Calculus II Week 4 Lecture

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The **definite integral of** f **on** [a, b] is the total signed area of f on [a, b], denoted

$$\int_a^b f(x) \, dx,$$

where *a* and *b* are the **bounds of integration**.

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Let f be continuous on [a, b] and let $F(x) = \int_a^x f(t) dt$. Then F is a differentiable function on (a, b), and

$$F'(x)=f(x).$$

Example Let $F(x) = \int_{-5}^{x} (t^2 + \sin t) dt$. What is F'(x)?

F is called an antiderivative of f

Let f be continuous on [a, b] and let F be any antiderivative of f. Then

$$\int_a^b f(x) \ dx = F(b) - F(a).$$

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Properties

$$\int_{a}^{a} f(x) dx = 0$$

$$\int_{a}^{b} f(x) dx + \int_{b}^{c} f(x) dx = \int_{a}^{c} f(x) dx$$

$$\int_{a}^{b} f(x) dx = -\int_{b}^{a} f(x) dx$$

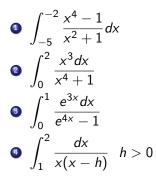
$$\int_{a}^{b} (f(x) \pm g(x)) dx = \int_{a}^{b} f(x) dx \pm \int_{a}^{b} g(x) dx$$

$$\int_{a}^{b} k \cdot f(x) dx = k \cdot \int_{a}^{b} f(x) dx$$

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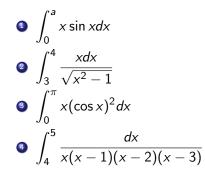
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The average value of a function f(x) over the interval [a, b] is given by

$$f_{avg} = \frac{1}{b-a} \int_{a}^{b} f(x) \, dx$$

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If f(x) is is a continuous function on [a, b] then there is a number c in [a, b] such that

$$\int_{a}^{b} f(x) dx = f(c)(b-a)$$

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Example

Determine the number c that satisfies the Mean Value Theorem for Integrals for the function f(x) = sinx on the interval $[0, \pi]$

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