Electric Fields in Matter

Most everyday objects are either **conductors** or **insulators** (or **dielectrics**).

Conductor: one or more electrons per atom are free to roam (e.g. metals).

Dielectric: each electron is attached to a particular atom (e.g. glass or rubber);
all they can do is move a bit within the atom or molecule.
Electric fields can distort the charge distribution of a dielectric atom or molecule by *stretching* and *rotating*.

Induced Dipoles:

What happens to a neutral atom when it is placed in an electric field **E**?

The atom **polarized,** with the shift of charges.
The atom now has a tiny dipole moment **p**, which points in the *same direction as* **E**.

The induced dipole moment:

$$\mathbf{p} = \alpha \mathbf{E}$$

α is called **atomic polarizability.**

H	He	Li	Be	C	Ne	Na	Ar	K	Cs
0.667	0.205	24.3	5.60	1.67	0.396	24.1	1.64	43.4	59.4

Atomic Polarizabilities ($\alpha/4\pi\epsilon_0$, in units of 10^{-30} m^3).
Data from: Handbook of Chemistry and Physics, 91st ed.
(Boca Raton: CRC Press, 2010).

E.g.: A primitive model for an atom consists of a point nucleus ($+q$) surrounded by a uniformly charged spherical cloud ($-q$) of radius a. Calculate the atomic polarizability of such an atom.

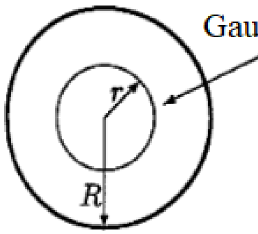
Figure 1

In **E** field, the nucleus will be shifted slightly to the right and the electron cloud to the left →

Figure 2

The field at a distance d from the center of a uniformly charged sphere is:

$$E_e = \frac{1}{4\pi\epsilon_0}\frac{qd}{a^3}$$



$$\oint \mathbf{E}\cdot d\mathbf{a} = E\cdot 4\pi r^2 = \frac{1}{\epsilon_0}Q_{enc} = \frac{1}{\epsilon_0}\frac{4}{3}\pi r^3\rho$$

$$\mathbf{E} = \frac{1}{3\epsilon_0}\rho r\hat{\mathbf{r}}$$

$$Q_{tot} = \frac{4}{3}\pi R^3\rho, \qquad \mathbf{E} = \frac{r}{4\pi\epsilon_0}\frac{Q}{R^3}\,\hat{\mathbf{r}}$$

At equilibrium;

$$E = \frac{1}{4\pi\epsilon_0}\frac{qd}{a^3}$$

$$\mathbf{p} = \alpha\mathbf{E} \longrightarrow p = qd = (4\pi\epsilon_0 a^3)E$$

$$\alpha = 4\pi\epsilon_0 a^3 = 3\epsilon_0 v$$

(v is the volume of the atom.)

Alignment of Polar Molecules

Water molecules are **polar.**

Figure 4

In a uniform electric field:

Figure 5

The *force* on the positive end, $\mathbf{F}_+ = q\mathbf{E}$, exactly cancels the force on the negative end, $\mathbf{F}_- = -q\mathbf{E}$. However, there will be a *torque:*

$$\mathbf{N} = (\mathbf{r}_+ \times \mathbf{F}_+) + (\mathbf{r}_- \times \mathbf{F}_-)$$
$$= \left[(\mathbf{d}/2) \times (q\mathbf{E})\right] + \left[(-\mathbf{d}/2) \times (-q\mathbf{E})\right] = q\mathbf{d} \times \mathbf{E}$$

Thus a dipole $\mathbf{p} = q\mathbf{d}$ in a uniform field $\mathbf{E}$ experiences a torque;

$$\mathbf{N} = \mathbf{p} \times \mathbf{E}$$

Polarization

What happens to a piece of dielectric material when it is placed in an electric field?

If the substance consists of neutral atoms (or nonpolar molecules), the field will induce in each a tiny dipole moment, pointing in the same direction as the field.

If the material is made up of polar molecules, each permanent dipole will experience a torque, tending to line it up along the field direction.

Result: *a lot of little dipoles pointing along the direction of the field* — the material becomes **polarized**. A convenient measure of this effect is

$\mathbf{P} \equiv$ *dipole moment per unit volume*

which is called the **polarization**.

THE FIELD OF A POLARIZED OBJECT

Bound Charges

What is the field produced by polarized material (not the field that may have *caused* the polarization, but the field the polarization *itself* causes)?

For a single dipole **p**;

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{\mathscr{r}}}}{\mathscr{r}^2}$$

Figure 8

The dipole moment for each infinitesimal volume:

$$\mathbf{p} = \mathbf{P}\, d\tau'$$

Then the total potential is

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} \frac{\mathbf{P}(\mathbf{r}') \cdot \hat{\mathbf{\mathscr{r}}}}{\mathscr{r}^2}\, d\tau'$$

$$\nabla'\left(\frac{1}{\mathscr{r}}\right) = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$V = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \mathbf{P}\cdot\nabla'\left(\frac{1}{\mathscr{r}}\right) d\tau'$$

$$\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$$

$$V = \frac{1}{4\pi\epsilon_0}\left[\int_{\mathcal{V}} \nabla'\cdot\left(\frac{\mathbf{P}}{\mathscr{r}}\right) d\tau' - \int_{\mathcal{V}} \frac{1}{\mathscr{r}}(\nabla'\cdot\mathbf{P})\, d\tau'\right]$$

Invoking the divergence theorem;

$$V = \frac{1}{4\pi\epsilon_0}\oint_{\mathcal{S}} \frac{1}{\mathscr{r}}\mathbf{P}\cdot d\mathbf{a}' - \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{1}{\mathscr{r}}(\nabla'\cdot\mathbf{P})\, d\tau'$$

looks like the potential of a surface charge

$$\sigma_b \equiv \mathbf{P}\cdot\hat{\mathbf{n}}$$

looks like the potential of a volume charge

$$\rho_b \equiv -\nabla\cdot\mathbf{P}$$

(*b*; bound charges)

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\oint_{\mathcal{S}} \frac{\sigma_b}{\mathscr{r}}\, da' + \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{\rho_b}{\mathscr{r}}\, d\tau'$$

Physical Interpretation of Bound Charges

Consider a long string of dipoles;

Figure 11

The net charge at the ends called *bound* charge to remind that it cannot be removed; in a dielectric every electron is attached to a specific atom or molecule. But apart from that, bound charge is no different from any other kind.

Uniform Polarization in one-dimension (total bound charge is zero):

Figure

$$\sigma_b \neq 0$$

$$\rho_b = 0$$

Non-Uniform Polarization in one-dimension (total bound charge is zero):

Figure

$\sigma_b \neq 0$

$\rho_b \neq 0$