BME 211 Circuit Analysis Laboratory

Experiment #9: RLC Circuits

Objective

The objective of this experiment is to understand and analyze circuits that contain both capacitors and inductors, which are components that can store energy. The concept of impedance will be introduced and use of phasor diagrams that were studied for first order circuits will be extented to second order circuits.

Background

Each element has a unique phase response: for resistors, the voltage is always in phase with the current, for capacitors the voltage always lags the current by 90 degrees, and for inductors the voltage always leads the current by 90 degrees. Consequently, a series combination of R, L, and C components will yield a complex impedance with a phase angle between +90 and -90 degrees. Due to the phase response, circuit analysis must be carried out using vector (phasor) sums rather than simply relying on the magnitudes. In phasor domanin, the relation between terminal voltage and terminal current for basic circuit elements can be expressed using their impedance. These relations are summarized in Table 9.1.

Table 9.1 Impedance for basic circuit elements

Circuit Element	Resistance, (R)	Reactance, (X)	Impedance, (Z)
Resistor	R	0	$Z_R = R = R\angle 0^\circ$
Inductor	0	ωL	$Z_L = j\omega L = \omega L \angle{+90}^\circ$
Capacitor	0	$-1/\omega C$	$Z_c = 1/j\omega C = (1/\omega C)\angle{-90}^\circ$

1. Series RLC Circuits

Series RLC circuits are classified as second-order circuits because they contain two energy storage elements, an inductance L and a capacitance C.

The series RLC circuit in Figure 9.1 has a single loop with the instantaneous current flowing through the loop being the same for each circuit element. Since the inductive and capacitive reactance's X_L and X_C are a function of the supply frequency, the sinusoidal response of a series RLC circuit will therefore vary with frequency, *ƒ*. Then the individual voltage drops across each circuit element of R, L and C element will be "out-of-phase" with each other.

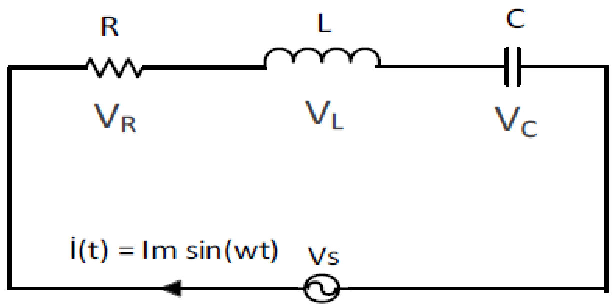

Figure 9.1 A series RLC circuit

Considering the current as reference:

- The instantaneous voltage across a pure resistor, V_R is "in-phase" with the current.
- The instantaneous voltage across a pure inductor, V_L "leads" the current by 90°.
- The instantaneous voltage across a pure capacitor, V_C "lags" the current by 90°.
- Therefore, V_L and V_C are 180° "out-of-phase" and in opposition to each other.

The amplitude of the source voltage across all three components in a series RLC circuit is the combination of the three individual component voltages, VR, VL and VC with the current common to all three components. The vector diagrams will therefore have the current vector as their reference with the three voltage vectors being plotted with respect to this reference. This means then that we cannot simply add together V_R, V_L and V_C to find the supply voltage, V_S across all three components as all three voltage vectors point in different directions with regards to the current vector. Therefore we will have to find the supply voltage, V_S as the **Phasor Sum** of the three component voltages combined together as vectors.

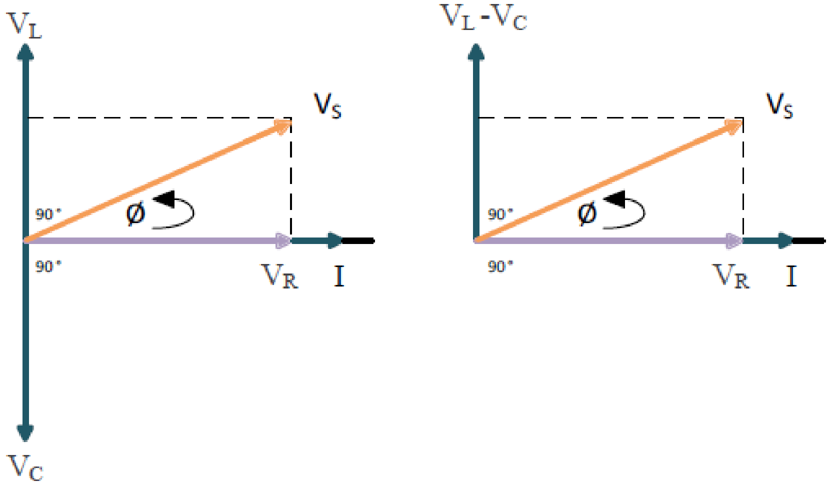

Figure 9.2 Phasor diagrams for series RLC circuits

Using the phasor diagram shown in Figure 9.2 (right), the voltage vectors produce a rectangular triangle, comprising of hypotenuse V_S, horizontal axis V_R and vertical axis $V_L - V_C$ (for $V_L > V_C$), and this is called the **Voltage Triangle**. To mathematically obtain the value of V_S, apply the Pythagoras's theorem on this voltage triangle as shown below

$$Vs^2 = VR^2 + (VL - VC)^2 \qquad Vs = \sqrt{VR^2 + (VL - VC)^2} \qquad (9.1)$$

The voltage across each component can also be described mathematically according to the current flowing through, and the voltage across each element as

$$V_R = I.R \sin(wt + 0^\circ) = I.R \qquad (9.2)$$

$$V_L = I.X_L \sin(wt + 90^\circ) = I.jwL \qquad (9.3)$$

$$V_C = I.X_C \sin(wt - 90^\circ) = I / jwC \qquad (9.4)$$

By substituting these values into Pythagoras's equation above for the voltage triangle will give us

$$V_R = I.R,\ V_L = I.X_L,\ V_C = I.X_C \qquad (9.5)$$

$$Vs = \sqrt{(I.R)^2 + (I.XL - I.XC)^2} \quad (9.6)$$

$$Vs = I.\sqrt{R^2 + (XL - XC)^2}$$

$$Vs = I.Z \text{ where } Z = \sqrt{R^2 + (XL - XC)^2}$$

The impedance Z of a series RLC circuit depends upon the angular frequency, ω as do X_L and X_C. If the capacitive reactance is greater than the inductive reactance, $X_C > X_L$ then the overall circuit reactance is capacitive giving a leading phase angle. Likewise, if the inductive reactance is greater than the capacitive reactance, $X_L > X_C$ then the overall circuit reactance is inductive giving the series circuit a lagging phase angle. If the two reactance's are the same and $X_L = X_C$ then the angular frequency at which this occurs is called the resonant frequency and produces the effect of resonance. The impedance triangle for series RLC circuit is given in Figure 9.3 and the impedance is expressed as

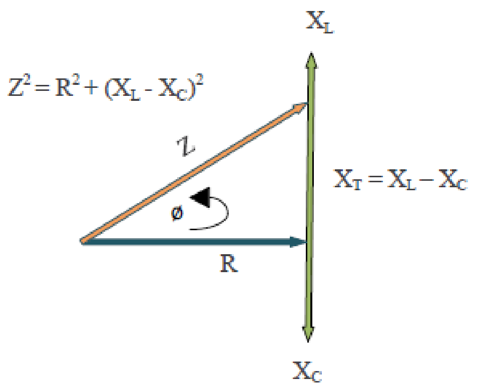

Figure 9.3 Impedance triangle for series RLC circuits

$$Z = \sqrt{R^2 + (wL - 1/wC)^2} \qquad (9.7)$$

The phase angle, Ø between the source voltage, V_S and the current, i is the same as for the angle between Z and R in the impedance triangle. This phase angle may be positive or negative in value depending on whether the source voltage leads or lags the circuit current and can be calculated mathematically from the ohmic values of the impedance triangle as

$$\cos Ø = R/Z \qquad (9.8)$$

2. Parallel RLC Circuit

In Figure 9.4, the supply voltage, V_S is common to all three components, while the supply current I_S consists of three parts: the current flowing through the resistor, I_R, the current flowing through the inductor, I_L and the current through the capacitor, I_C. The total current drawn from the supply will not be the mathematical sum of the three individual branch currents but their vector sum.

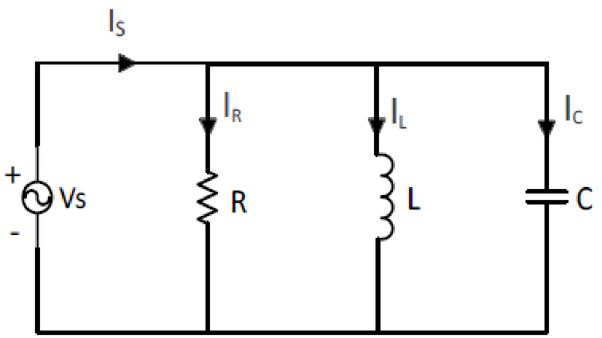

Figure 9.4 A parallel RLC circuit

The resulting vector I_S is obtained by adding together two of the vectors, I_L and I_C and then adding this sum to the remaining vector I_R. The resulting angle obtained between V and I_S will be the circuits phase angle as shown in Figure 9.5.

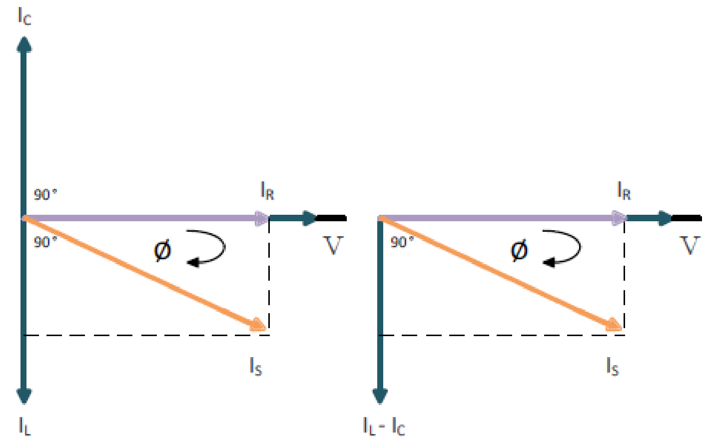

Figure 9.5 Phasor diagrams for parallel RLC circuits

Since the voltage across the circuit is common to all three circuit elements, the current through each branch can be found using Kirchoff's Current Law, (KCL). Kirchoff's current law or junction law states that "the total current entering a junction or node is exactly equal to the current leaving that node",

$$\text{Is}^2 = \text{IR}^2 + (\text{IL} - \text{IC})^2 \qquad (9.9)$$

$$\text{Is} = \sqrt{\text{IR}^2 + (\text{IL} - \text{IC})^2}$$

$$\text{Is} = \sqrt{(\text{V}/\text{R})^2 + (\text{V}/\text{XL} - \text{V}/\text{XC})^2}$$

where

$$\text{IR} = V/R,\ \text{IL} = \text{V}/\text{XL},\ \text{IC} = \text{V}/\text{XC}$$

$$\text{R} = V/IR \qquad \text{XL} = \text{V}/\text{IL} \qquad \text{XC} = \text{V}/\text{IC}$$

$$\text{Z} = 1/\sqrt{(1/\text{R})^2 + (1/\text{XL} - 1/\text{XC})^2}$$

$$\cos\text{Ø} = \text{R}/\text{Z}$$

Preliminary Work

1) For **a series RLC circuit**, component values are given as R = 1 kΩ, L = 1 mH and C = 1 μF. The frequency of the voltage supply is given as 160 Hz and the amplitude is given as 10 V peak-to-peak.

a) Calculate the impedance (Z).
b) Calculate $i_s(t)$.
c) Calculate voltage for each component.
d) Plot the phasor diagrams and calculate the phase angle between source voltage and current.

2) For **a parallel RLC circuit**, component values are given as R = 1 kΩ, L = 1 mH and C = 22 nF. The frequency of the voltage supply is given as 10 kHz and the amplitude is given as 10 V peak-to-peak.

a) Calculate the impedance (Z).
b) Calculate the currents for all components.
c) Plot the phasor diagrams and calculate the phase angle between source voltage and total current.

Procedure

1) For given circuit in Preliminary Work Q1,
    a) Measure the current using multimeter and compare your result with Preliminary Work Q1(b).
    b) Measure the voltage for all components using multimeter and compare your results with Preliminary Work Q1(c).
    c) Measure the phase difference between source voltage and current using oscilloscope and compare your result with Preliminary Work Q1(d).
    d) Change the frequency between 100 Hz - 200 kHz and observe the effect of frequency on the resistor voltage.
2) For given circuit in Preliminary Work Q2, measure the current for each component using multimeter and compare your results with Preliminary Work Q2(b).

List of Equipment and Components

Equipment: Function Generator, Oscilloscope, Digital Multimeter

Components: a resistor of $1\ \mathrm{k\Omega}$, an inductor of $1\ \mathrm{mH}$, capacitors of $1\ \mu$F and $22\ \mathrm{nF}$

BME 211 Report #9
RLC Circuits

Objective

Results

1. Comparision of calculated and measured values.

a)

$I_{calculated}$	
$I_{measured}$	

Comments:

b)

	V_R	V_C	V_L
Calculated			
Measured			

c)

	Phase Difference
Calculated	
Measured	

2. Comparision of calculated and measured values.

	I_R	I_C	I_L
Calculated			
Measured			

Comments: