The effect of damping ratio on process response[1-5]

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$\tau = 4$ and $\zeta = 1$

$$\frac{Y(s)}{X(s)} = \frac{1}{\tau^2 s^2 + 2\zeta\tau s + 1} = \frac{1}{16s^2 + 8s + 1}$$

$$\tau^2 s^2 + 2\zeta\tau s + 1 = 0$$

ξ: **the damping coefficient**

$$s_1 = -\frac{\zeta}{\tau} + \frac{\sqrt{\zeta^2 - 1}}{\tau} \qquad s_2 = -\frac{\zeta}{\tau} - \frac{\sqrt{\zeta^2 - 1}}{\tau} \qquad \zeta = 1$$

$$s_1 = -\frac{1}{4} = -0.25 \qquad s_2 = -\frac{1}{4} = -0.25 \qquad \text{(repeated roots)}$$

$$X(t) = 1 \quad \rightarrow \quad X(s) = \frac{1}{s}$$

$$Y(s) = \frac{1}{s}\frac{1/\tau^2}{(s-s_1)(s-s_2)} = \frac{1/16}{s\,(s+0.25)(s+0.25)} = \frac{0.0625}{s\,(s+0.25)^2}$$

$$Y(s) = \frac{0.0625}{s\,(s+0.25)^2} = \frac{A}{s} + \frac{B}{(s+0.25)^2} + \frac{C}{(s+0.25)}$$

To obtain A, multiply equation by s and set s = 0

$$s\frac{0.0625}{s\,(s+0.25)^2} = s\frac{A}{s} + s\frac{B}{(s+0.25)^2} + s\frac{C}{(s+0.25)}$$

$A = 1$

To obtain B, multiply equation by $(s+0.25)^2$ and set $s = -0.25$

$$(s+0.25)^2\frac{0.0625}{s\,(s+0.25)^2} = (s+0.25)^2\frac{A}{s} + (s+0.25)^2\frac{B}{(s+0.25)^2} + (s+0.25)^2\frac{C}{(s+0.25)}$$

$B = -0.25$

To obtain C,

$$\frac{0.0625}{s\,(s+0.25)^2} = \frac{1}{s} - \frac{0.25}{(s+0.25)^2} + \frac{C}{(s+0.25)}$$

$$\frac{0.0625}{s\,(s+0.25)^2} = \frac{(s+0.25)^2 - 0.25s + C\,s\,(s+0.25)}{s\;\;(s+0.25)^2}$$

$$\frac{0.0625}{s\,(s+0.25)^2} = \frac{s^2 + 0.5s + 0.0625 - 0.25s + Cs^2 + Cs0.25)}{s\;\;(s+0.25)^2}$$

$1 + C = 0 \qquad\qquad C = -1$

$$Y(s) = \frac{1}{s} - \frac{0.25}{(s+0.25)^2} - \frac{1}{(s+0.25)}$$

$$Y(t) = 1 - 0.25t\,e^{-0.25\,t} - e^{-0.25\,t}$$

$$Y(t) = 1 - (1 + 0.25t)\,e^{-0.25\,t}$$

four conditions for the damping coefficient

ξ: **the damping coefficient**

ξ >1 The roots of the denominator of output function are **real**, negative but not equal

ξ =1 The roots of the denominator of output function are **real**, negative but equal

ξ <1 The roots of the denominator of output function are **complex**

ξ =0 The roots of the denominator of output function are **complex with no real part**

$$\theta_L(s) = \frac{F}{s}$$

$$\theta_1(s) = \frac{F / s(K_L \tau_R s)}{L\tau_R s^2 + \tau_R(1+K)s + K}$$

$$\theta_1(s) = \frac{FK_L\tau_R}{K}\left[\frac{1}{\frac{\tau_R L s^2}{K} + \frac{\tau_R(1+K)s}{K} + 1}\right]$$

complex roots $\xi<1$

$$W_m{}^2 = \frac{K}{\tau_R L} = \frac{K+1}{2}(\tau_R / K_L)^{0.5}$$

$$\theta_1(s) = \frac{FK_L\tau_R}{K}\left[\frac{W_m}{(1-\rho^2)^{0.5}} \frac{W_m(1-\rho^2)^{0.5}}{(s+W_m\rho^2)^2 + W_m{}^2(1-\rho^2)}\right]$$

$$\theta_1(t) = \frac{FK_L\tau_R}{K}\left[\frac{W_m}{(1-\rho^2)^{0.5}} \exp(-W_m\rho t)\sin(1-\rho t)^{0.5}\right]$$

real roots $\xi>1$

$$\frac{\tau_R L}{K}s^2+\frac{\tau_R(1+K)}{K}s+1=(L_f s+1)(L_g s+1)$$

$$\theta_1(t)=\frac{FK_L\tau_R}{K}\left[\frac{1}{(L_f s+1)(L_g s+1)}\right]$$

$$\theta_1(t)=\frac{FK_L\tau_R}{K}\frac{1}{L_f-L_g}(\exp(-t/L_f)-\exp(-t/L_g))$$