CONDUCTORS

Insulator: each electron is attached to a particular atom (e.g. glass or rubber).

Conductor: one or more electrons per atom are free to roam (e.g. metals).

The basic electrostatic properties of ideal conductors:

(i) $\mathbf{E} = 0$ inside a conductor.

put a conductor into an external electric field $\mathbf{E}_0$

induced charges produce a field of their own, $\mathbf{E}_1$,

Figure 42

→ the field of the induced charges *tends to cancel the original field.*

the resultant field inside the conductor: $E_0 – E_1 = 0$

(ii) $\boldsymbol{\rho} = \mathbf{0}$ inside a conductor**.**

Gauss's law: $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$. ⟹ If $\mathbf{E}$ is zero, so also is ρ.

(iii) Any net charge resides on the surface. That's the only place left.

(iv) A conductor is an equipotential.

For if **a** and **b** are any two points within (or at the surface of) a given conductor,

$$V(\mathbf{b}) - V(\mathbf{a}) = -\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E} \cdot d\mathbf{l} = 0 \quad \Longrightarrow \quad V(\mathbf{a}) = V(\mathbf{b})$$

(v) E is perpendicular to the surface, just outside a conductor.

Figure 43

Induced Charges

If you hold a charge $+q$ near an uncharged conductor, the two will attract one another:

Figure 44

If there is some hollow *cavity* in the conductor, and within that cavity you put some charge, then the field *in the cavity* will *not* be zero.

Figure 45

Example 10. An uncharged spherical conductor centered at the origin has a cavity of some weird shape carved out of it (Fig. 46). Somewhere within the cavity is a charge q. *Question:* What is the field outside the sphere?

Figure 46

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

HW-4: **Page 94;** **Pr.** 32, 33, 38, 39

Capacitors

Two conductors; having +Q *charge* on one and −Q on the other. Since V is constant over a conductor, the potential difference between them:

$$V = V_+ - V_- = -\int_{(-)}^{(+)} \mathbf{E} \cdot d\mathbf{l}$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\rho}{\mathscr{r}^2} \hat{\mathscr{r}} \, d\tau$$

Since **E** is proportional to Q → So, V is also proportional to Q.

The constant of proportionality is called the **capacitance** of the arrangement:

$$C \equiv \frac{Q}{V}$$

Capacitance is determined by the sizes, shapes, and separation of the two conductors. In SI units, C is measured in **farads** (F); a farad is a coulomb-per-volt.

Example. Find the capacitance of a **parallel-plate capacitor** consisting of two metal surfaces of area *A* held a distance *d* apart.

$$E = \sigma/\epsilon_0 = (1/\epsilon_0)\, Q/A$$

The potential difference between the plates;

Figure 52

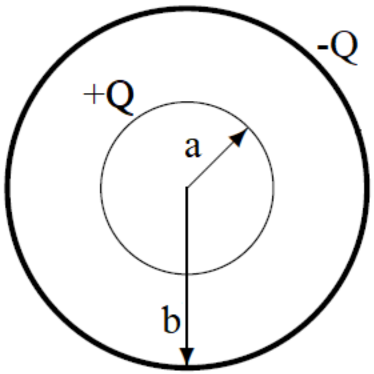
$$V = E\, d \longrightarrow V = \frac{Q}{A\epsilon_0} d$$

$$(C = Q/V) \Longrightarrow C = \frac{A\epsilon_0}{d}$$

E.g.: A square plates with sides 1 cm long, and held 1 mm apart, then the capacitance is

$$C = 9 \times 10^{-13}\ \text{F}.$$

Example. Find the capacitance of two concentric spherical metal shells, with radii *a* and *b*.



The field between the spheres is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}$$

The potential difference between the spheres;

$$V = -\int_b^a \mathbf{E}\cdot d\mathbf{l} = -\frac{Q}{4\pi\epsilon_0}\int_b^a \frac{1}{r^2}\,dr = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

The capacitance is

$$C = \frac{Q}{V} = 4\pi\epsilon_0\frac{ab}{(b-a)}$$

How much work does it take to charge a capacitor up to a final amount *Q*?

Suppose that at some intermediate stage in the process the charge on the positive plate is *q*, so that the potential difference is *q*/*C*.

The work one must do to transport (the next) piece of charge, *dq*, is

$$dW = \left(\frac{q}{C}\right) dq$$

The total work necessary to go from $q = 0$ to $q = Q$, is

$$W = \int_0^Q \left(\frac{q}{C}\right) dq = \frac{1}{2}\frac{Q^2}{C}$$

$$Q = CV,$$

$$W = \frac{1}{2}CV^2$$