Signals and Systems

Lecture 6. Systems - 1

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Fall Semester

Introduction

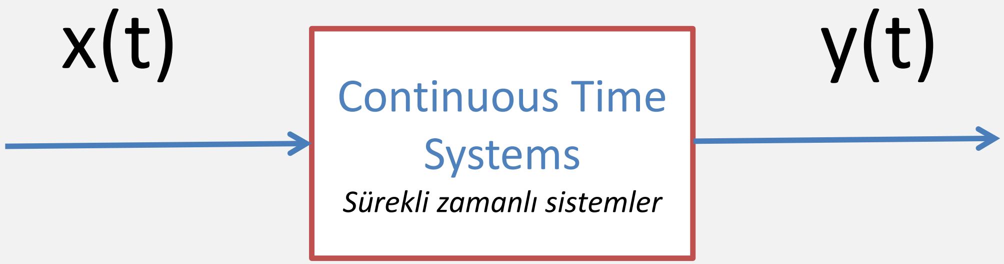
- Definition of a System
- Interconnection of Systems *(Series, Feedback, etc.)*
- Examples of Systems
- Properties of Systems

  Memory (Bellek)
  Invertibility (Tersi Alınabilirlik)
  Causality (Nedensellik)
  Stability (Kararlılık)
  Linearity (Doğrusallık)
  Time invariance (Zaman Değişmezlik)

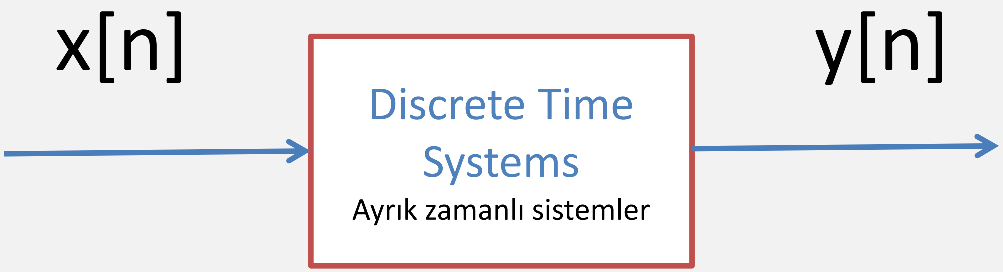
- *Linear Time Invariant (LTI) Systems*

  Doğrusal Zamanda Değişmeyen Sistemler

Definition of a System

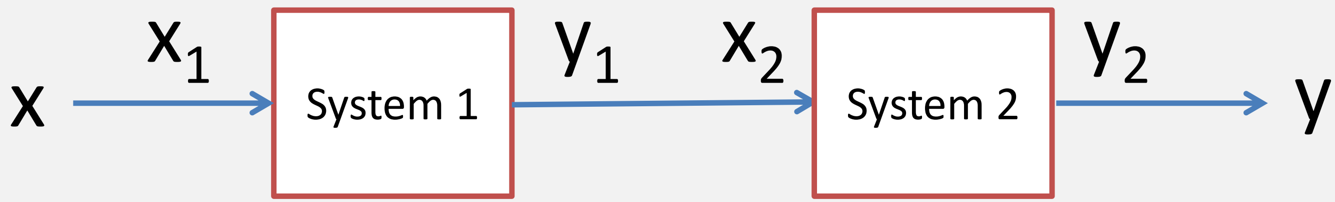


$$x(t) \longrightarrow y(t)$$

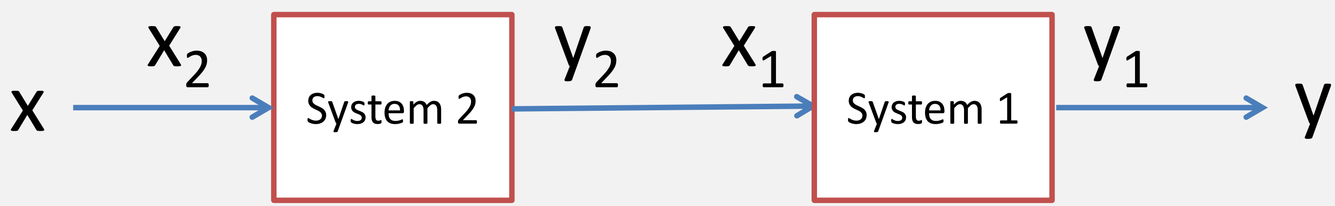


$$x[n] \longrightarrow y[n]$$

Series Systems (Cascade Systems)

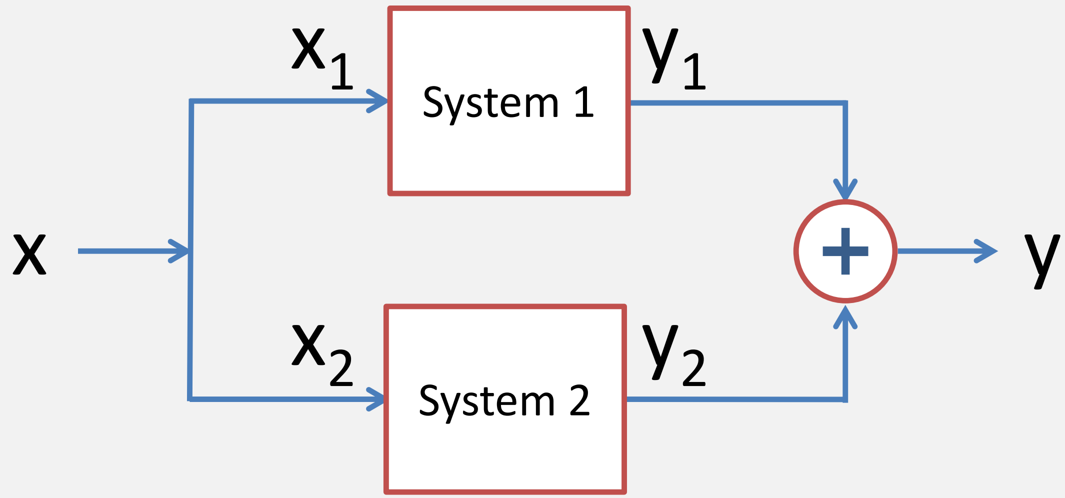


$x_1 \rightarrow y_1$ $\qquad$ $x_2 = y_1 \rightarrow y_2$



$x_2 \rightarrow y_2$ $\qquad$ $y_2 = x_1 \rightarrow y_1$

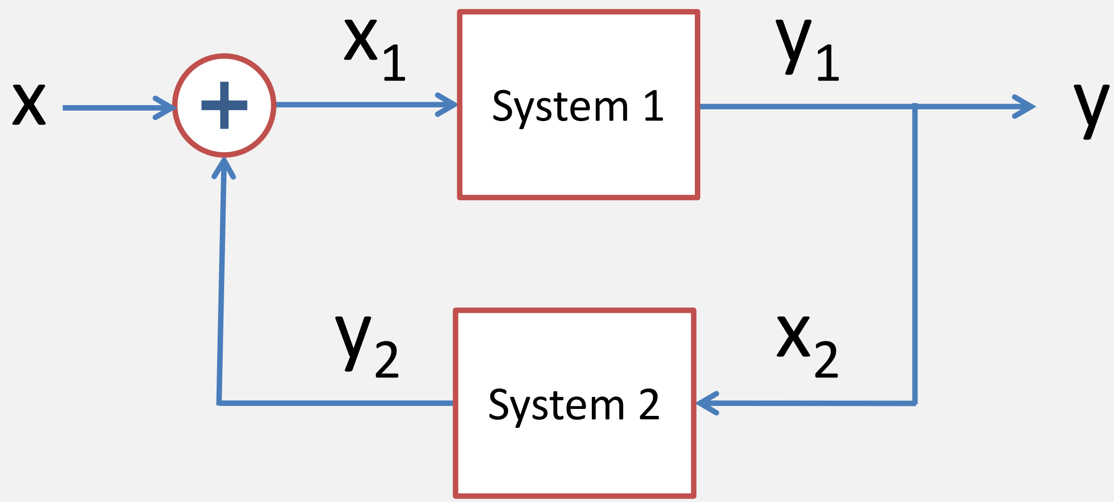
Parallel Systems



$$x_1 = x_2 = x$$

$$y = y_1 + y_2$$

Feedback Systems

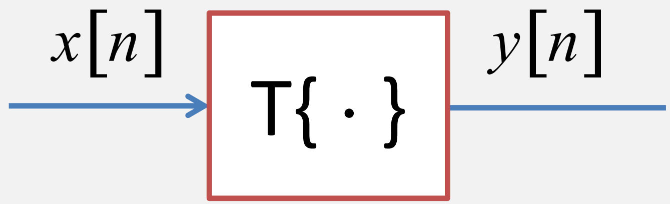


$$x_1 = x + y_2$$

$$y = y_1$$

$$X_2 = y_1$$

Examples of Systems



$$y[n] = T\{x[n]\}$$

e.g. ideal delay

$$y[n] = x[n - n_d] \qquad -\infty < n < \infty$$

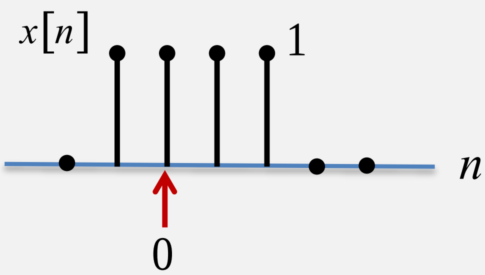
e.g. moving average

$$y[n] = \frac{1}{M_1 + M_2 + 1} \sum_{k=-M_1}^{M_2} x[n-k]$$

$$y[n] = \frac{1}{M_1 + M_2 + 1} \{x[n + M_1] + x[n + M_1 - 1] + \cdots + x[n] + x[n-1] + \cdots + x[n - M_2]\}$$

Examples of Systems

e.g. moving average, $M_1 = M_2 = 1$



$$y[n] = ?$$

solution:

$$y[n] = \frac{1}{1+1+1}\sum_{k=-1}^{1} x[n-k] = \tfrac{1}{3}\{x[n+1] + x[n] + x[n-1]\}$$

$n = 0 \quad y[0] = \tfrac{1}{3}\{x[1] + x[0] + x[-1]\} = \tfrac{1}{3}\{1+1+1\} = 1$

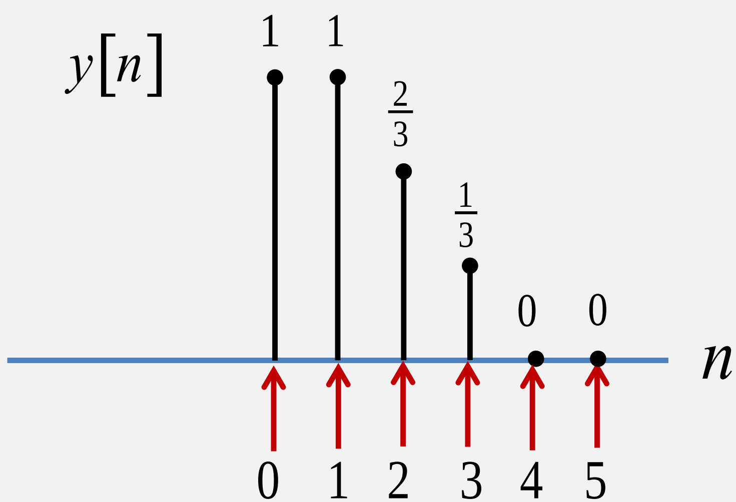
$n = 1 \quad y[1] = \tfrac{1}{3}\{x[2] + x[1] + x[0]\} = \tfrac{1}{3}\{1+1+1\} = 1$

$n = 2 \quad y[2] = \tfrac{1}{3}\{x[3] + x[2] + x[1]\} = \tfrac{1}{3}\{0+1+1\} = \tfrac{2}{3}$

$n = 3 \quad y[3] = \tfrac{1}{3}\{x[4] + x[3] + x[2]\} = \tfrac{1}{3}\{0+0+1\} = \tfrac{1}{3}$

$n = 4 \quad y[4] = \tfrac{1}{3}\{x[5] + x[4] + x[3]\} = \tfrac{1}{3}\{0+0+0\} = 0$

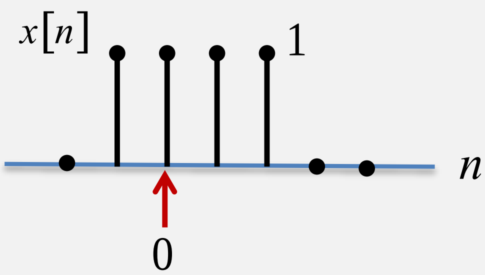
$n > 4 \quad y[n] = 0$



Examples of Systems

e.g. moving average, $M_1 = M_2 = 1$

$$y[n] = ?$$



solution:

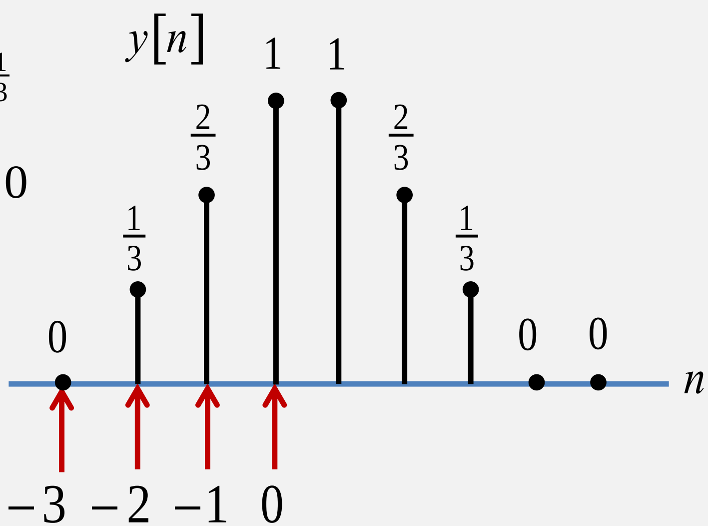
$$y[n] = \frac{1}{1+1+1}\sum_{k=-1}^{1} x[n-k] = \tfrac{1}{3}\left\{x[n+1] + x[n] + x[n-1]\right\}$$

$$n = -1 \quad y[-1] = \tfrac{1}{3}\left\{x[0] + x[-1] + x[-2]\right\} = \tfrac{1}{3}\left\{1+1+0\right\} = \tfrac{2}{3}$$

$$n = -2 \quad y[-2] = \tfrac{1}{3}\left\{x[-1] + x[-2] + x[-3]\right\} = \tfrac{1}{3}\left\{1+0+0\right\} = \tfrac{1}{3}$$

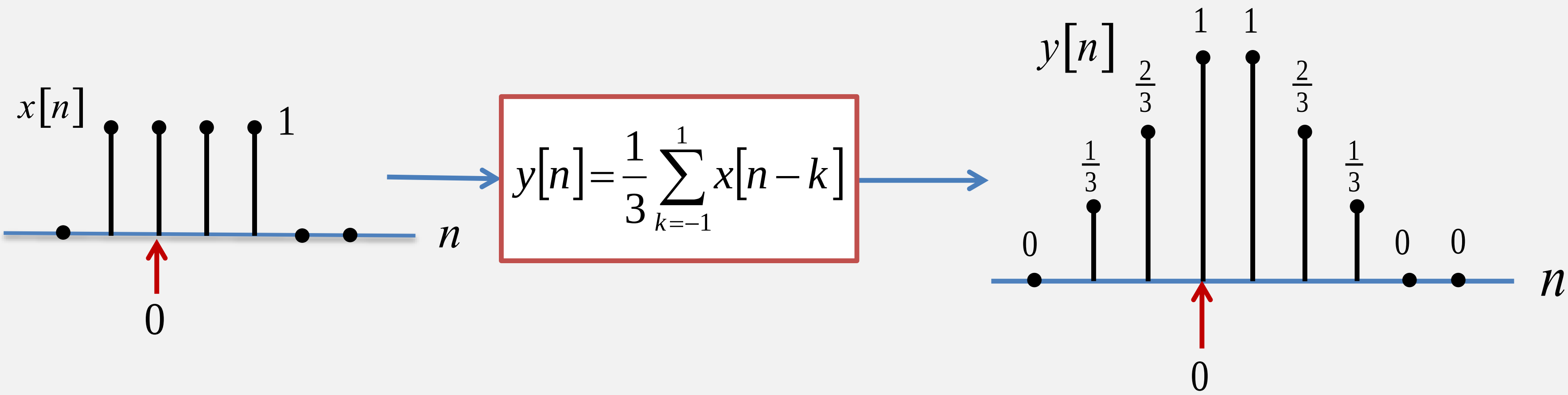
$$n = -3 \quad y[-3] = \tfrac{1}{3}\left\{x[-2] + x[-3] + x[-4]\right\} = \tfrac{1}{3}\left\{0+0+0\right\} = 0$$

$$n < -3 \quad y[n] = 0$$



Examples of Systems

moving average, $M_1 = M_2 = 1$



Comments: ?

Properties of Systems

- *Memory (Bellek)*
- *Invertibility (Tersi Alınabilirlik)*
- *Causality (Nedensellik)*
- *Stability (Kararlılık)*
- *Linearity (Doğrusallık)*
- *Time invariance (Zaman Değişmezlik)*

Memory/Memoryless Systems

- A system is *memoryless* if for every t_o (n_o), the output y(t) (y[n]) depends only on the value of x(t) (x[n]) at t_o (n_o).

Continuous Time: $y(t) @ t = t_o \leftarrow x(t) @ t = t_o$

Discrete Time: $y[n] @ t = t_o \leftarrow x[n] @ t = t_o$

Memoryless Systems

example:

$$y(t) = x^2(t) = \{x(t)\}^2$$

$$y[n] = x^2[n] = \{x[n]\}^2$$

with memory/memoryless?

example:

$$y(t) = \int_{-\infty}^{t} x^2(\tau)\,d\tau$$

with memory/memoryless?

example:

$$y[n] = x[n-1]$$

with memory/memoryless?