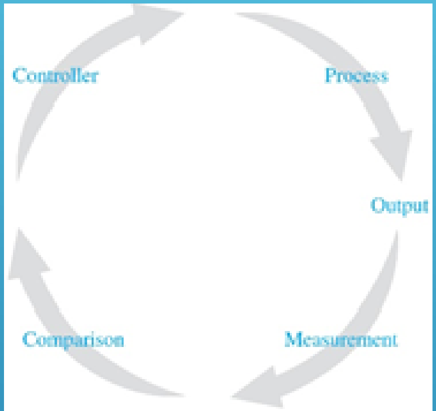
CONTROL SYSTEMS



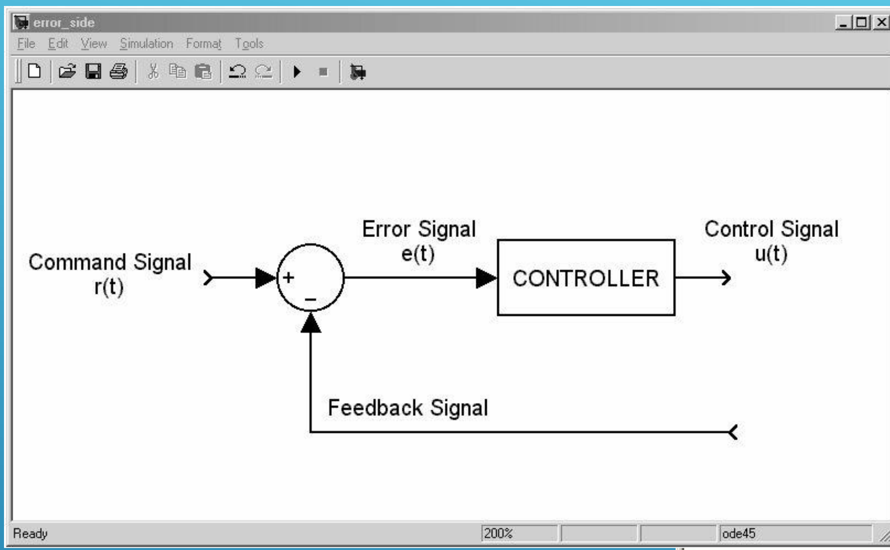
Doç. Dr. Murat Efe

WEEK 4

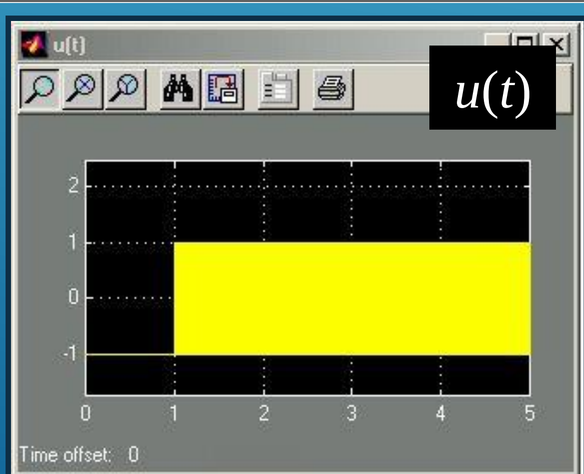
P-2 Basic Control Actions, P-I-D Effects

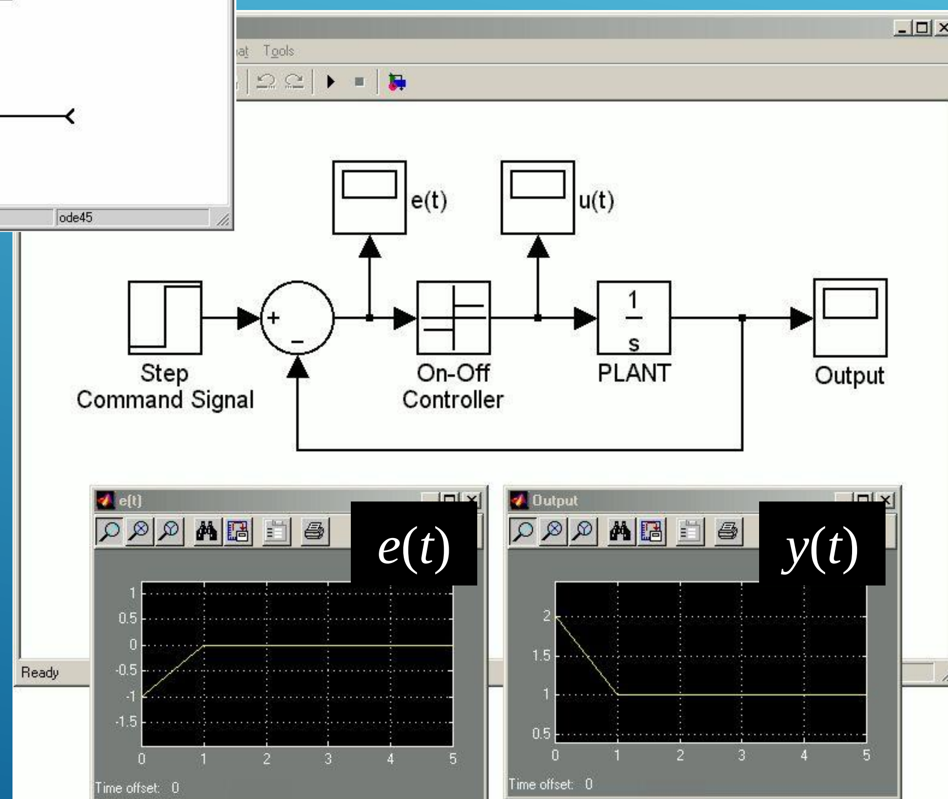
- On-Off Controller
- Proportional (P) Controller
- Integral (I) Controller
- Proportional-Integral (PI) Controller
- Proportional-Derivative (PD) Controller
- Proportional-Integral-Derivative (PID) Controller

On-Off Controller



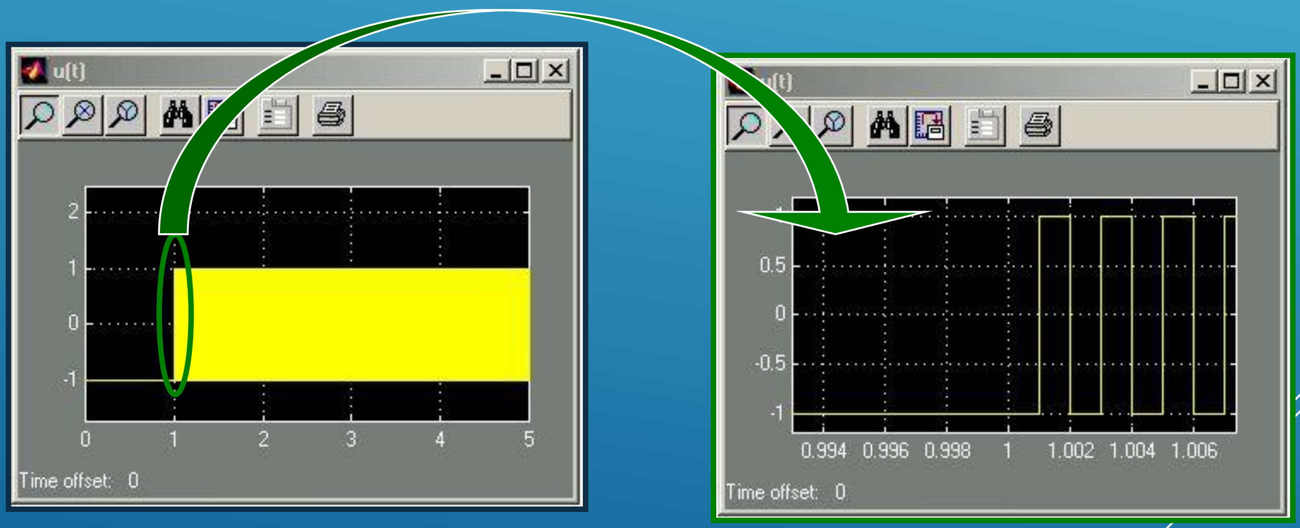
$$u(t) = \begin{cases} U_1 & if\ e(t) > 0 \\ U_2 & if\ e(t) < 0 \end{cases}$$





On-Off Controller

- Initial condition was 2.0
- Reference input was 1.0 (step input at t=0)
- Calculated error = -1.0 ($0 \leq t < 1$)
- Apply minimum (U_2) control value (U_1=1, U_2=-1)
- This would bring the output to 1 (Command Sig.)
- Around zero output, what is your control?

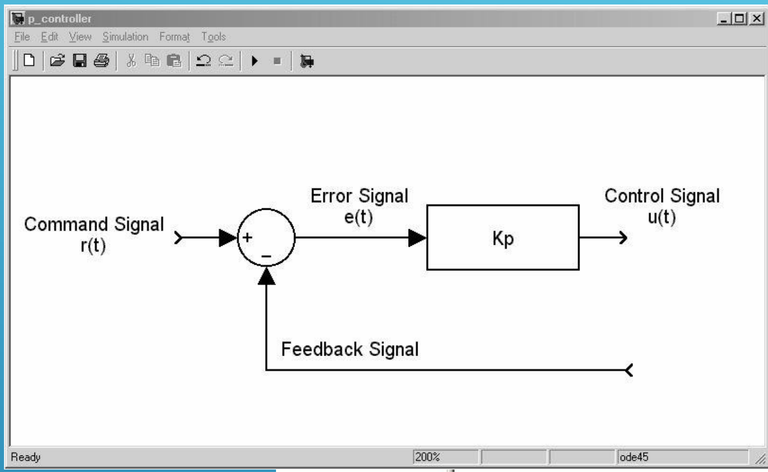


On-Off Controller
Remarks on Simulation

- Ideally, the switching frequency is infinity!
- Simulation step size was 1 msec
- This example shows how On-Off type controller works

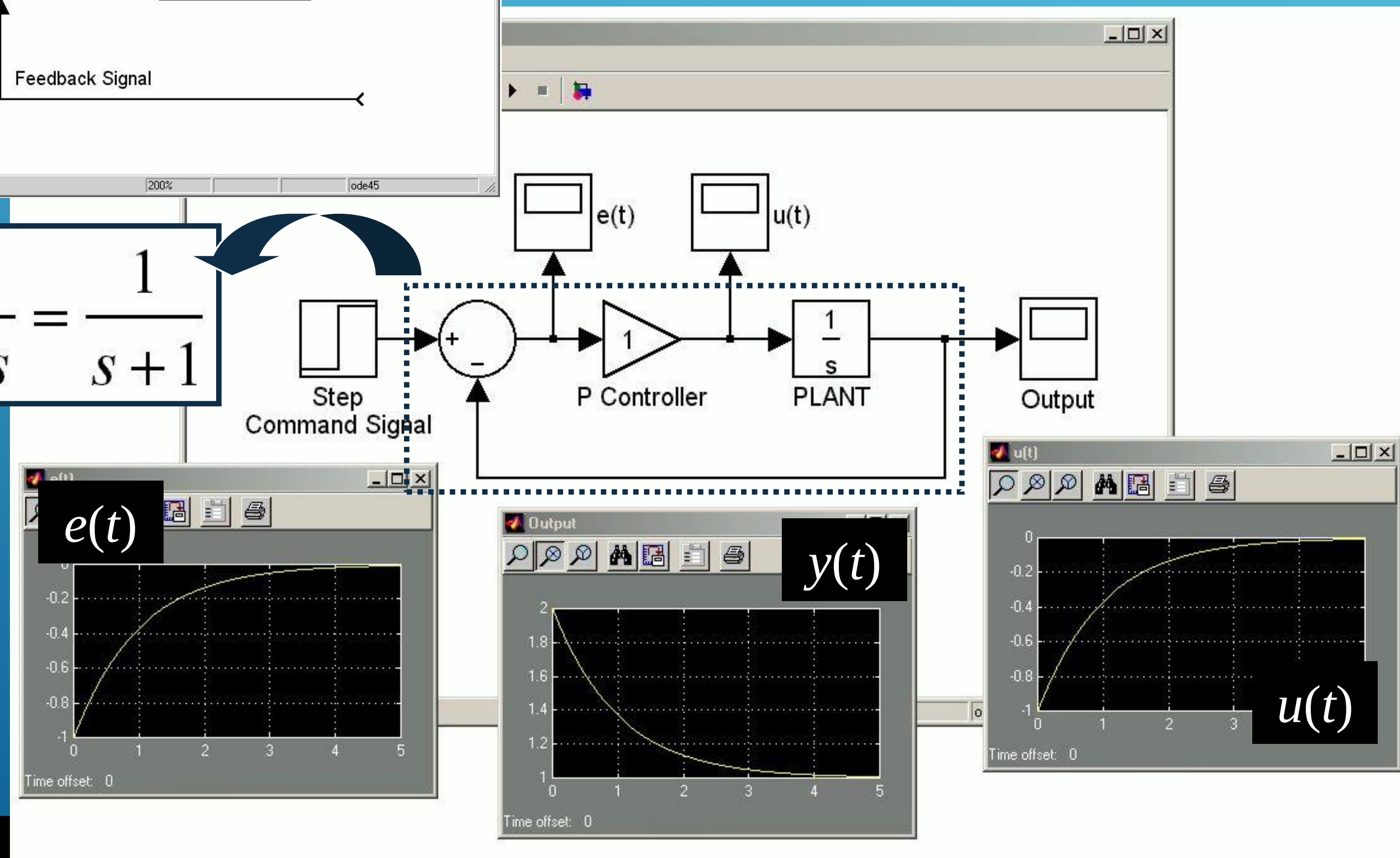
Proportional (P) Controller

$$u(t) = K_p e(t)$$



$$T(s) = \frac{1/s}{1+1/s} = \frac{1}{s+1}$$

Closed loop is stable!



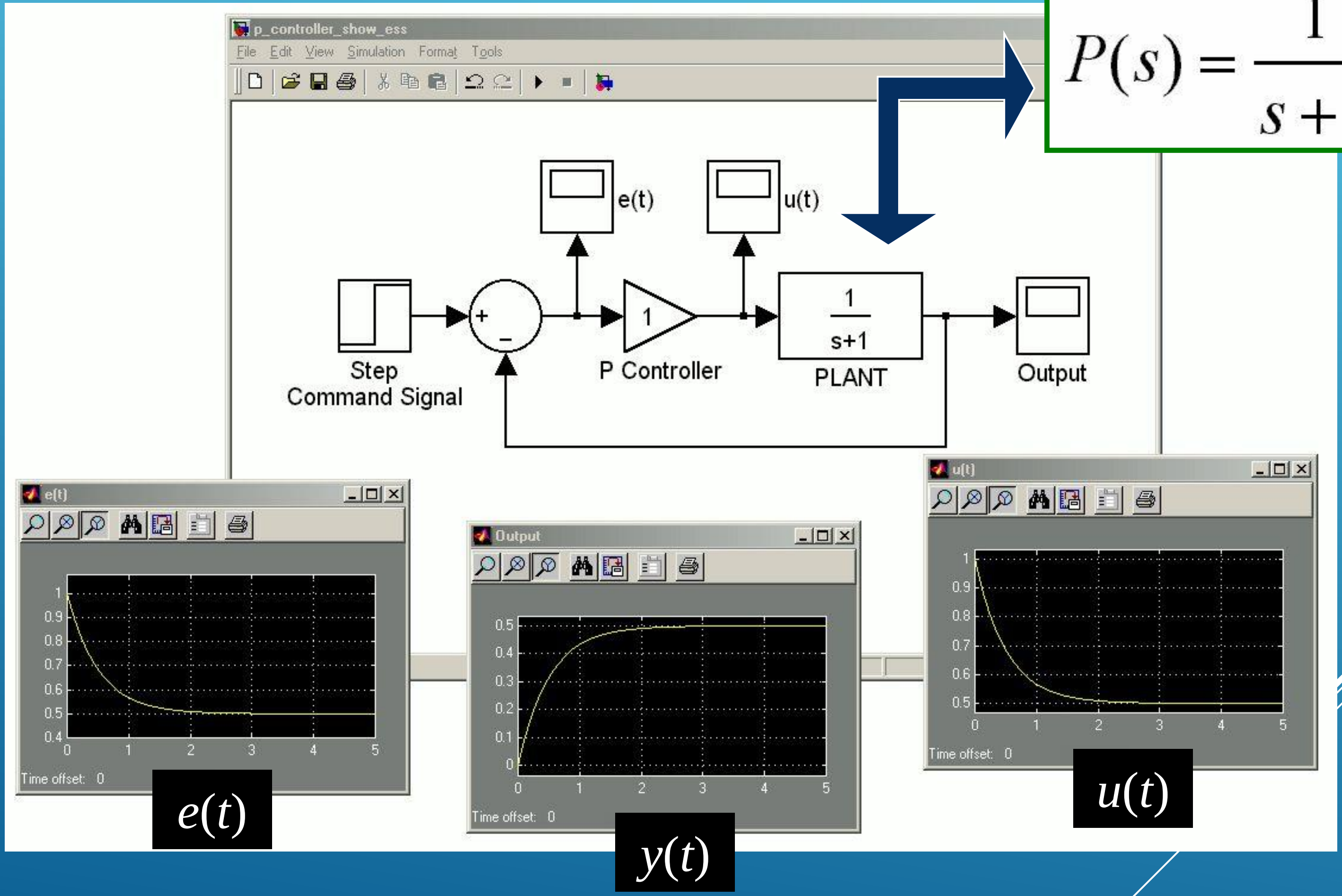
Proportional (P) Controller Remarks

- Initial condition was 2.0
- Reference input was 1.0 (step input at t=0)
- Calculated error converges to zero
- This would bring the output to 1 (Command Sig.)

Integral (I) Controller

First see what P controller performs with the plant

$$P(s) = \frac{1}{s+1}$$



Here is what happened inside...

$$T(s) = \frac{P(s)C(s)}{1 + P(s)C(s)} = \frac{K_p}{s + 1 + K_p}$$

$$Y(s) = \frac{K_p}{s + 1 + K_p} R(s), \quad R(s) = \frac{1}{s}$$

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) = \frac{K_p}{1 + K_p}$$

$$\lim_{t \to \infty} y(t) = \frac{1}{2} \text{ for } K_p = 1$$

When is this stable?

PLANT

$$P(s) = \frac{1}{s + 1}$$

CONTROLLER

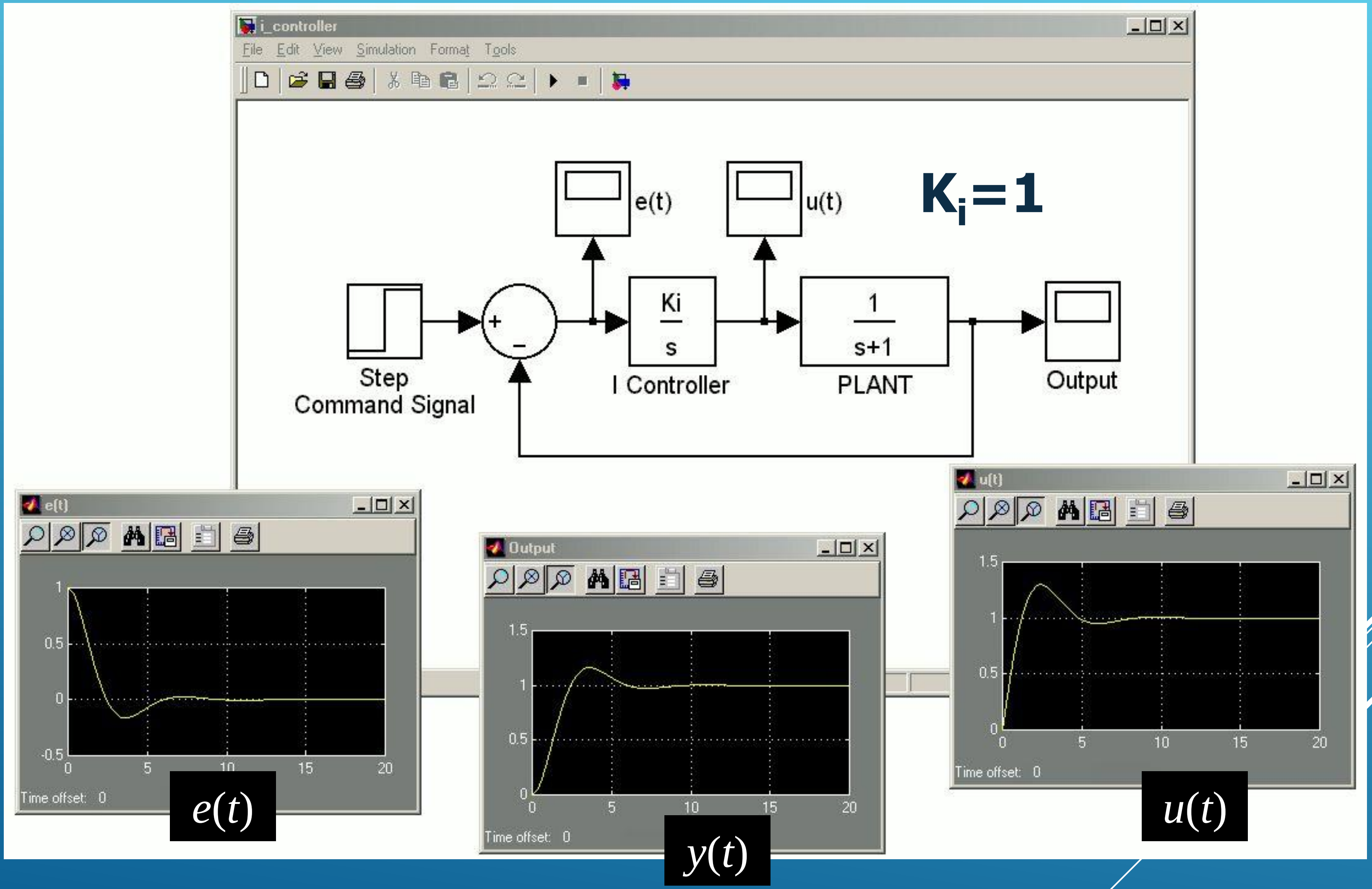
$$C(s) = 1$$

$\neq$ 1
Steady State Error!

Integral (I) Controller

- When there is steady state error, integral action is required
- Transfer functions having no integrator (no pole at s=0) would output steady state error to step input
- We will turn back to this later… Now consider the same simulation with $C(s)=K_i/s$ (Set $K_i=1$)

Integral (I) Controller



Integral (I) Controller

When is this TF stable?

$$T(s) = \frac{P(s)C(s)}{1+P(s)C(s)} = \frac{K_i}{s^2+s+K_i}$$

$$Y(s) = \frac{K_i}{s^2+s+K_i}R(s),\quad R(s) = \frac{1}{s}$$

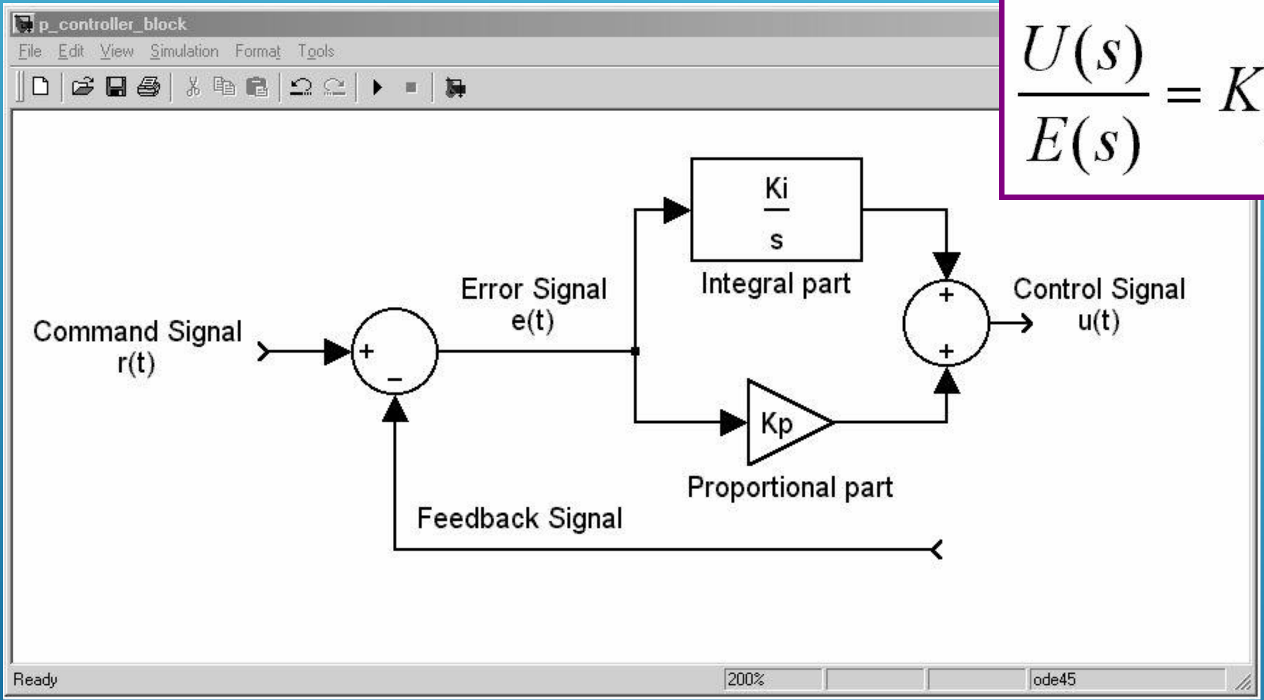
$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = 1$$

Note that

- P controller calculates the control input based on the current value of the error
- I controller calculates the control input based on the accumulated (integrated) value of the error

- A combination of both would possess the two properties collectively. This type of a controller is called PI controller

Proportional-Integral (PI) Controller



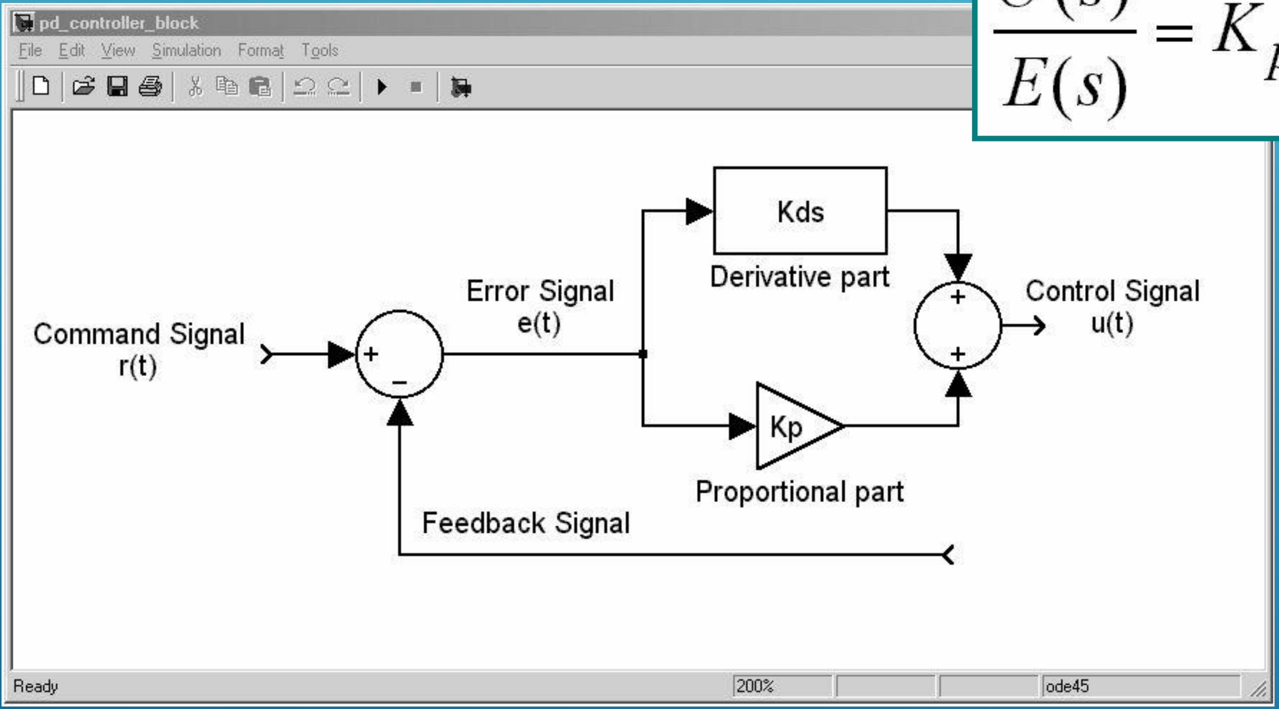
$$u(t) = K_p e(t) + K_i \int_0^t e(t)dt$$

$$\frac{U(s)}{E(s)} = K_p + \frac{K_i}{s}$$

Proportional-Derivative (PD) Controller

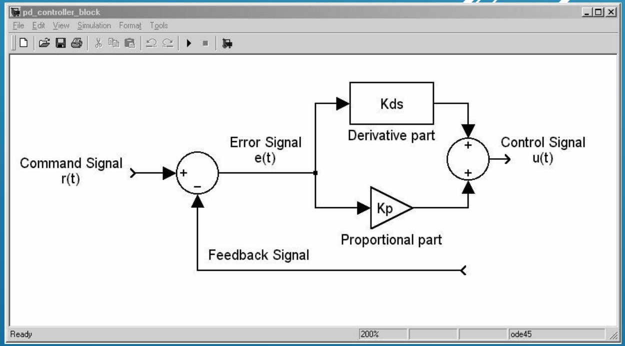
$$u(t) = K_p e(t) + K_d \frac{de(t)}{dt}$$

$$\frac{U(s)}{E(s)} = K_p + K_d s$$



Note that

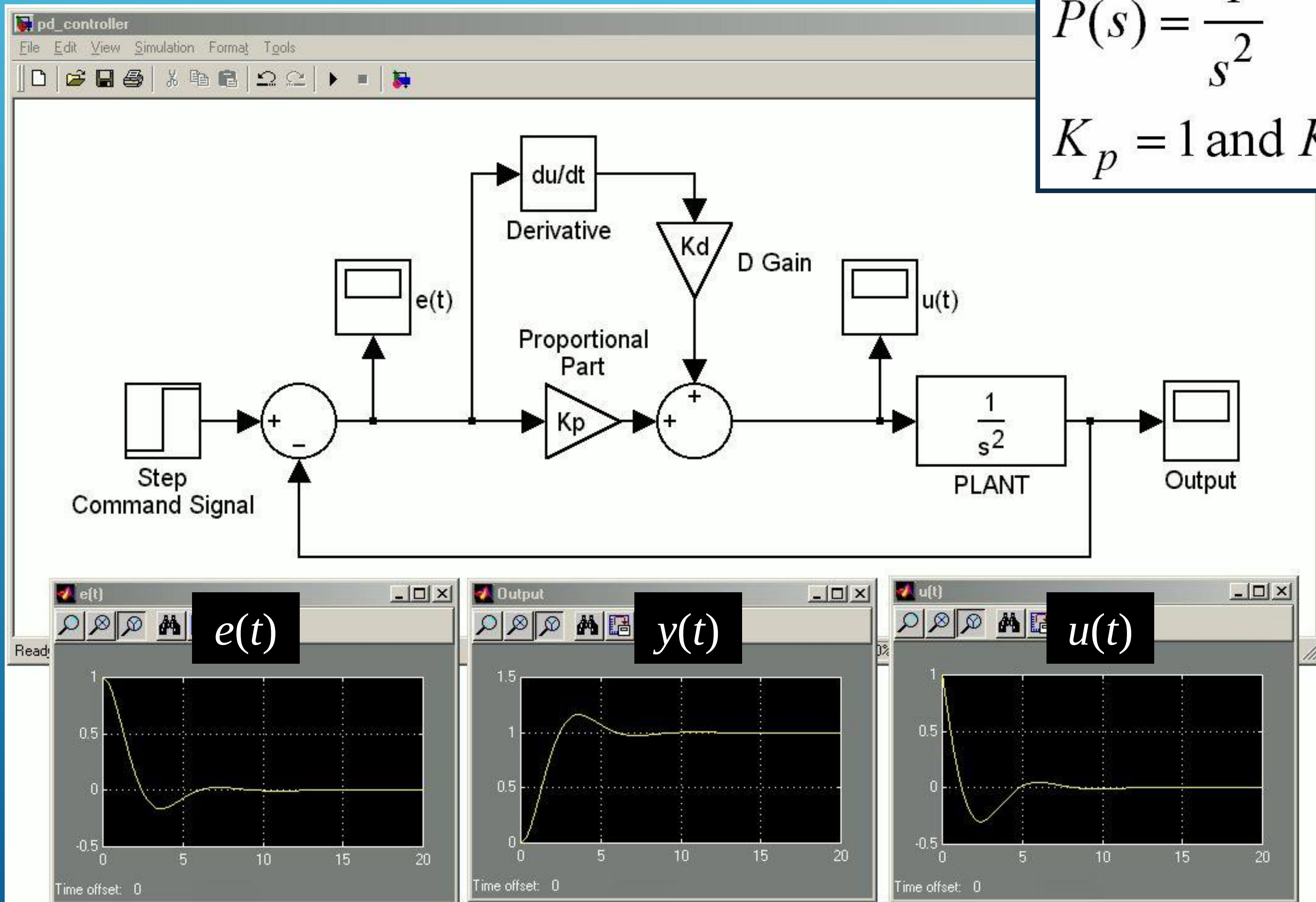
- If the feedback signal is noisy, e(t) will be noisy
- Differentiation of a noisy signal can lead to an excessively large output! Several modifications can be proposed...
- Derivative action introduces anticipatory behavior since it is based on the slope of the error signal
- A combination of P-D actions would possess the two properties collectively. This type of a controller is called PD controller



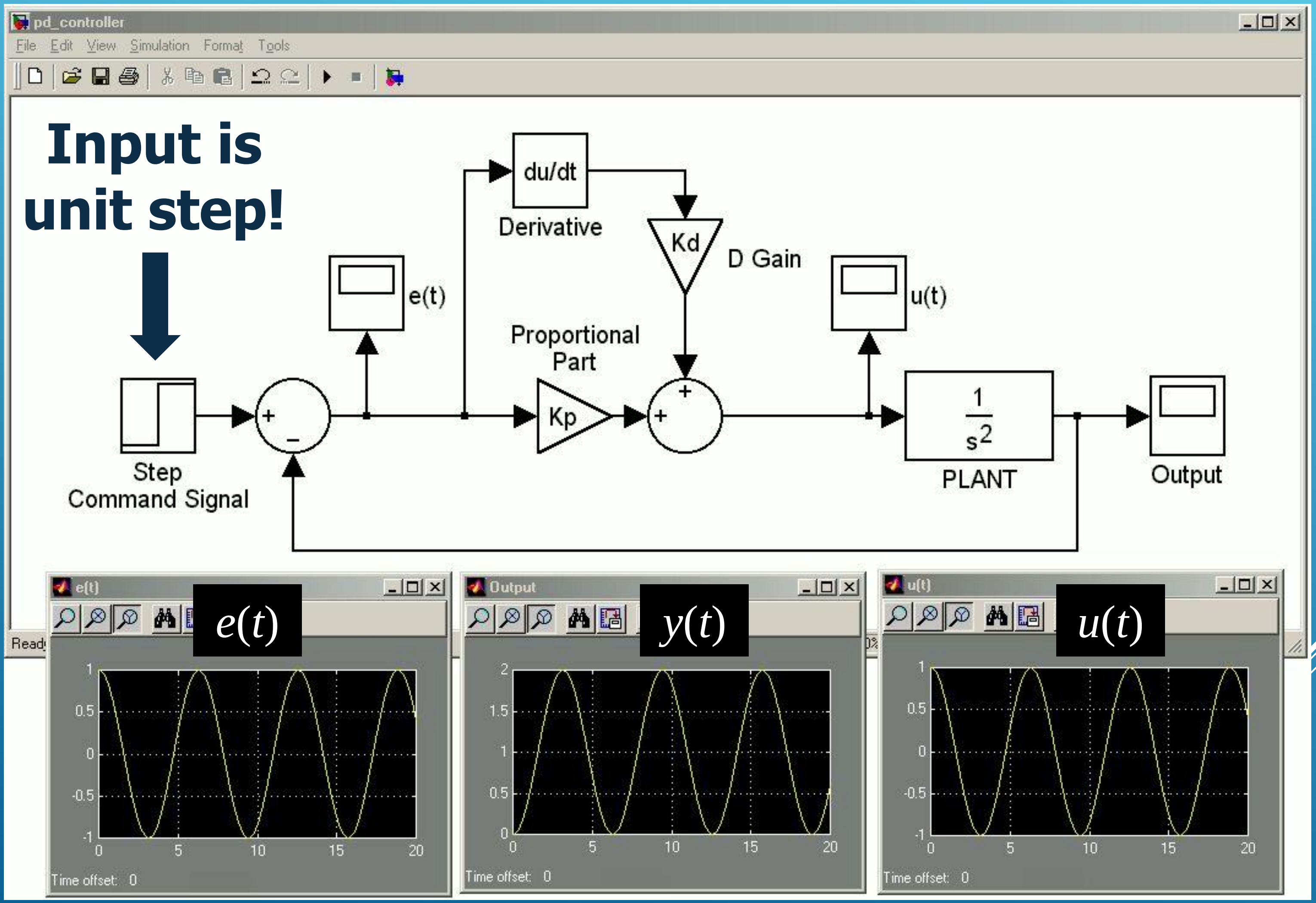
Proportional-Derivative (PD) Controller
An Example

$$P(s) = \frac{1}{s^2}$$

$$K_p = 1 \text{ and } K_d = 1$$



Proportional-Derivative (PD) Controller
An Example: **Now set K_d=0**



Proportional-Derivative (PD) Controller

An Example: Let's analyze what happened...

$$T(s) = \frac{K_d s + K_p}{s^2 + K_d s + K_p}$$

CL Transfer Function

$$T(s) = \frac{K_p}{s^2 + K_p}$$

CLTF with K_d=0

$$R(s) = \frac{1}{s}$$

Unit Step

$$y(t) = L^{-1}\left\{\frac{1}{s}\frac{K_p}{s^2 + K_p}\right\} = 1(t) - \cos\left(\sqrt{K_p}\, t\right)$$

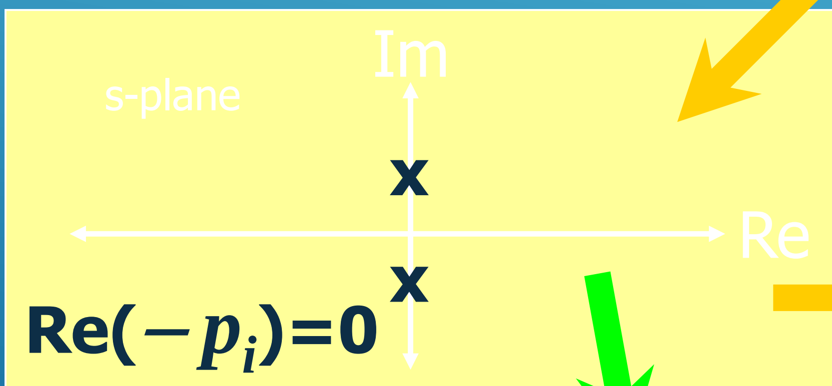
With only proportional controller, the output oscillates in response to constant input

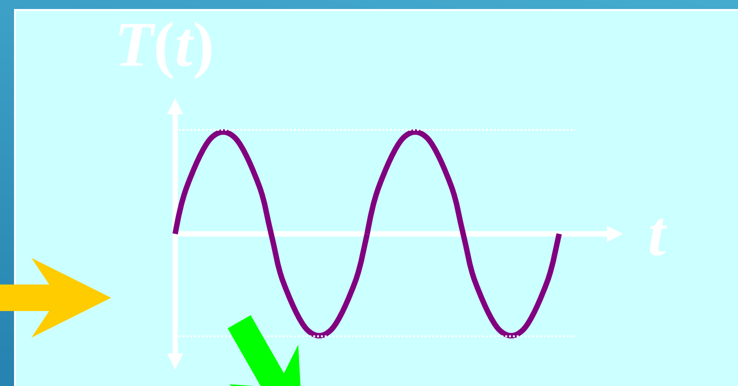
Proportional-Derivative (PD) Controller
An Example: **Let's see in terms of stability**

$$T(s)=\frac{K_d s+K_p}{s^2+K_d s+K_p}$$

$$T(s)=\frac{K_p}{s^2+K_p}$$

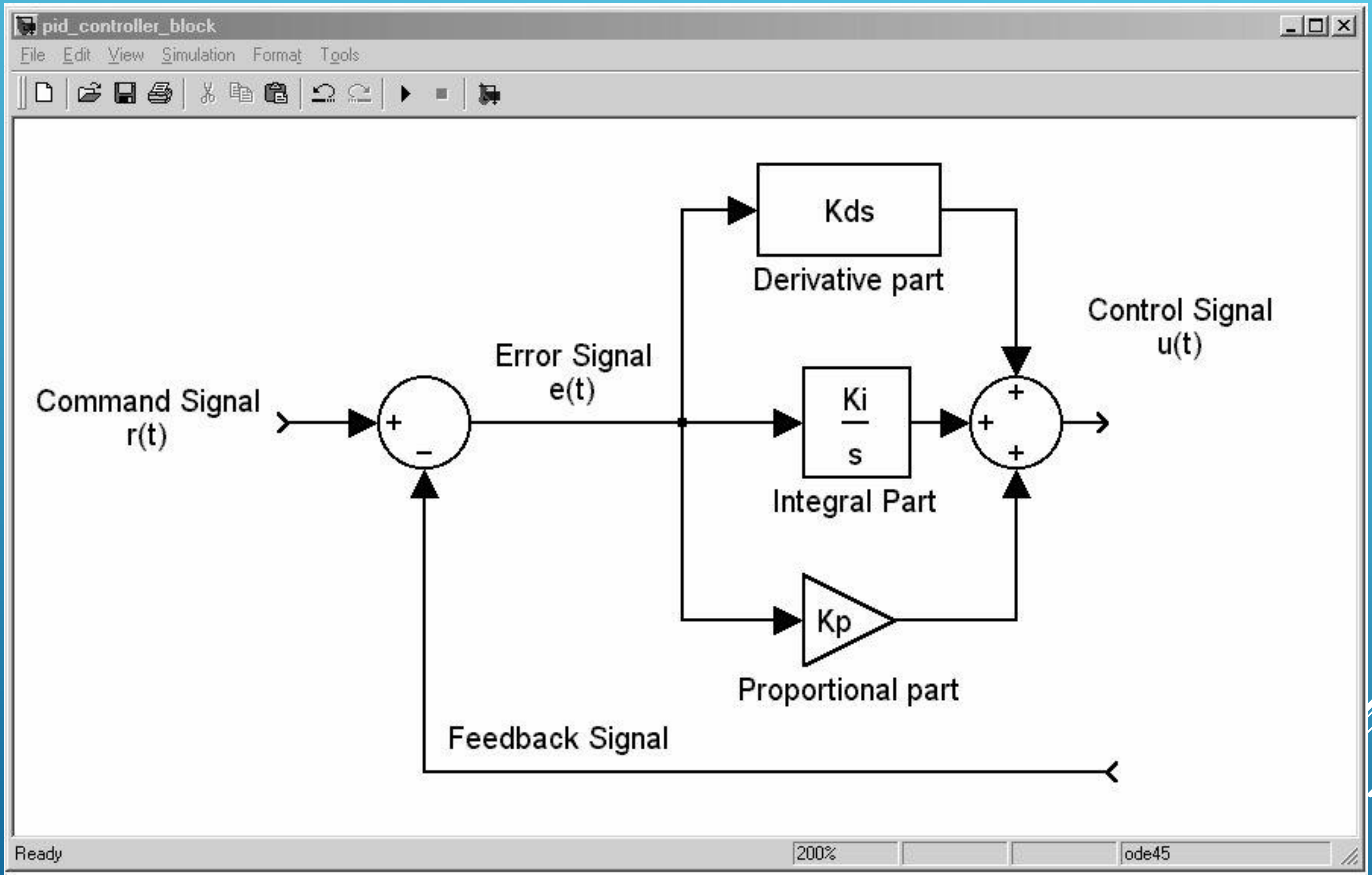
$$R(s)=\frac{1}{s}$$





$$y(t)=L^{-1}\left\{\frac{1}{s}\frac{K_p}{s^2+K_p}\right\}=1(t)-\cos\left(\sqrt{K_p}\,t\right)$$

Proportional-Integral-Derivative (PID) Controller



Proportional-Integral-Derivative (PID) Controller

- Over 95% of the controllers operating in industry are of type PID
- PID Controller utilizes the information contained in the current value, accumulated value and the tendency of the error signal
- Hardware/Software implementation of the PID controller is easy

Proportional-Integral-Derivative (PID) Controller

- If the plant transfer function is changing, PID controller may not account for the entire set of combinations

PID Controller
Questions & Answers

Q: Can we so freely assign the controller parameters?
A: NO

Q: What constraints do we have in designing a PID controller?
A: First requirement is the stability, then the design specifications must be met

Q: How to check stability compactly? What are design specifications?
A: Next week's agenda...