

CEN 207 Physical Chemistry

Text book:

Atkins' Physical Chemistry, Peter Atkins, Julio de Paula, James Keeler, 11th Edition, Oxford University Press.

Reference books

- . Physical Chemistry, [Robert J. Silbey](#), Robert A. Alberty, [Moungi G. Bawendi](#)
- . Physical Chemistry, Ira N. Levine

B. The kinetic model

The Maxwell-Boltzmann distribution of speeds: an expression for the distribution of the kinetic energy;

$$f(v) = K e^{-\epsilon/kT}$$

where K is a constant of proportionality. The kinetic energy is

$$\epsilon = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2$$

$$f(v) = K e^{-(mv_x^2+mv_y^2+mv_z^2)/2kT} = K e^{-mv_x^2/2kT} e^{-mv_y^2/2kT} e^{-mv_z^2/2kT}$$

$$f(v_x) = K_x e^{-mv_x^2/2kT} \text{ (for } x \text{ coordinate)}$$

B. The kinetic model

Determine the constants K_x , K_y and K_z

To determine the constant K_x , note that a molecule must have a velocity component somewhere in the range $-\infty < v_x < \infty$, so integration over the full range of possible values of v_x must be a total probability of 1:

$$\int_{-\infty}^{\infty} f(v_x) dv_x = 1$$

For $f(v_x)$

$K_x = (m/2\pi kT)^{1/2}$ and

$$f(v_x) = \left(\frac{m}{2\pi kT}\right)^{1/2} e^{-mv_x^2/2kT}$$

The expressions for $f(v_y)$ and $f(v_z)$ are analogous.

B. The kinetic model

Write a preliminary expression for $f(v_x) f(v_y) f(v_z) dv_x dv_y dv_z$.

The probability that a molecule has a velocity in the range v_x to v_x+dv_x , v_y to v_y+dv_y , v_z to v_z+dv_z is

$$\begin{aligned} f(v_x)f(v_y)f(v_z)dv_x dv_y dv_z &= \left(\frac{m}{2\pi kT}\right)^{3/2} \overbrace{e^{-mv_x^2/2kT} e^{-mv_y^2/2kT} e^{-mv_z^2/2kT}}^{e^{-m(v_x^2+v_y^2+v_z^2)/2kT}} * dv_x dv_y dv_z \\ &= \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-m v^2/2kT} dv_x dv_y dv_z \end{aligned}$$

where $v^2 = v_x^2 + v_y^2 + v_z^2$

B. The kinetic model

Calculate the probability that a molecule has a speed in the range **v to v+dv**

- . think three velocity components
- . three coordinate velocity space
- . think the volume of a spherical shell of radius r and thickness dr. That volume is $4\pi r^2 dr$. For velocity space (analogous to that volume) $4\pi v^2 dv$.

If probability is written for $f(v)dv$

$$f(v)dv = 4\pi v^2 dv \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-mv^2/2kT}$$

B. The kinetic model

and $f(v)$ itself, after minor rearrangements , is

$$f(v) = 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-mv^2/2kT}$$

Because $R=N_A k$, $m/k = mN_A/R=M/R$

$$f(v) = 4\pi \left(\frac{M}{2\pi RT} \right)^{3/2} v^2 e^{-mv^2/2kT}$$

Maxwell-Boltzmann distribution (KMT)