Odes and Problems [1-6]

References:

1. Chapra S.C. and Canale R.P. "Numerical Methods for Engineers", Sixth Edition,McGraw Hill, International Edition 2010.

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4. Mathews J.H. and Fınk K.D. "Numerical Methods using MATLAB", Fourth Edition, Pearson P. Hall, International Edition 2004.

5. Fausett L.V. "Applied Numerical Analysis Using MATLAB, Second Edition, PearsonP. Hall, International Edition, 2008.

6. Gilat A. And Subramaniam V. "Numerical Methods, An introduction with Applications Using MATLAB", Second Edition,John Wiley and Sons. Inc. 2011.

the fourth-order Runge-Kutta Method.

$$\frac{dy_1}{dx} = y_1 + 2y_2$$

$$\frac{dy_2}{dx} = 3y_1 + 2y_2$$

at $x = 0$, $y_1 = 6$, $y_2 = 4$

step size h = 0.02.

$$y_{1,i+1} = y_{1,i} + \frac{h}{6}(k_{1,1} + 2k_{2,1} + 2k_{3,1} + k_{4,1})$$

$$y_{2,i+1} = y_{2,i} + \frac{h}{6}(k_{1,2} + 2k_{2,2} + 2k_{3,2} + k_{4,2})$$

$$k_{1,1} = f_1(x_i, y_{1,i}, y_{2,i})$$

$$k_{1,2} = f_2(x_i, y_{1,i}, y_{2,i})$$

$$k_{2,1} = f_1\left(x_i + \frac{h}{2}, \quad y_{1,i} + \frac{h}{2}k_{1,1}, \quad y_{2,i} + \frac{h}{2}k_{1,2}\right)$$

$$k_{2,2} = f_2\left(x_i + \frac{h}{2}, \quad y_{1,i} + \frac{h}{2}k_{1,1}, \quad y_{2,i} + \frac{h}{2}k_{1,2}\right)$$

$$k_{3,1} = f_1\left(x_i + \frac{h}{2}, \quad y_{1,i} + \frac{h}{2}k_{2,1}, \quad y_{2,i} + \frac{h}{2}k_{2,2}\right)$$

$$k_{3,2} = f_1\left(x_i + \frac{h}{2}, \quad y_{1,i} + \frac{h}{2}k_{2,1}, \quad y_{2,i} + \frac{h}{2}k_{2,2}\right)$$

$$k_{4,1} = f_1(x_i + h, \quad y_{1,i} + k_{3,1}h \quad y_{2,i} + k_{3,2}h)$$

$$k_{4,2} = f_2(x_i + h, \quad y_{1,i} + k_{3,1}h \quad y_{2,i} + k_{3,2}h)$$

$$k_{1,1} = f(0.0, 6.0, 4.0) = y_1 + 2 * y_2 = 6.0 + 2 * 4 = 14.0$$

$$k_{1,2} = g(0.0, 6.0, 4.0) = 3 * y_1 + 2 * y_2 = 3 * 6 + 2 * 4 = 26.0$$

$$y_{1,i} + \frac{h}{2} k_{1,1} = 6.0 + \frac{0.02}{2} * 14.0 = 6.14$$

$$y_{2,i} + \frac{h}{2} k_{1,2} = 4 + \frac{0.02}{2} * 26.0 = 4.26$$

$$k_{2,1} = f(0.01, 6.14, 4.26) = y_1 + 2 * y_2 = 6.14 + 2 * 4.26 = 14.66$$

$$k_{2,2} = g(0.01, 6.14, 4.26) = 3 * y_1 + 2 * y_2 = 3 * 6.14 + 2 * 4.26 = 26.94$$

$$y_{1,i} + \frac{h}{2} k_{2,1} = 6.0 + \frac{0.02}{2} * 14.66 = 6.1466$$

$$y_{2,i} + \frac{h}{2} k_{2,2} = 4 + \frac{0.02}{2} * 26.94 = 4.2694$$

$$k_{3,1} = f(0.01, 6.1466, 4.2694) = y_1 + 2 * y_2 = 6.1466 + 2 * 4.2694 = 14.6854$$

$$k_{3,2} = g(0.01, 6.1466, 4.2694) = 3 * y_1 + 2 * y_2 = 3 * 6.1466 + 2 * 4.2694 = 26.9786$$

$$y_{1,i} + k_{3,1}h = 6.0 + 0.02 * 14.6854 = 6.293708$$
$$y_{2,i} + k_{3,2}h = 4 + 0.02 * 26.9786 = 4.539572$$

$$k_{4,1} = f(0.02, 6.293708, 4.539575) = y_1 + 2 * y_2 = 6.293708 + 2 * 4.539575 = 15.372858$$

$$k_{4,2} = g(0.02, 6.293708, 4.539575) = 3 * y_1 + 2 * y_2 = 3 * 6.293708 + 2 * 4.539575 = 27.960274$$

$$y_{1,i} = 6 + \frac{0.02}{6}(14.0 + 2(14.66) + 2(14.6854) + 15.372858) = 6.29354553$$
$$y_{2,i} = 4 + \frac{0.02}{6}(26.0 + 2(26.94) + 2(26.9786) + 27.960274) = 4.53932491$$

at $x = 0$, $y_1 = 6$, $y_2 = 4$

x interval: $[0.0, 0.02]$

step size h = 0.02

Solve the following set of differential equations

$$\frac{dy_1}{dx} = y_1 + 2y_2$$

$$\frac{dy_2}{dx} = 3y_1 + 2y_2$$

biricitur.m

```
function yp = biricitur(x,y)
yp=[y(1)+2*y(2);3*y(1)+2*y(2)];
end
```

Command window

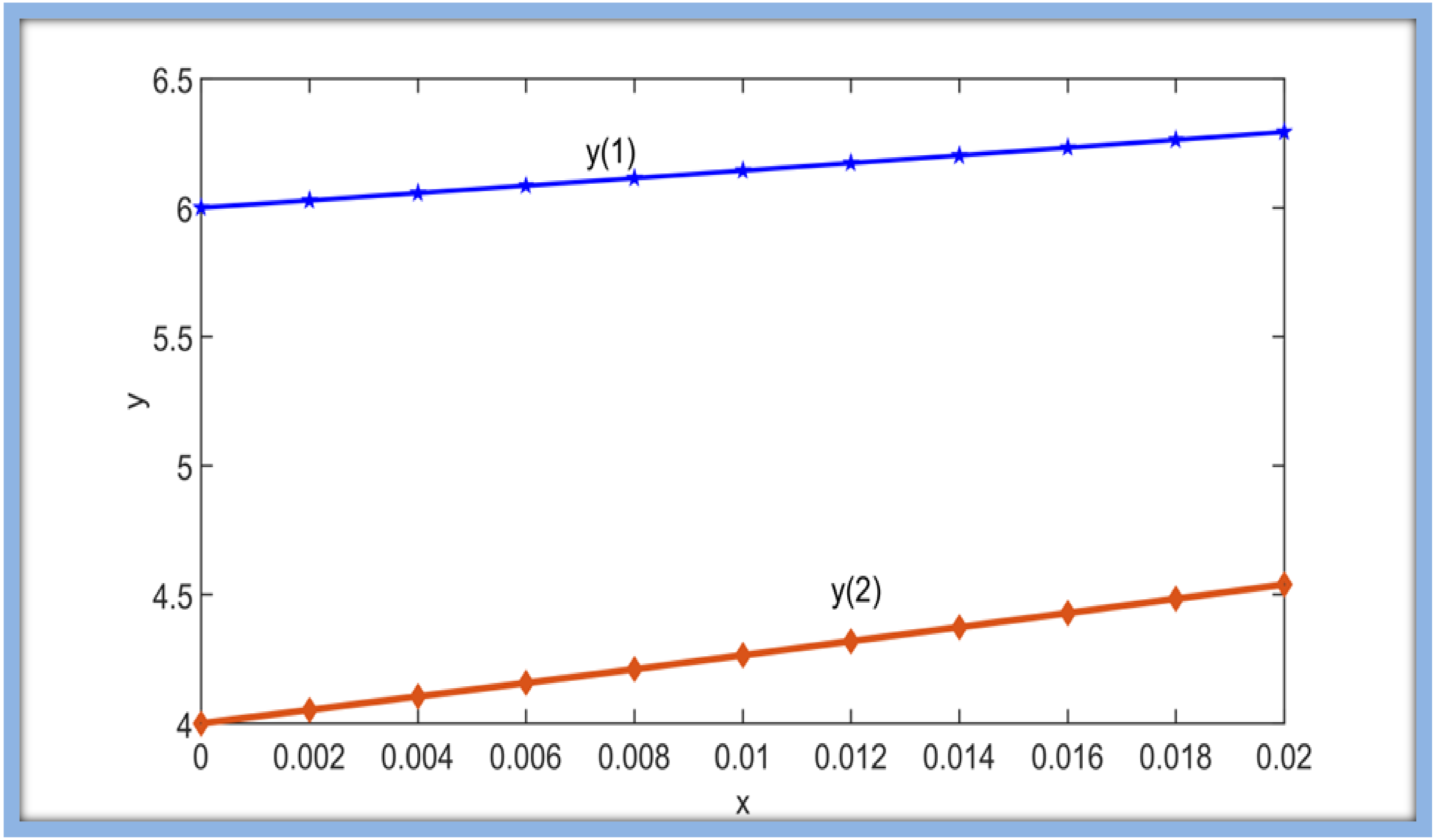
```
>> xs=[0,0.02];
>> y0=[6,4];
>> [x,y]=ode23('biricitur', xs,y0);
```

y(1)	y(2)
6	4
6,02813233866667	4,05218851466667
6,05653071893120	4,10475613168349
6,08519719758201	4,15770597611329
6,11413384808782	4,21104119795967
6,14334276073131	4,26476497236765
6,17282604274384	4,31888049982589
6,20258581844128	4,37339100637066
6,23262422936093	4,42829974379126
6,26294343439949	4,48360998983724
6,29354560995219	4,53932504842723

$$y_{1,i} = 6 + \frac{0.02}{6}(14.0 + 2(14.66) + 2(14.6854) + 15.372858) = 6.29354553$$

$$y_{2,i} = 4 + \frac{0.02}{6}(26.0 + 2(26.94) + 2(26.9786) + 27.960274) = 4.53932491$$

>> plot(x,y)



6.5
6
5.5
5
4.5
4
y
y(1)
y(2)
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02
x