIMSAL SONUÇLARIN ÖZETİ

Şimdi bu kesimin dağılımsal sonuçlarını özetleyeceğiz.

$$\text{Model: } Y = \alpha + \beta x + e, \quad e \sim \mathcal{N}(0, \sigma^2)$$

$$\text{Veri: } (x_i, Y_i), \quad i = 1, 2, \dots, n$$

Üzerinde Kanıtlama İçin	Dağılımsal Sonucunu Kullan
β	$\sqrt{\frac{(n-2)S_{xxx}}{SS_r}}(B - \beta) \sim t_{n-2}$
α	$\sqrt{\frac{n(n-2)S_{xxx}}{\sum_i x_i^2 SS_R}}(A - \alpha) \sim t_{n-2}$



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DAĞILIMSAL SONUÇLARIN ÖZETİ

Üzerinde Kanıtlama
İçin

Dağılımsal Sonucunu Kullan

$\alpha + \beta x_0$

$$\frac{A + Bx_0 - \alpha - \beta x_0}{\sqrt{\left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right) \left(\frac{SS_R}{n-2}\right)}} \sim t_{n-2}$$

$Y(x_0)$

$$\frac{Y(x_0) - A - Bx_0}{\sqrt{\left(1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right) \left(\frac{SS_R}{n-2}\right)}} \sim t_{n-2}$$



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BELİRLEME KATSAYISI VE ÖRNEK KORELASYON KATSAYISI



$$S_{YY} = \sum_{i=1}^n (Y_i - \bar{Y})^2$$



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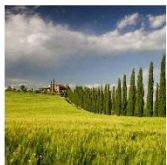


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ARTIKLARIN ANALİZİ: MODEL TAYİNİ

$$Y = \alpha + \beta x + e, \quad e \sim \mathcal{N}(0, \sigma^2)$$



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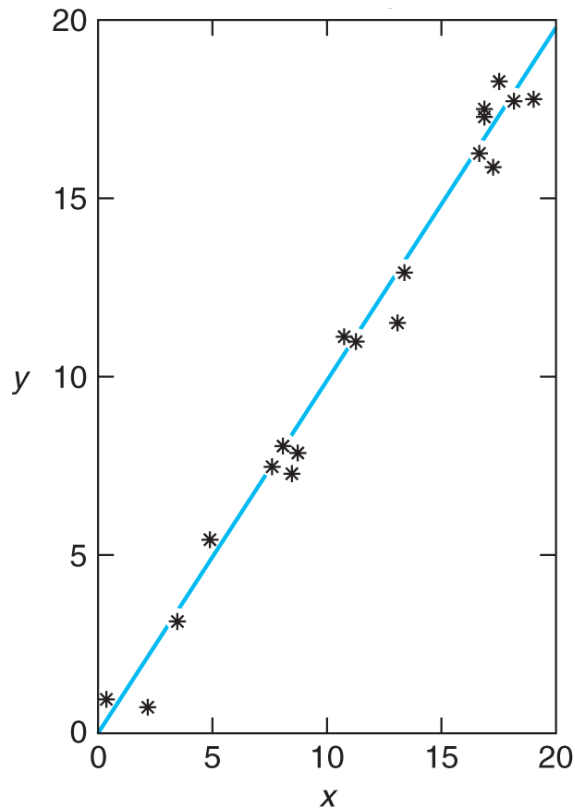


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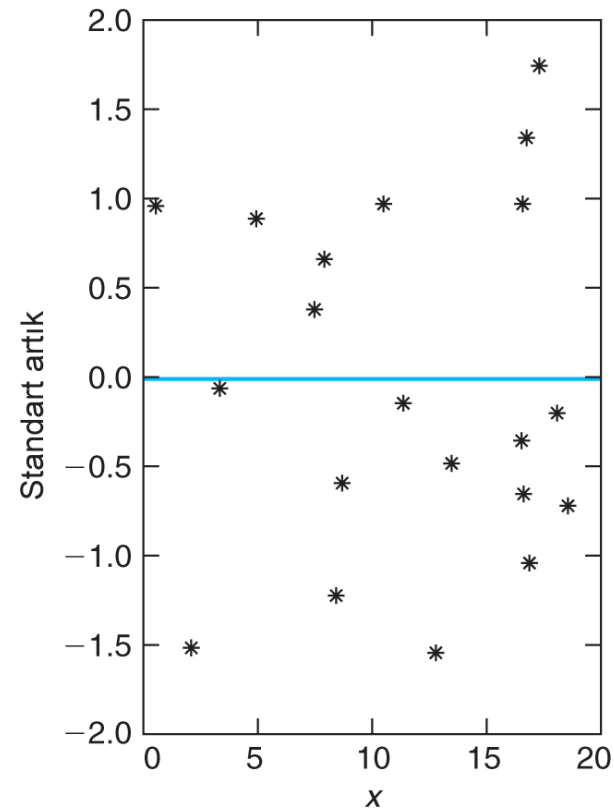


ARTIKLARIN ANALİZİ: MODEL TAYİNİ

(a) Rastgele veri ve regresyon doğrusu



Artıklar



Şeklin devamı bir sonraki slaytta yer almaktadır!



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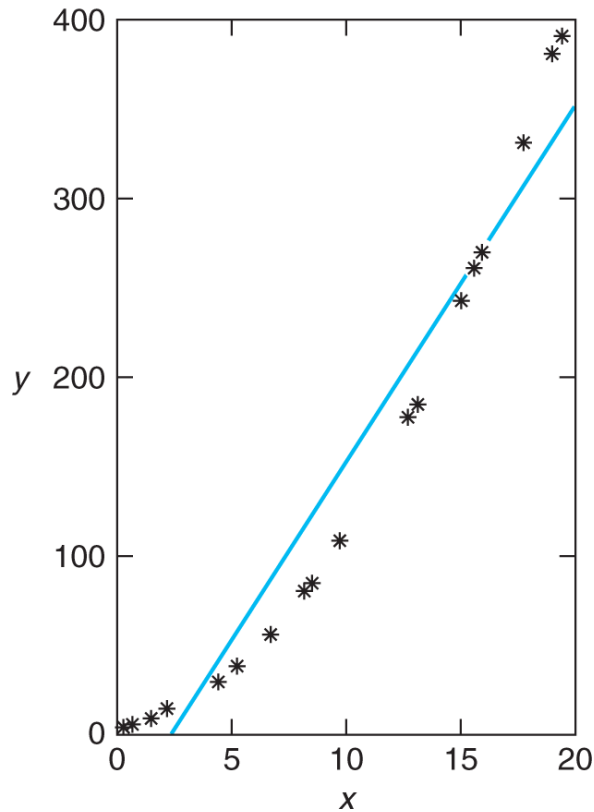


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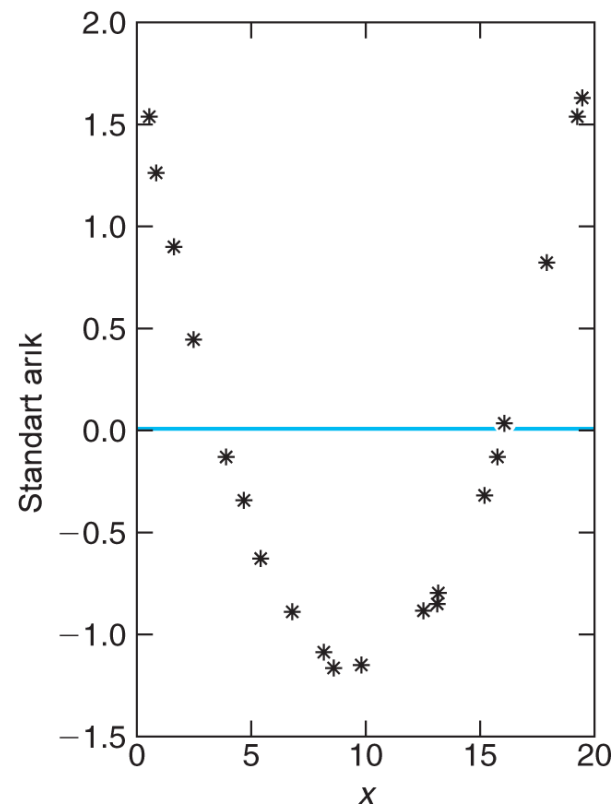


ARTIKLARIN ANALİZİ: MODEL TAYİNİ

(b) Rastgele veri ve regresyon doğrusu



Artıklar



Şeklin devamı bir sonraki slaytta yer almaktadır!



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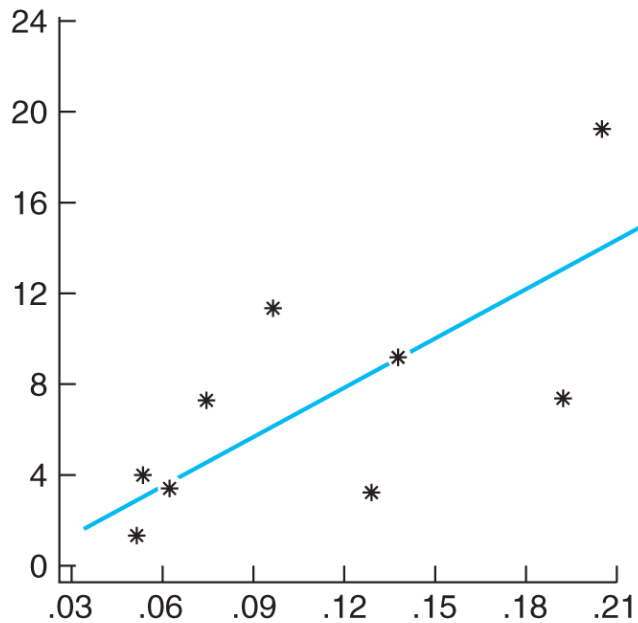


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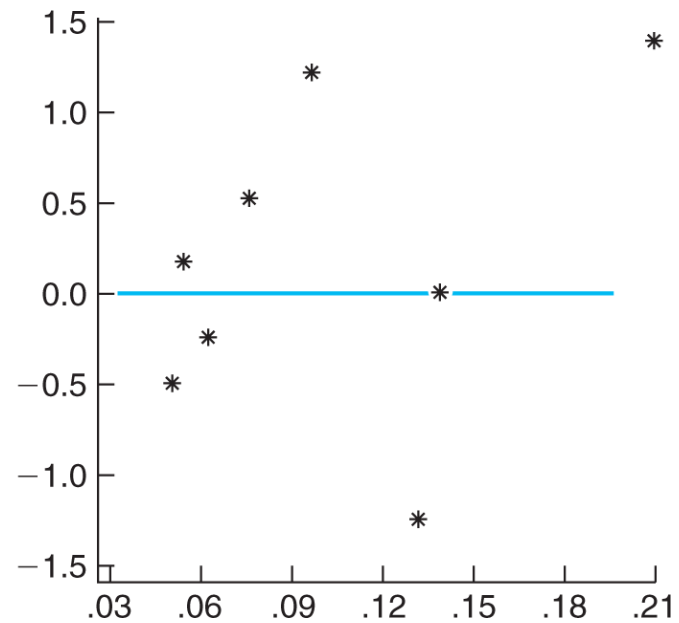


ARTIKLARIN ANALİZİ: MODEL TAYİNİ

(c) Veri ve regresyon doğrusu



Artıklar



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DOĞRUSALLIĞA DÖNÜŞTÜRME

$$W(t) \approx ce^{-dt}$$



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DOĞRUSALLIĞA DÖNÜŞTÜRME

Sıcaklık	$-\log(1 - P)$
5°	.063
10°	.120
20°	.213
30°	.300
40°	.414
50°	.512
60°	.618
80°	.801



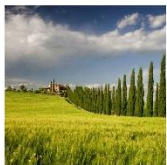
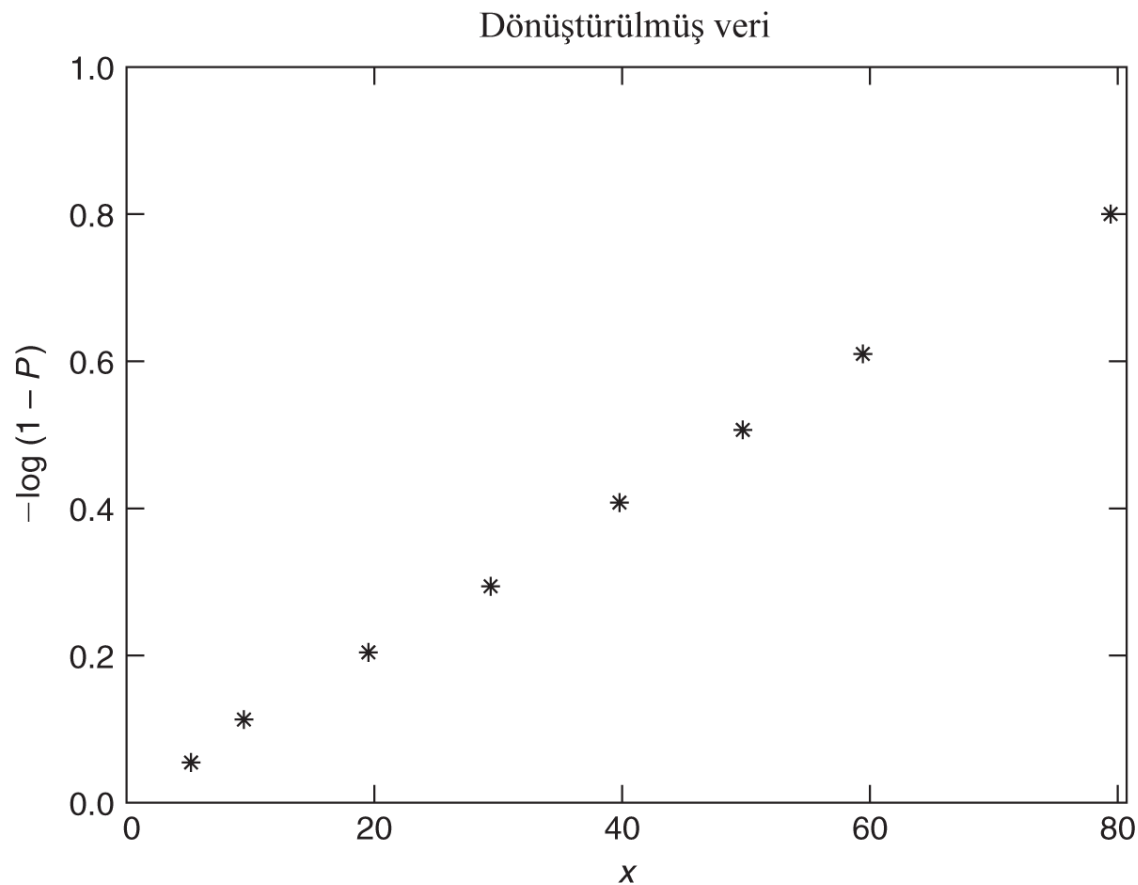
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DOĞRUSALLIĞA DÖNÜŞTÜRME



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AĞIRLIKLI EN KÜÇÜK KARELER

$$Y = \alpha + \beta x + e$$

$$\text{Var}(Y_i) = \frac{\sigma^2}{w_i}$$



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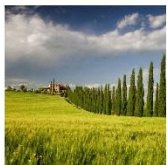
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POLİNOMİYAL REGRESYON

$$Y = \beta_0 + \beta_1x + \beta_2x^2 + \dots + \beta_r x^r + e$$

$$\sum_{i=1}^n (Y_i - B_0 - B_1x_i - B_2x_i^2 - \dots - B_r x_i^r)^2$$



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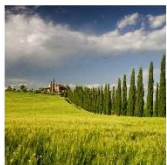
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POLİNOMİYAL REGRESYON

Aşağıdaki veriye bir polinom uydurunuz

x	Y
1	20.6
2	30.8
3	55
4	71.4
5	97.3
6	131.8
7	156.3
8	197.3
9	238.7
10	291.7



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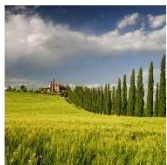
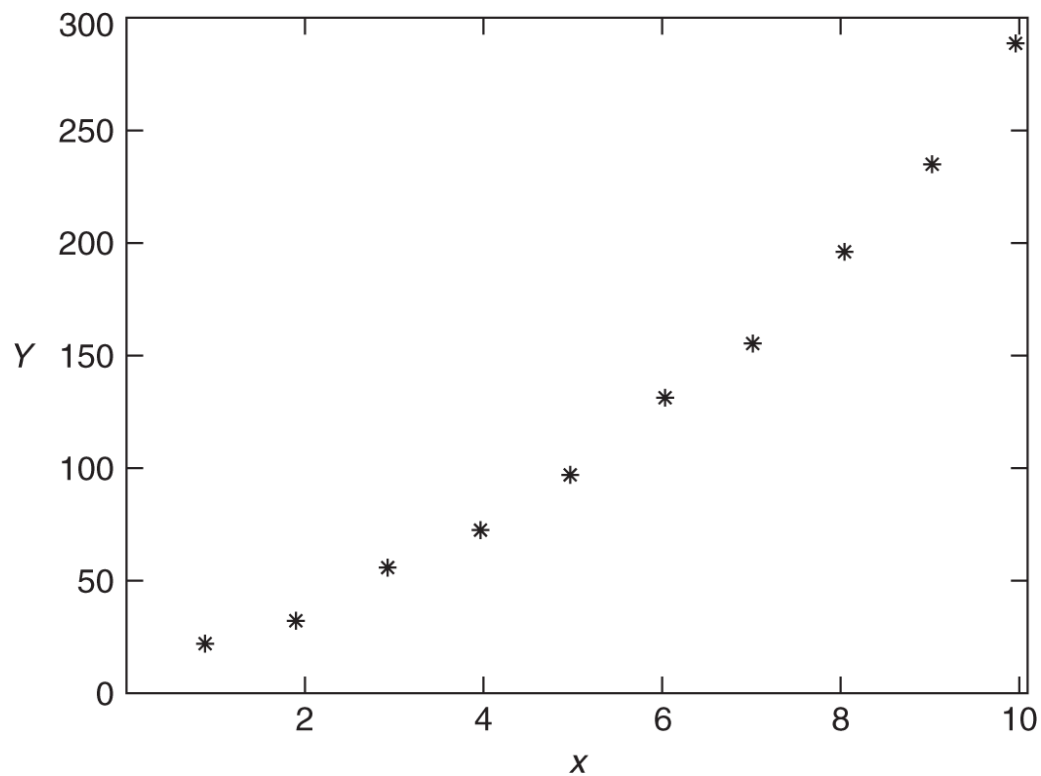
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POLİNOMİYAL REGRESYON

Bu verinin grafiği

$$Y = \beta_0 + \beta_1x + \beta_2x^2 + e$$



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ÇOKLU DOĞRUSAL REGRESYON

$$Y = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + e;$$

$$E[Y_i] = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_k x_{ik}$$

$$\sum_{i=1}^n (Y_i - B_0 - B_1 x_{i1} - B_2 x_{i2} - \cdots - B_k x_{ik})^2$$



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ÇOKLU DOĞRUSAL REGRESYON

Multiple Linear Regression

Enter the number of rows of the X -matrix:

Enter the number of columns of the X -matrix:

Begin Data Entry

Quit



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ÇOKLU DOĞRUSAL REGRESYON

Multiple Linear Regression

	A	B	C
1	1	679	30.4
2	1	1420	34.1
3	1	1349	17.2
4	1	296	26.8
5	1	6975	29.1
6	1	323	18.7

Compute Inverse

Back 1 Step



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ÇOKLU DOĞRUSAL REGRESYON

Multiple Linear Regression

Enter 8 response values:

8.4

Add This Value To List

Remove Selected Value From List

Response Values

11.6

16.1

9.3

9.1

8.4

7.7

Compute coeffs.

Back 1 Step

Estimates of the regression coefficients:

$B(0) = 3.5073534$
 $B(1) = -0.0002477$
 $B(2) = 0.2609466$

Display Inverse

Interval Estimates

Inverse Matrix (X'X)⁻¹

2.78312	0.00002	-9.73E-0	↑
0.00002	2.70E-08	-2.55E-0	
-9.73E-02	-2.55E-06	0.0037	↓

The sum of the squares of the residuals is $SS_R = 34.1212$



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GELECEK YANITLARI ÖNGÖRME

$$E[Y|\mathbf{x}] = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k$$



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GELECEK YANITLARI ÖNGÖRME

	A	B	C		↑
1	1	.02	1.05		
2	1	.03	1.20		
3	1	.03	1.25		
4	1	.04	1.30		
5	1	.10	1.30		
6	1	.15	1.00		↓

Compute Inverse

Back 1 Step



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GELECEK YANITLARI ÖNGÖRME

Multiple Linear Regression

Enter 10 response values:

49.8

Add This Value To List

Remove Selected Value From List

Response Values

79.2	↑
64	
55.7	
56.3	
58.6	
84.3	↓

Compute coeffs.

Back 1 Step

Estimates of the regression coefficients:

$B(0) = 160.2928774$
 $B(1) = 16.6528513$
 $B(2) = -80.8074296$

Display Inverse

Interval Estimates

Inverse Matrix (X'X)⁻¹

9.42764	-5.22E+00	-7.29E+00	↑
-5.22E+00	43.74856	1.30478	
-7.29E+00	1.30478	5.88687	
			↓

The sum of the squares of the residuals is $SS_R = 66.6593$



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GELECEK YANITLARI ÖNGÖRME

Multiple Linear Regression

Enter in the 3 input levels to estimate future responses for this experiment

Data value =

Response Vector

1
0.15
1.15

The value $\text{Sqr}(X'(X'X)^{-1}x) = 0.55946$

$\sum x(i)B(i) = 69.86226$

The value $\text{Sqr}(SSr/(n - k - 1)) = 3.0859$



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İKİLİ ÇIKTI VERİSİ İÇİN LOJİSTİK REGRESYON MODELLERİ



$$p(x) = \frac{e^{a+bx}}{1 + e^{a+bx}}$$



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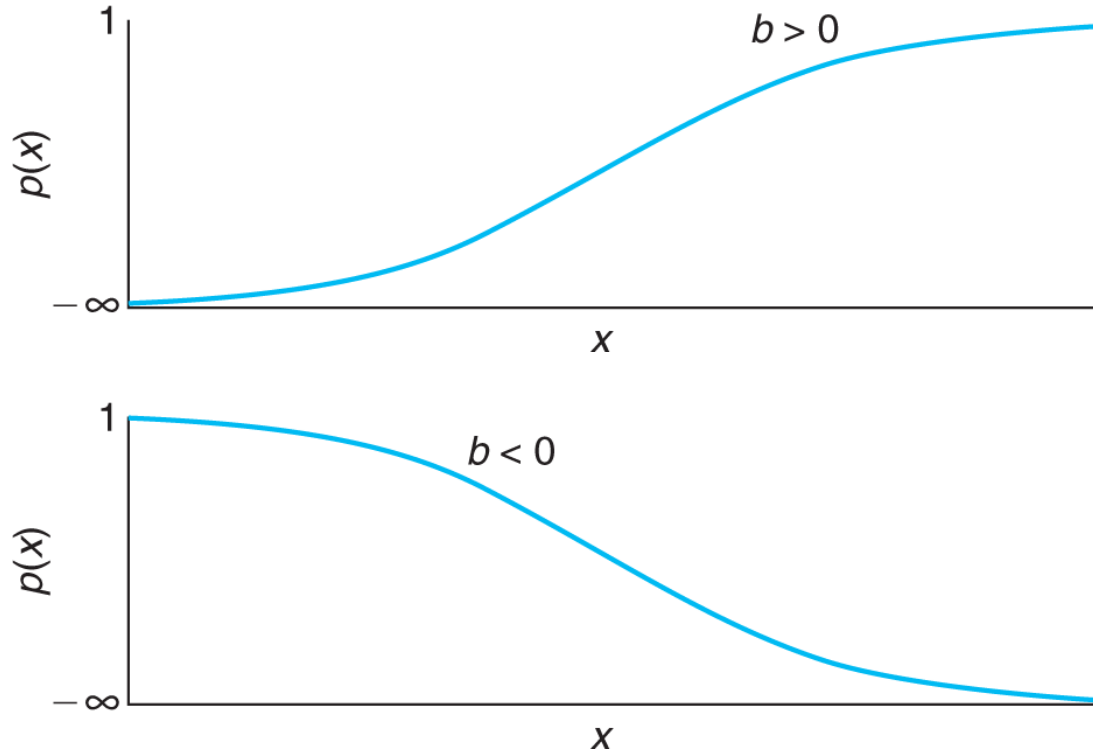
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İKİLİ ÇIKTI VERİSİ İÇİN LOJİSTİK REGRESYON MODELLERİ



Lojistik regresyon fonksiyonları



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