Calculus II
Week 9 Lecture

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Series

Let $\{a_n\}_{n=1}^{\infty}$ be a sequence.

$$\begin{aligned} s_1 &= a_1 \\ s_2 &= a_1 + a_2 \\ s_3 &= a_1 + a_2 + a_3 \\ s_4 &= a_1 + a_2 + a_3 + a_4 \\ &\vdots \\ s_n &= a_1 + a_2 + a_3 + a_4 + \cdots + a_n = \sum_{i=1}^{n} a_i \end{aligned}$$

The s_n are called partial sums and they will also form a sequence, $\{s_n\}_{n=1}^{\infty}$

Series

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty} \sum_{i=1}^{n} a_i = \sum_{i=1}^{\infty} a_i$$

We will call

$$\sum_{i=1}^{\infty} a_i$$

an infinite series.

If the sequence $\{s_n\}_{n=1}^{\infty}$ is convergent and and its limit is finite then $\sum_{i=1}^{\infty} a_i$ is convergent. Otherwise the series is divergent.

Theorem

If $\sum a_n$ *and* $\sum b_n$ *are both convergent and* c *is a constant then*

$$\sum c a_n = c \sum a_n$$

and

$$\sum_{n=k}^{\infty} a_n \pm \sum_{n=k}^{\infty} b_n = \sum_{n=k}^{\infty} (a_n \pm b_n)$$

Examples

Determine if the following series is convergent or divergent. If it converges determine its value.

1. $\sum_{n=1}^{\infty} n$
2. $\sum_{n=0}^{\infty} (-1)^n$
3. $\sum_{n=1}^{\infty} \frac{1}{2^{n-1}}$

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Conclusion

$\lim_{n\to\infty} n = \infty$ this series diverged

$\lim_{n\to\infty} (-1)^n$ doesn't exist this series diverged

$\lim_{n\to\infty} \frac{1}{2^{n-1}} = 0$ this series converged

Theorem

If $\sum a_n$ converges then

$$\lim_{n\to\infty} a_n = 0$$

Determine if the following series is convergent or divergent.

$$\sum_{n=0}^{\infty} \frac{3n^2 - n}{1 + 2n^2}$$

Series: absolutely convergent series

A series $\sum a_n$ is said to converge absolutely if $\sum |a_n|$ also converges. Absolute convergence is stronger than convergence in the sense that a series that is absolutely convergent will also be convergent, but a series that is convergent may or may not be absolutely convergent.
In fact if $\sum a_n$ converges and $\sum |a_n|$ diverges the series $\sum a_n$ is called conditionally convergent.

Series: Simple Facts

If $\sum a_n$ is absolutely convergent and its value is s then any rearrangement of $\sum a_n$ will also have a value of s.
If $\sum a_n$ is conditionally convergent and r is any real number then there is a rearrangement of $\sum a_n$ whose value will be r