

Book I

1. {*Preface*}⁴

The true philosophers, Syrus,⁵ were, I think, quite right to distinguish the theoretical part of philosophy from the practical. For even if practical philosophy, before it is practical, turns out to be theoretical,⁶ nevertheless one can see that there is a great difference between the two: in the first place, it is possible for many people to possess some of the moral virtues even without being taught, whereas it is impossible to achieve theoretical understanding of the universe without instruction; furthermore, one derives most benefit in the first case [practical philosophy] from continuous practice in actual affairs, but in the other [theoretical philosophy] from making progress in the theory. Hence we thought it fitting to guide our actions (under the impulse of our actual ideas [of what is to be done]) in such a way as never to forget, even in ordinary affairs, to strive for a noble and disciplined disposition, but to devote most of our time to intellectual matters, in order to teach theories, which are so many and beautiful, and especially those to which the epithet 'mathematical' is particularly applied. For Aristotle divides theoretical philosophy too, very fittingly, into three primary categories, physics, mathematics and theology.⁷ For everything that exists is composed of matter, form and motion; none of these [three] can be observed in its substratum by itself, without the others: they can only be imagined. Now the first cause of the first motion of the universe, if one considers it simply, can be thought of as an invisible and motionless deity; the division [of theoretical philosophy] concerned with investigating this [can be called] 'theology', since this kind of activity, somewhere up in the highest reaches of the universe, can only be imagined, and is completely separated from

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⁴ This 'philosophical' preface and its relationship to Ptolemy's attitude to philosophy is discussed by Boll, *Studien* 68-76, to which the reader is referred for the relevant passages in ancient literature. The general standpoint is Aristotelian.

⁵ Syrus is also the addressee of a number of other works by Ptolemy (see Toomer[5] 187). Nothing is known about him. The name is very common in (but not confined to) Greco-Roman Egypt. The statement in a scholion to the *Tetrabiblos* (quoted by Boll, *Studien* 67, n. 2) that some say he was a fictitious person, others that he was a doctor, merely reveals that he was equally unknown in late antiquity.

⁶ Theon in his commentary (Rome II 320, 13-14) gives φησὶ . . . συμβεβηκέναι τῷ πρακτικῷ τὸ πρότερον αὐτοῦ τοῦ θεωρητικοῦ τυγχάνειν. This is a paraphrase rather than a different reading, but shows that he understood the text as I have translated it. By this obscure expression I take Ptolemy to mean that before actually practising virtues one must have some concept of them (even though this is innate rather than taught).

⁷ E. g. *Metaphysics* E 1, 1026a 18 ff., ὥστε τρεῖς ἀν εἶεν φιλοσοφίαι θεωρητικαί, μαθηματικὴ, φυσικὴ, θεολογικὴ.

H6 perceptible reality. The division [of theoretical philosophy] which investigates material and ever-moving nature, and which concerns itself with 'white', 'hot', 'sweet', 'soft' and suchlike qualities one may call 'physics'; such an order of being is situated (for the most part) amongst corruptible bodies and below the lunar sphere. That division [of theoretical philosophy] which determines the nature involved in forms and motion from place to place, and which serves to investigate shape, number, size, and place, time and suchlike, one may define as 'mathematics'. Its subject-matter falls as it were in the middle between the other two, since, firstly, it can be conceived of both with and without the aid of the senses, and, secondly, it is an attribute of all existing things without exception, both mortal and immortal: for those things which are perpetually changing in their inseparable form, it changes with them, while for eternal things which have an aethereal⁸ nature, it keeps their unchanging form unchanged.

H7 From all this we concluded:⁹ that the first two divisions of theoretical philosophy should rather be called guesswork than knowledge, theology because of its completely invisible and ungraspable nature, physics because of the unstable and unclear nature of matter; hence there is no hope that philosophers will ever be agreed about them; and that only mathematics can provide sure and unshakeable knowledge to its devotees, provided one approaches it rigorously. For its kind of proof proceeds by indisputable methods, namely arithmetic and geometry. Hence we were drawn to the investigation of that part of theoretical philosophy, as far as we were able to the whole of it, but especially to the theory concerning divine and heavenly things. For that alone is devoted to the investigation of the eternally unchanging. For that reason it too can be eternal and unchanging (which is a proper attribute of knowledge) in its own domain, which is neither unclear nor disorderly. Furthermore it can work in the domains of the other [two divisions of theoretical philosophy] no less than they do. For this is the best science to help theology along its way, since it is the only one which can make a good guess at [the nature of] that activity which is unmoved and separated: [it can do this because] it is familiar with the attributes of those beings¹⁰ which are on the one hand perceptible, moving and being moved, but on the other hand eternal and unchanging, [I mean the attributes] having to do with motions and the arrangements of motions. As for physics, mathematics can make a significant contribution. For almost every peculiar attribute of material nature becomes apparent from the peculiarities of its motion from place to place. [Thus one can distinguish] the corruptible from the incorruptible by [whether it undergoes] motion in a straight line or in a circle, and heavy from light, and passive from active, by [whether it moves] towards the centre or away from the centre. With

⁸ 'aethereal' (αἰθερώδης) has a precise meaning in Aristotelian physics: everything above the sphere of the moon is composed of an 'incorruptible' substance, unlike anything known on earth in its consistency (very thin) and in its natural motion (circular). See I 3 p. 40. One of the names for this substance is 'aether', another 'fifth essence'. See Campanus IV n. 56, pp. 394-5.

⁹ In this exaltation of mathematics above the other two divisions of philosophy Ptolemy parts company with Aristotle, for whom theology was the most noble pursuit for the human mind.

¹⁰ The heavenly bodies.

regard to virtuous conduct in practical actions and character, this science, above all things, could make men see clearly; from the constancy, order, symmetry and calm which are associated with the divine, it makes its followers lovers of this divine beauty, accustoming them and reforming their natures, as it were, to a similar spiritual state.

It is this love of the contemplation of the eternal and unchanging which we constantly strive to increase, by studying those parts of these sciences which have already been mastered by those who approached them in a genuine spirit of enquiry, and by ourselves attempting to contribute as much advancement as has been made possible by the additional time between those people and ourselves.¹¹ We shall try to note down¹² everything which we think we have discovered up to the present time; we shall do this as concisely as possible and in a manner which can be followed by those who have already made some progress in the field.¹³ For the sake of completeness in our treatment we shall set out everything useful for the theory of the heavens in the proper order, but to avoid undue length we shall merely recount what has been adequately established by the ancients. However, those topics which have not been dealt with [by our predecessors] at all, or not as usefully as they might have been, will be discussed at length, to the best of our ability. H8

2. {On the order of the theorems}

In the treatise which we propose, then, the first order of business is to grasp the relationship of the earth taken as a whole to the heavens taken as a whole.¹⁴ In the treatment of the individual aspects which follows, we must first discuss the position of the ecliptic¹⁵ and the regions of our part of the inhabited world and also the features differentiating each from the others due to the [varying] latitude at each horizon taken in order.¹⁶ For if the theory of these matters is treated first it will make examination of the rest easier. Secondly, we have to go through the motion of the sun and of the moon, and the phenomena accompanying these [motions];¹⁷ for it would be impossible to examine the theory of the stars¹⁸ thoroughly without first having a grasp of these matters. Our final task in this way of approach is the theory of the stars. Here too it would be appropriate to deal first with the sphere of the so-called 'fixed stars',¹⁹ H9

¹¹ This notion of the advancement of science, and particularly astronomy, by the additional time available is one to which Ptolemy recurs in the epilogue (XIII 11 p. 647), and also, in a specifically astronomical context, at VII 1 p. 321 and VII 3 p. 329.

¹² ὑπομνηματίσασθαι. A ὑπόμνημα is a 'memoir', usually implying summary brevity. Ptolemy recurs to this too in the epilogue (XIII 11 p. 647).

¹³ Ptolemy assumes that his readers will have a certain competence. See Introduction p. 6.

¹⁴ I 3-8. On the logic of Ptolemy's order see Introduction pp. 5-6.

¹⁵ I 12-16. The mathematical section I 10-11 is not specifically mentioned here.

¹⁶ Book II.

¹⁷ Books III-VI.

¹⁸ 'Stars' here and throughout chs. 3-8 includes both fixed stars and planets (see Introduction p. 21) and also, sometimes, sun and moon.

¹⁹ Books VII-VIII.

and follow that by treating the five 'planets', as they are called.²⁰ We shall try to provide proofs in all of these topics by using as starting-points and foundations, as it were, for our search the obvious phenomena, and those observations made by the ancients and in our own times which are reliable. We shall attach the subsequent structure of ideas to this [foundation] by means of proofs using geometrical methods.

H10 The general preliminary discussion covers the following topics: the heaven is spherical in shape, and moves as a sphere; the earth too is sensibly spherical in shape, when taken as a whole; in position it lies in the middle of the heavens very much like its centre; in size and distance it has the ratio of a point to the sphere of the fixed stars; and it has no motion from place to place. We shall briefly discuss each of these points for the sake of reminder.

3. {That the heavens move like a sphere}²¹

It is plausible to suppose that the ancients got their first notions on these topics from the following kind of observations. They saw that the sun, moon and other stars were carried from east to west along circles which were always parallel to each other, that they began to rise up from below the earth itself, as it were, gradually got up high, then kept on going round in similar fashion and getting lower, until, falling to earth, so to speak, they vanished completely, then, after remaining invisible for some time, again rose afresh and set; and [they saw] that the periods of these [motions], and also the places of rising and setting, were, on the whole, fixed and the same.

H11 What chiefly led them to the concept of a sphere was the revolution of the ever-visible stars, which was observed to be circular, and always taking place about one centre, the same [for all]. For by necessity that point became [for them] the pole of the heavenly sphere: those stars which were closer to it revolved on smaller circles, those that were farther away described circles ever greater in proportion to their distance, until one reaches the distance of the stars which become invisible. In the case of these, too, they saw that those near the ever-visible stars remained invisible for a short time, while those farther away remained invisible for a long time, again in proportion [to their distance]. The result was that in the beginning they got to the aforementioned notion solely from such considerations; but from then on, in their subsequent investigation, they found that everything else accorded with it, since absolutely all phenomena are in contradiction to the alternative notions which have been propounded.

For if one were to suppose that the stars' motion takes place in a straight line towards infinity, as some people have thought,²² what device could one

²⁰ Books IX-XIII.

²¹ See Pedersen 36-7.

²² According to Theon's commentary (Rome II 338) this belief was Epicurean, but I know of no other evidence. The only other relevant passage appears to be Xenophanes, Diels-Kranz A41a (the sun really moves towards infinity).

conceive of which would cause each of them to appear to begin their motion from the same starting-point every day? How could the stars turn back if their motion is towards infinity? Of, if they did turn back, how could this not be obvious? [On such a hypothesis], they must gradually diminish in size until they disappear, whereas, on the contrary, they are seen to be greater at the very moment of their disappearance, at which time they are gradually obstructed and cut off, as it were, by the earth's surface.

But to suppose that they are kindled as they rise out of the earth and are extinguished again as they fall to earth is a completely absurd hypothesis.²³ For even if we were to concede that the strict order in their size and number, their intervals, positions and periods could be restored by such a random and chance process; that one whole area of the earth has a kindling nature, and another an extinguishing one, or rather that the same part [of the earth] kindles for one set of observers and extinguishes for another set; and that the same stars are already kindled or extinguished for some observers while they are not yet for others: even if, I say, we were to concede all these ridiculous consequences, what could we say about the ever-visible stars, which neither rise nor set? Those stars which are kindled and extinguished ought to rise and set for observers everywhere, while those which are not kindled and extinguished ought always to be visible for observers everywhere. What cause could we assign for the fact that this is not so? We will surely not say that stars which are kindled and extinguished for some observers never undergo this process for other observers. Yet it is utterly obvious that the same stars rise and set in certain regions [of the earth] and do neither at others.

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To sum up, if one assumes any motion whatever, except spherical, for the heavenly bodies, it necessarily follows that their distances, measured from the earth upwards, must vary, wherever and however one supposes the earth itself to be situated. Hence the sizes and mutual distances of the stars must appear to vary for the same observers during the course of each revolution, since at one time they must be at a greater distance, at another at a lesser. Yet we see that no such variation occurs. For the apparent increase in their sizes at the horizons²⁴ is caused, not by a decrease in their distances, but by the exhalations of moisture surrounding the earth being interposed between the place from which we observe and the heavenly bodies, just as objects placed in water appear bigger than they are, and the lower they sink, the bigger they appear.

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The following considerations also lead us to the concept of the sphericity of the heavens. No other hypothesis but this can explain how sundial constructions produce correct results; furthermore, the motion of the heavenly bodies is the most unhampered and free of all motions, and freest motion belongs among

²³ Theon (Rome II 340) ascribes this to Heraclitus. Otherwise it is attested for Xenophanes (Diels-Kranz A38), and was admitted as one possible explanation by Epicurus (e.g. *Letter to Pythocles* 92) and his followers.

²⁴ Ptolemy refers to the well-known phenomenon that the sun and moon appear larger when close to the horizon. He gives an incorrect physical and optical explanation here. In a later work (*Optics* III 60, ed. Lejeune p. 116) he correctly explains it as a purely psychological phenomenon. No doubt instrumental measurement of the apparent diameters had convinced him that the enlargement is entirely illusory.

plane figures to the circle and among solid shapes to the sphere; similarly, since of different shapes having an equal boundary those with more angles are greater [in area or volume], the circle is greater than [all other] surfaces, and the sphere greater than [all other] solids;²⁵ [likewise] the heavens are greater than all other bodies.

H14 Furthermore, one can reach this kind of notion from certain physical considerations. E.g., the aether is, of all bodies, the one with constituent parts which are finest and most like each other; now bodies with parts like each other have surfaces with parts like each other; but the only surfaces with parts like each other are the circular, among planes, and the spherical, among three-dimensional surfaces. And since the aether is not plane, but three-dimensional, it follows that it is spherical in shape. Similarly, nature formed all earthly and corruptible bodies out of shapes which are round but of unlike parts, but all aethereal and divine bodies out of shapes which are of like parts and spherical. For if they were flat or shaped like a disc²⁶ they would not always display a circular shape to all those observing them simultaneously from different places on earth. For this reason it is plausible that the aether surrounding them, too, being of the same nature, is spherical, and because of the likeness of its parts moves in a circular and uniform fashion.

4. {*That the earth too, taken as a whole, is sensibly spherical*}²⁷

H15 That the earth, too, taken as a whole,²⁸ is sensibly spherical can best be grasped from the following considerations. We can see, again, that the sun, moon and other stars do not rise and set simultaneously for everyone on earth, but do so earlier for those more towards the east, later for those towards the west. For we find that the phenomena at eclipses, especially lunar eclipses,²⁹ which take place at the same time [for all observers], are nevertheless not recorded as occurring at the same hour (that is at an equal distance from noon) by all observers. Rather, the hour recorded by the more easterly observers is always later than that recorded by the more westerly. We find that the differences in the hour are proportional to the distances between the places [of observation]. Hence one can reasonably conclude that the earth's surface is spherical, because its evenly curving surface (for so it is when considered as a whole) cuts off [the heavenly bodies] for each set of observers in turn in a regular fashion.

If the earth's shape were any other, this would not happen, as one can see from the following arguments. If it were concave, the stars would be seen rising first by those more towards the west; if it were plane, they would rise and set

²⁵ These propositions were proved in a work by Zenodorus (early second century B.C., see Toomer[1]) from which extensive excerpts are given by (among others) Theon (Rome II 355-79). There is a good summary in Heath *HGM* II 207-13.

²⁶ The only relevant passage I know is Empedocles, Diels-Kranz A60, who maintained that the moon is disk-shaped.

²⁷ See Pedersen 37-9.

²⁸ 'taken as a whole': ignoring local irregularities such as mountains, which are negligible in comparison to the total mass.

²⁹ The timings for solar eclipses are complicated by parallax.