# MTH312 ALGEBRA II MIDTERM EXAM

**1.** Let consider $\mathbb{Z}_2[i] = \{a + ib \mid a, b \in \mathbb{Z}_2\}$ with the usual addition and multiplication of complex numbers.

$(i)$ Is $\mathbb{Z}_2[i]$ a ring?

$(ii)$ Is $\mathbb{Z}_2[i]$ an integral domain?

$(iii)$ Is $\mathbb{Z}_2[i]$ a field?

**2.** Find the characteristic of the following rings.

$(i)$ $R = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \in \mathbb{Z}_3 \right\}.$

$(ii)$ $(\mathbb{Z}/20\mathbb{Z}) \,/\, (5\mathbb{Z}/20\mathbb{Z})\,.$

**3.** $(i)$ Find all idempotents of the ring $\mathbb{Z} \times \mathbb{Z}$.

$(ii)$ Determine all ring homomorphisms from $\mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$.

**5.** Determine all prime and maximal ideals of $\mathbb{Z}_{30}$.

**6.** Let $R$ be an integral domain and let $a \in R$. Show that $R[x]/\left\langle x - a \right\rangle \simeq R$.

**7.** Indicate whether each of the following statements is **True (T),** or **False (F)**. Explain your answers.

a) $\mathbb{Q}/\mathbb{Z}$ is an integral domain......

b) $\mathbb{Q} \times \mathbb{Z}_3$ is a field......

c) There exists a field with six elements......

d) The characteristic of an infinite ring is zero......

e) $\mathbb{R}$ is a field of quotients of $\mathbb{Z}$......

f) Every ring with unity has at least two units.......

g) In $\mathbb{Z}_7[x]$, $\left(x+\overline{1}\right)^7 = x^7 + \overline{1}$......

h) Let $R$ be a commutative ring with unity. Then every prime ideal in $R$ is maximal......

i) If $R[x]$ has zero divisors, so does $R$......

j) If $F$ is a field, then $F[x]$ is a field......

# Bibliography

[1] J. B. Fraleigh, A First Course In Abstract Algebra, Addison Wesley. (7th Edition).

[2] D. S. Malik, J. M. Mordeson and M. K. Sen, Fundamentals of Abstract Algebra, Mc Graw Hill, 1997.