BME 211 Circuit Analysis Laboratory

Experiment #6: Capacitors and Inductors,

RC and RL Circuits

Objective

The objective of this experiment is to understand and analyze circuits that contains capacitors or inductors which are components that can store energy. The relation between terminal voltage and terminal current for these components and first order circuits that are composed of resistors and either capacitors or inductors will be studied. The consept of phasors will be introduced and applied to first order circuits.

Background

1. Capacitors

The capacitor is one of the three basic passive circuit components (resistor, capacitor, inductor) of any electronic or electrical circuit. Resistance in a circuit gives rise to ohmic or watt losses, and its current is in phase with the applied voltage waveform. Inductance or a capacitance gives rise to currents out of phase with voltage by 90° in AC circuits, and is the cause of transient currents in many circuits. A capacitor works in electric field. It stores energy when a steady voltage is applied. It gets charged to the applied voltage and keeps the energy as well as the voltage even after removal of external voltage. This factor makes handling of capacitors quite dangerous at times, and caution must be exercised when working with them. A capacitor offers an open circuit to the flow of DC current in steady state. Current in ideal capacitor leads the voltage by 90° in AC circuits [3].

A capacitor can be constructed by using two parallel conducting plates separated by distance d as shown in Figure 6.1. Electric charge is stored on the plates, and a uniform electric field exists between the conducting plates whenever there is a voltage across the capacitor. The space between the plates is filled with a dielectric material [5]. Impregnated paper, mica sheets, ceramics, metal films, or air can be used for a dielectric. The capacitance value of the paralel plate capacitor is given as

$$C = \frac{\varepsilon A}{d}$$

where;

- ε : Dielectric constant of the plates
- A : Surface area of the plates
- d : Distance between the plates

and the unit is Farad (F). Circuit symbols of a capacitor are given in figure x.

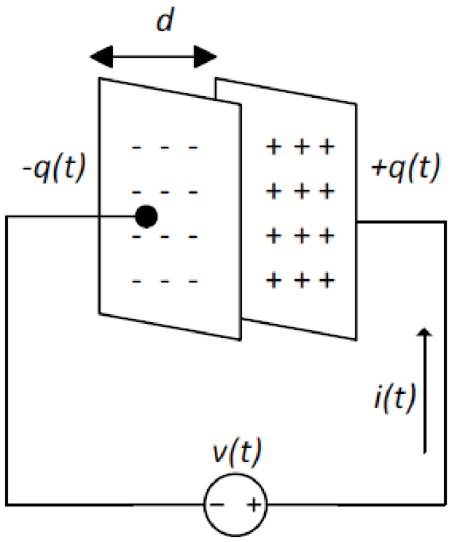

Figure 6.1

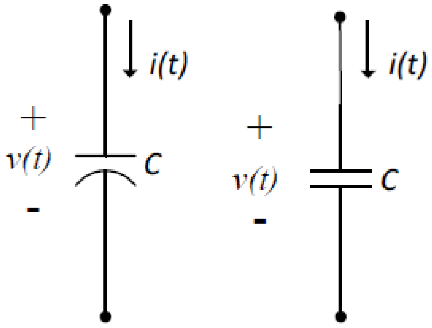

Figure 6.2

Capacitor voltage v(t) deposits a charge +q(t) on one plate and a charge –q(t) on the other plate. The charge q(t) is stored by the capacitor is given as.

$$q(t) = Cv(t)$$

In general, the capacitor voltage v(t) varies as a function of time. Consequently, q(t), the charge stored by the capacitor, also varies as a function of time. The variation of the capacitor charge with respect to time implies a capacitor current, i(t), and is given as:

$$i(t) = \frac{d}{dt}q(t)$$

$$i(t) = C\frac{d}{dt}v(t)$$

Voltage v(t) can be obtained in terms of i(t) using:

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$

$$v(t) = \frac{1}{C}\int_{t_o}^{t} i(\tau)d\tau + \frac{1}{C}\int_{-\infty}^{t_o} i(\tau)d\tau = \frac{1}{C}\int_{t_o}^{t} i(\tau)d\tau + v(t_o)$$

The power and energy stored in the capacitor are given as:

$$p = vi = v\left(C\frac{dv}{dt}\right)$$

$$w_c(t) = \frac{1}{2}Cv^2(t)$$

If the capacitor is connected to the terminals of a resistor, a current flows until all the energy is dissipated as heat by the resistor. After all the energy dissipates, the current is zero and the voltage across the capacitor is zero. Voltage and charge on a capacitor cannot change instantaneously which is summarized by the equation x where t = 0 is called $t = 0^-$ and the time immediately after t = 0 is called $t = 0^+$ [5].

$$v(0^+) = v(0^-)$$

1.1 Capacitor Types

Capacitors are divided into two groups which are fixed and variable capacitors [4].

1.1.1 Fixed Capacitors

a) Paper Capacitors

A paper capacitor is one that uses paper as its dielectric. It consists of flat thin strips of metal foil conductors, seperated by the dielectric material. In this capacitor the dielectric used is waxed paper. Paper capacitors usually range in value from about 300 picofarads to about 4 microfarads. Normally, the voltage limit across the plates rarely exceeds 600 volts.

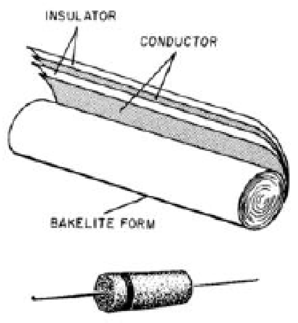

Figure 6.3 Paper capacitor[4]

b) Mica Capacitors

A mica capacitor is made of metal foil plates that are seperated by sheets of mica, which from the dielectric. Mica is an excellent dielectric and will withstand higher voltages than paper without allowing arching between the plates. Common values of mica capacitors range from approximately 50 picofarads to about 0.02 microfarad. Some typical mica capacitors are shown in Figure 6.4

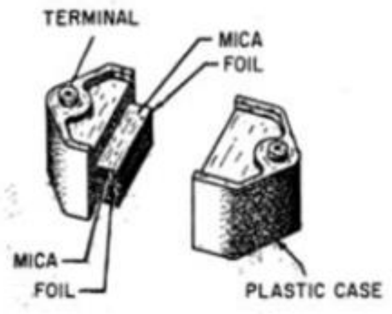

Figure 6.4. Mica capacitors[4]

d) Electrolytic Capacitors

Electrolytic capacitors are used where a large amount of capacitance is required. As the name implies, electrolytic capacitors contain an electrolyte. This electrolyte can be in the form of either a liquid (wet electrolytic capacitor) or a paste (dry electrolytic capacitor). Wet electrolytic capacitors are no longer in popular use due to the care needed to prevent spilling of the electrolyte. Dry electrolytic capacitors consists of two metal plates between which is placed the electrolyte[4].

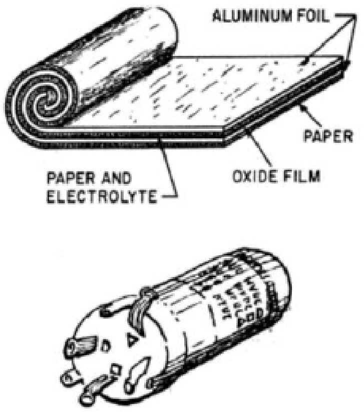

Figure 6.6 Electrolytic capacitor[4]

e) Oil Capacitors

Oil capacitors are often used in radio transmitters where high output power is desired. Oil-filled capacitors are nothing more than paper capacitors that are immersed in oil [4].

1.1.2 Variable Capacitors

Variable capacitors are constructed in such manner that their value of capacitance can be varied. A typical variable capacitor (adjustable capacitor) is the rotor/stator type. It consists of two sets of metal plates move between the stator plates. Air is the dielectric. As the position of the rotor is changed, the capacitance value is likewise changed. This is the type capacitor used for tuning most radio receivers and it is shown in figure x. Another type variable (trimmer) capacitor is shown in figure x. It consist of two plates seperated by a sheet of mica. A screw adjustment is used to change the distance between the plates, thereby changing the capacitance [4].

2. Inductors

An inductor is a circuit element that stores enegry in a magnetic filed. An inductor can be constructed by winding a coil of wire around a magnetic core as shown in Figure 6.9 [5]. An ideal inductor is a short circuit path to a steady DC current. In AC circuits, its current lags behind the voltage by 90° [3].

Inductors are represented by a parameter called the inductance which is given below. The unit of inductance is henry (H).

$$L = \frac{\mu N^2 A}{l}$$

N : The number of turns
l : The length of the winding
A : Cross sectional area of the core
μ : Permeability

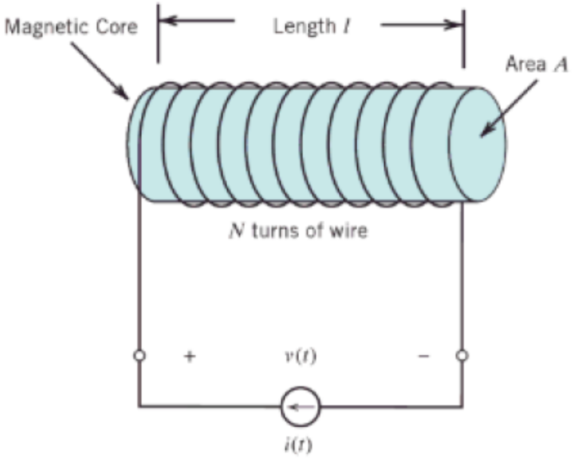

Figure 6.9

The voltage v(t) across the coil and the inductor current i(t) are given as:

$$v(t) = L\frac{d}{dt}i(t)$$

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau$$

$$i(t) = \frac{1}{L}\int_{t_o}^{t} v(\tau)d\tau + \frac{1}{L}\int_{-\infty}^{t_o} v(\tau)d\tau = \frac{1}{L}\int_{t_o}^{t} v(\tau)d\tau + i(t_o)$$

The power and energy stored in the inductor are given below:

$$p = vi = \left(L\frac{di}{dt}\right)i$$

$$w = \frac{1}{2}Li^2$$

3. First Order Circuits

3.1 Series RC Circuits

A circuit that consists of pure resistance connected in series with pure capacitor is shown in Figure 6.10.

Voltage drop across the resistor : $V_R = I \; x \; R$

Voltage drop across the capacitor : $V_C = I \; x \; X_C$

Capacitive reactance : $X_C = 1/2\pi f C$

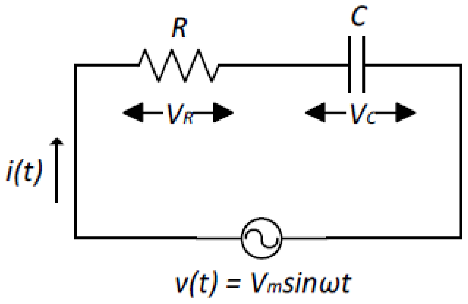

Figure 6.10

where I and V_R, V_C are rms values of current drawn and the voltage drops respectively. If the Kirchhoff's voltage law is applied to the AC RC circuit, phasor (vector) addition should be realized. Phasor addition of voltages are given in the equations below:

$$\bar{V} = \overline{V_R} + \overline{V_C}$$

$$\bar{V} = \overline{IR} + \overline{IX_C}$$

Phasor diagram, voltage, impedance and power triangle of the RC circuit are given in the Figures 6.11 and 6.12.

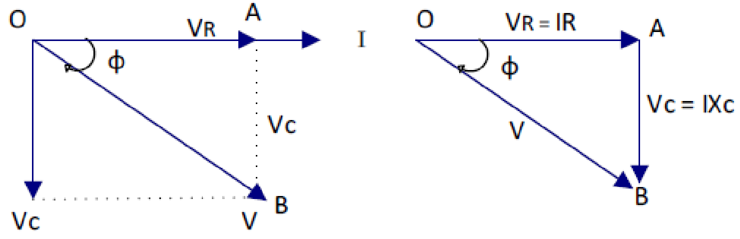

Figure 6.11 Phasor diagram and voltage triangle

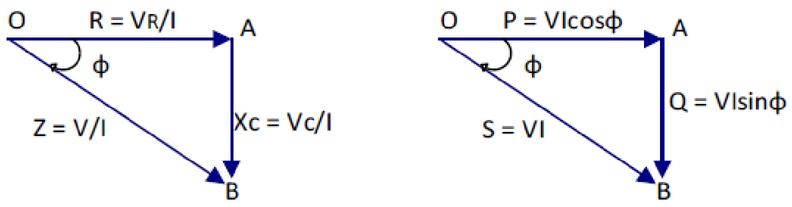

Figure 6.12. Impedance and power triangle

Equations given above are derived in terms of voltage triangle.

$$V = \sqrt{V_R^2 + V_C^2}$$

$$V = \sqrt{(IR)^2 + (IX_C)^2} = I\sqrt{(R)^2 + (X_C)^2}$$

$$V = IZ$$

Impedance : $Z = \sqrt{(R)^2 + (X_C)^2}$

The current leads the applied voltage by an angle Φ given as:

$$\cos\Phi = \frac{V_R}{V} = \frac{R}{Z}$$

Apparent power, true power and reactive power formulas are given in the equations below:

$$S = VI, \quad (\text{VA})$$

$$P = VI \cos\Phi, \quad (\text{W})$$

$$Q = VI \sin\Phi, \quad (\text{VAR})$$

3.2 Parallel RC Circuits

When a circuit consists of pure resistance connected in parallel with pure capacitor, equations given above are derived in terms of the current triangle,

$$I = \sqrt{I_R^2 + I_C^2}$$

$$I = \sqrt{(V/R)^2 + (V/X_C)^2} = V\sqrt{(1/R)^2 + (1/X_C)^2}$$

Impedance :

$$Z = V/I$$

The current leads the applied voltage by an angle Φ given as

$$\cos\Phi = \frac{I_R}{I}$$

3.3 Series RL Circuits

A circuit which consists of pure resistance connected in series with pure inductance is shown in Figure 6.13.

Voltage drop across the resistance : $V_R = I\ x\ R$

Voltage drop across the inductance : $V_L = I\ x\ X_L$

Inductive reactance : $X_L = 2\pi f l$

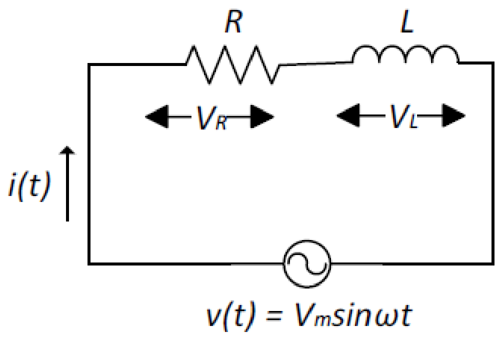

Figure 6.13. Series RL circuit

where I and V_R , V_L are rms values of current drawn and the voltage drops respectively.

Phasor sum of voltages are given in the following equations.

$$\bar{V} = \overline{V_R} + \overline{V_L}$$

$$\bar{V} = \overline{IR} + \overline{IX_L}$$

Phasor diagram, voltage, impedance and power triangle of the series RL circuit are given in the figure below.

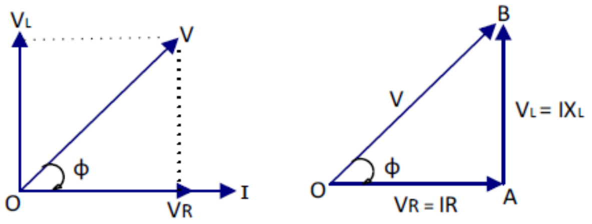

Figure 6.14. Phasor diagram and voltage triangle

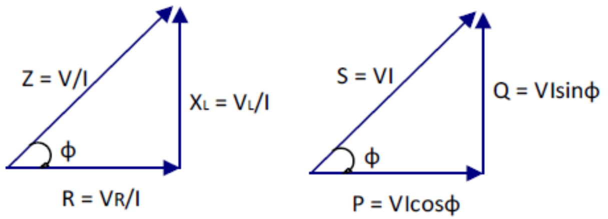

Figure 6.15 Impedance and power triangle

Equations given below are derived in terms of the voltage triangle.

$$V = \sqrt{V_R^2 + V_L^2}$$

$$V = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{(R)^2 + (X_L)^2}$$

$$V = IZ$$

Impedance :

$$Z = \sqrt{(R)^2 + (X_L)^2}$$

The current lags behind the applied voltage by an angle Φ given in the equation x.

$$\cos\Phi = \frac{V_R}{V} = \frac{R}{Z}$$

Apparent power, true power and reactive power formulas are given in the equations below.

$$S = VI \ \ (\mathrm{VA})$$

$$P = VI \ \cos\Phi \quad (\mathrm{W})$$

$$Q = VI \ \sin\Phi \quad (\mathrm{VAR})$$

3.4 Parallel RL Circuits

When a circuit consists of pure resistance connected in parallel with pure inductor, equations given above are derived in terms of current triangle,

$$I = \sqrt{I_R^2 + I_L^2}$$

$$I = \sqrt{(V/R)^2 + (V/X_L)^2} = V\sqrt{(1/R)^2 + (1/X_L)^2}$$

The sides of the triangle representing the conductance, susceptance and admittance of the circuit, it is known as the admittance triangle. The figure 6.16 shows admittance triangles.

$$\frac{I^2}{V^2} = \frac{I_R^2}{V^2} + \frac{I_L^2}{V^2}$$

$$Z = \frac{V}{I}$$

$$R = \frac{V}{I_R}$$

$$X_L = \frac{V}{I_L}$$

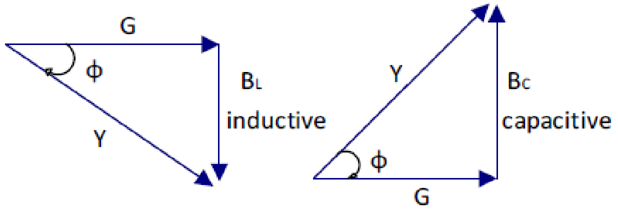

Figure 6.16 Admittance triangles

Admittance : $Y = \frac{1}{Z}$

Conductance : $G = \frac{1}{R}$

Susteptance : $B = \frac{1}{X_L}$

$$Y^2 = G^2 + B^2$$

The current lags behind the applied voltage by an angle Φ given as.

$$\cos\Phi = \frac{I_R}{I}$$

Preliminary Work

1. Find and plot the voltage v(t) for a capacitor C = 1/4 F if the current is as shown in figure 6.17 and V(0)=0.

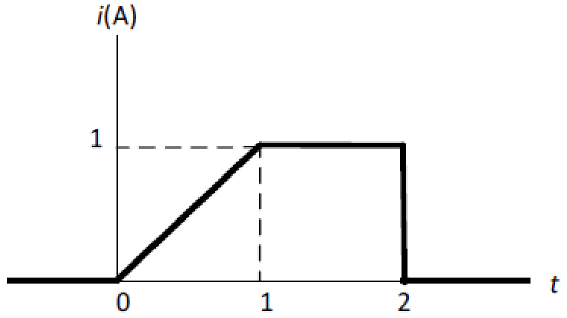

Figure 6.17

2. If the voltage across the 2mF capacitor is represented by the signal shown in figure 6.18, find and plot the capacitor current.

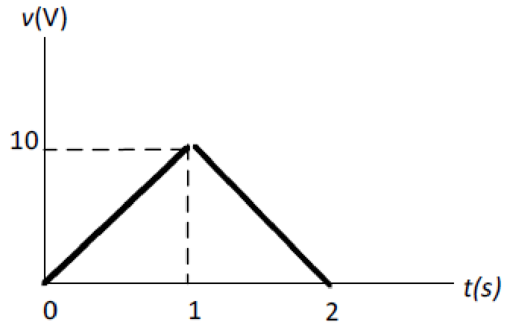

Figure 6.18

3. Find the expression for the instantaneous values of the voltages across the resistance (V_R), inductance (V_L) and combination (V) if $i = 414\sin(2\pi f t)\ A, f = 50Hz, R = 100\Omega\ , L = 0.31831\ H$.

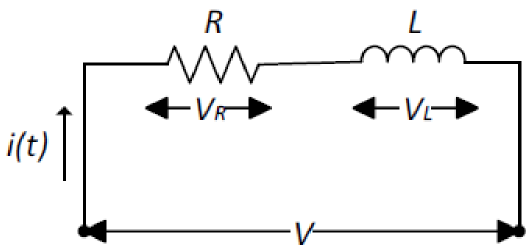

Figure 6.19

4. If voltage and current waveforms of the circuit given in the figure are $v = 120\sin(314t)$ and $i = 10\sin(314t + \pi/6)$ respectively,
a) Calculate the values of the resistance and capacitance.
b) Draw the waveforms for current, voltage and phasor diagram.
c) Calculate the power consumed by the circuit.

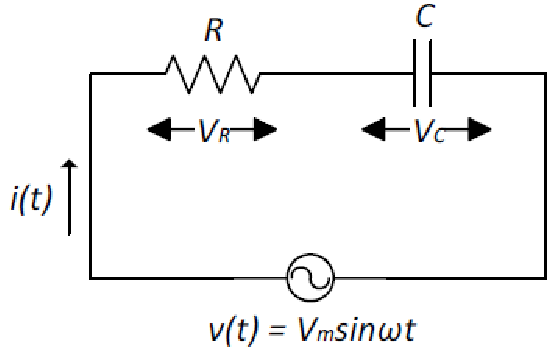

Figure 6.20

Procedure

1. Measure the internal resistance of the 1mH inductor. Set up the given circuit in figure 6.21 with R = 270Ω and L = 1mH. Apply a sinusoidal voltage as input. Adjust $V_s = 4V_{pp}$ and set the frequency as 20kHz. Observe the input voltage and the voltage across the resistor simultaneously using an oscilloscope and plot both signals. Find the phase difference between these signals and comment on the results. Make sure that Vs is maintaining at $4V_{pp}$ during measurements.

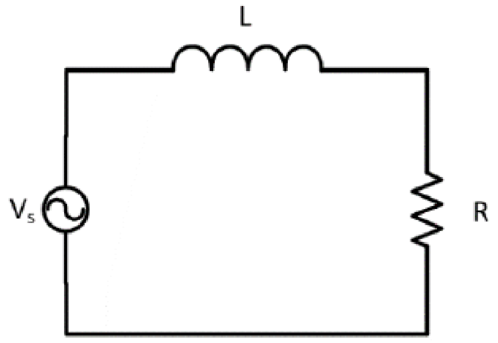

Figure 6.21

2. For the same circuit (in figure 6.21), set the amplitude as $V_s = 4V_{pp}$ and the frequency as 100Hz, 1KHz, 10kHz, 20kHz, 50kHz, 100kHz and 500kHz. Measure the voltage across R and L using multimeter and calculate I_{rms} for each frequency value. Comment on the results.

3. Calculate the impedance of the inductor at 20kHz and obtain the phasor diagram using the related values you measured in the previous step. Calculate the apparent power, true power and reactive power related to each component in the circuit.

4. Set up the given circuit in figure 6.22 with R = 270Ω and C=1uF. Apply a sinusoidal voltage as input. Adjust V_{AB} = 4Vpp and set the frequency as 1kHz. Observe the input voltage and the voltage across the resistor simultaneously using an oscilloscope and plot both signals. Find the phase difference between these signals and comment on the result. Make sure that Vs is maintaining at $4V_{pp}$ during measurements.

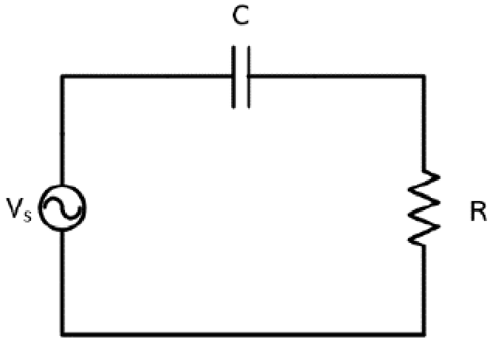

Figure 6.22

5. For the same circuit in step 4, set the amplitue as V_{pp} =4V and frequency as 100Hz, 1KHz, 10kHz, 20kHz, 50kHz, 100kHz and 500kHz. Measure the voltage across R and C using multimeter and calculate I_{rms} for each frequency value. Comment on the results.

6. Calculate the impedance of the capacitor at 1kHz and obtain the phasor diagram using the related values you measured in the previous step. Calculate the apparent power, true power and reactive power related to each component in the circuit.

List of Equipment and Components

Equipment: Function Generator, Oscilloscope, Multimeter

Components: 270Ω resistor, 1mH inductor, 1µF capacitor

References

[1] Boylestad, Introductory Circuit Analysis (Tenth Edition)

[2] James W. Nilsson, Susan A. Riedel, Electric Circuits (Ninth Edition)

[3] R. P. Deshpande, Capacitors: Technology and Trends

[4] United States. Dept. of the Army,Naval Education and Training Program Development Center, Basic Electricity

[5] Richard C. Dorf, James A. Svoboda, Introduction to Electric Circuits

[6] A.V.Bakshi U.A.Bakshi, Circuit Theory

5. For the same circuit in step 4, set the amplitue as V_{pp} =4V and frequency as 100Hz, 1KHz, 10kHz, 20kHz, 50kHz, 100kHz and 500kHz. Measure the voltage across R and C using multimeter and calculate I_{rms} for each frequency value. Comment on the results.

6. Calculate the impedance of the capacitor at 1kHz and obtain the phasor diagram using the related values you measured in the previous step. Calculate the apparent power, true power and reactive power related to each component in the circuit.

List of Equipment and Components

Equipment: Function Generator, Oscilloscope, Multimeter

Components: 270Ω resistor, 1mH inductor, 1µF capacitor

References

[1] Boylestad, Introductory Circuit Analysis (Tenth Edition)

[2] James W. Nilsson, Susan A. Riedel, Electric Circuits (Ninth Edition)

[3] R. P. Deshpande, Capacitors: Technology and Trends

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[5] Richard C. Dorf, James A. Svoboda, Introduction to Electric Circuits

[6] A.V.Bakshi U.A.Bakshi, Circuit Theory

CAPACITORS AND INDUCTORS, RC AND RL CIRCUITS

Objective

Results

1.

$R_{inductor}$	
Phase difference	

Comment:

2.

	100Hz	1KHz	10kHz	20kHz	50kHz	100kHz	500kHz
V_R							
V_L							
I_{rms}							

Comment:

3. Impedance of Inductor:

Comment:

	Apparent Power	True Power	Reactive Power
Power supply			
Resistance			
Inductanse			

4.

Comment:

5.

	100Hz	1KHz	10kHz	20kHz	50kHz	100kHz	500kHz
V_R							
V_C							
I_{rms}							

Comment:

6. Impedance of Capacitor:

Comments :

	Apparent Power	True Power	Reactive Power
Power supply			
Resistance			
Capasitance			