Gauss-Seidel [1-6]

References:

1. Chapra S.C. and Canale R.P. “Numerical Methods for Engineers”, Sixth Edition,McGraw Hill, International Edition 2010.

2. Chapra S.C. and Canale R. P. “Yazılım ve programlama Uygulamalarıyla Mühendisler için Sayısal Yöntemler” 4.Basımdan Çevirenler: Hasan Heperkan ve Uğur Kesgin 2003.

3.Chapra S.C. “Applied Numerical Methods with MATLAB for engineers and Scientists” Third Edition, McGraw Hill, International Edition 2012.

4. Mathews J.H. and Fınk K.D. “Numerical Methods using MATLAB”, Fourth Edition, Pearson P. Hall, International Edition 2004.

5. Fausett L.V. “Applied Numerical Analysis Using MATLAB, Second Edition, PearsonP. Hall, International Edition, 2008.

6. Gilat A. And Subramaniam V. “Numerical Methods, An introduction with Applications Using MATLAB”, Second Edition,John Wiley and Sons. Inc. 2011.

Gauss Seidel iterative method:

$$8X_1 + 3X_2 + 2X_3 = 38$$

$$X_1 + 5X_2 + 2X_3 = 21$$

$$-4 + 2X_2 + 6X_3 = 16$$

True values:

```
>> A=[8 3 2; 1 5 2; -4 2 6];
>> b=[38;21;16];
>> C=b/A
>> C=A\b
C =
3
2
4
>> 8*C(1)+3*C(2)+2*C(3)
ans =
38
>> C(1)+5*C(2)+2*C(3)
ans =
21
>> C(1)+5*C(2)+2*C(3)
ans =
21
```

$$X_1 = \frac{38 - 3X_2 - 2X_3}{8}$$

$$X_2 = \frac{21 - X_2 - 2X_3}{5}$$

$$X_3 = \frac{16 + 4X_1 - 2X_2}{6}$$

Initial guesses X1=0, X2=0, X3=0

```
>> x1=0;x2=0;x3=0;
>> x1=(38-3*x2-2*x3)/8
x1 =
4.7500

>> x1=4.7500;x2=0;x3=0;
>>  x2=(21-x1-2*x3)/5
x2 =
3.2500
>> x1=4.7500;x2=3.2500;x3=0;
>> x3=(16+4*x1-2*x2)/6
x3 =
4.7500
```

Second iteration

```
>> x1=4.7500;x2=3.2500;x3=4.7500;
>> x1=(38-3*x2-2*x3)/8
x1 =
2.3438
>> x1=2.3438;x2=3.2500;x3=4.7500;
>> x2=(21-x1-2*x3)/5
x2 =
1.8312
>> x1=2.3438;x2=1.8312;x3=4.7500;
>> x3=(16+4*x1-2*x2)/6
x3 =
3.6188
```

3rd iteration

```
>> x1=2.3438;x2=1.8312;x3=3.6188;
>> x1=(38-3*x2-2*x3)/8
x1 =
3.1586
>> x1=3.1586;x2=1.8312;x3=3.6188;
>> x2=(21-x1-2*x3)/5
x2 =
2.1208
>> x1=3.1586;x2=2.1208;x3=3.6188;
>> x3=(16+4*x1-2*x2)/6
x3 =4.0655
```

4td iteration

```
>> x1=3.1586;x2=2.1208;x3=4.0655;
>> x1=(38-3*x2-2*x3)/8
x1 =  2.9383
>> x1=2.9383;x2=2.1208;x3=4.0655;
>> x2=(21-x1-2*x3)/5
x2 =
1.9861
>> x1=2.9383;x2=1.9861;x3=4.0655;
>> x3=(16+4*x1-2*x2)/6
x3 =
3.9635
```

5th iteration

```
>> x1=2.9383;x2=1.9861;x3=3.9635;
>> x1=(38-3*x2-2*x3)/8
x1 =
3.0143
>> x1= 3.0143;x2=1.9861;x3=3.9635;
>> x2=(21-x1-2*x3)/5
x2 =
2.0117
>> x1= 3.0143;x2=2.0117;x3=3.9635;
>> x3=(16+4*x1-2*x2)/6
x3 =
4.0056
```

6th iteration

```
>> x1= 3.0143;x2=2.0117;x3=4.0056;
>> x1=(38-3*x2-2*x3)/8
x1 =
2.9942
>> x1= 2.9942;x2=2.0117;x3=4.0056;
>> x2=(21-x1-2*x3)/5
x2 =
1.9989
>> x1= 2.9942;x2=1.9989;x3=4.0056;
>> x3=(16+4*x1-2*x2)/6
x3 =
3.9965
```

7th iteration

```
>> x1= 2.9942;x2=1.9989;x3= 3.9965;
>> x1=(38-3*x2-2*x3)/8
x1 =
3.0013
>> x1= 3.0013;x2=1.9989;x3= 3.9965;
>> x2=(21-x1-2*x3)/5
x2 =
2.0011
>> x1= 3.0013;x2=2.0011;x3= 3.9965;
>> x3=(16+4*x1-2*x2)/6
x3 =
4.0005
```

Absolute approximate percent relative errors :

$$|\varepsilon_a| = \left|\frac{X1current - X1previous}{X1current} * 100\%\right| = \left|\frac{3.0013 - 2.9942}{3.0013} * 100\%\right| = 0.237\%$$

$$|\varepsilon_a| = \left|\frac{X2current - X2previous}{X2current} * 100\%\right| = \left|\frac{2.0011 - 1.9989}{2.0011} * 100\%\right| = 0.1099\%$$

$$|\varepsilon_a| = \left|\frac{X3current - X3previous}{X3current} * 100\%\right| = \left|\frac{4.0005 - 3.9965}{4.0005} * 100\%\right| = 0.09999\%$$

Stopping Criteria= $|\varepsilon_{stop}|$ =0.3%

The Jacobi iterative method

$$8X_1 + 3X_2 + 2X_3 = 38$$

$$X_1 + 5X_2 + 2X_3 = 21$$

$$-4 + 2X_2 + 6X_3 = 16$$

$$X_1 = \frac{38 - 3X_2 - 2X_3}{8}$$

$$X_2 = \frac{21 - X_2 - 2X_3}{5}$$

$$X_3 = \frac{16 + 4X_1 - 2X_2}{6}$$

Initial guesses X1=0, X2=0, X3=0

```
>> x1=0;x2=0;x3=0;
>> x1=(38-3*x2-2*x3)/8
x1 =4.7500
>>  x2=(21-x1-2*x3)/5
x2 = 3.2500
>> x3=(16+4*x1-2*x2)/6
x3 = 4.7500
```

Second İteration

```
>> x1=4.7500;x2=3.2500;x3=4.7500;
>> x1=(38-3*x2-2*x3)/8
x1 =
2.3438
>> x2=(21-x1-2*x3)/5
x2 =
1.8313
x3 =
3.6187
```

Third iteration

```
>> x1=2.3438;x2=1.8313;x3=3.6187;
>> x1=(38-3*x2-2*x3)/8
x1 =
3.1586
>> x2=(21-x1-2*x3)/5
x2 =
2.1208
>> x3=(16+4*x1-2*x2)/6
x3 =
4.0655
```

CONVERGENCE