Principles of Chemical Equilibrium


## The Concept of Equilibrium and the Equilibrium Constant

Few chemical reactions proceed in only one direction. Most are reversible, at least to some extent. At the start of a reversible process, the reaction proceeds toward the formation of products. As soon as some product molecules are formed, the reverse process begins to take place and reactant molecules are formed from product molecules.

Chemical equilibrium is achieved when the rates of the forward and reverse reactions are equal and the concentrations of the reactants and products remain constant.

$$
\begin{gathered}
\mathrm{CO}(\mathrm{~g})+2 \mathrm{H}_{2}(\mathrm{~g}) \xrightarrow{k_{1}} \mathrm{CH}_{3} \mathrm{OH}(\mathrm{~g}) \quad R_{\mathrm{fwrd}}=k_{1}[\mathrm{CO}]\left[\mathrm{H}_{2}\right]^{2} \\
\mathrm{CH}_{3} \mathrm{OH}(\mathrm{~g}) \xrightarrow{k_{1}} \mathrm{CO}(\mathrm{~g})+2 \mathrm{H}_{2}(\mathrm{~g}) \quad R_{\mathrm{rvrs}}=k_{-1}\left[\mathrm{CH}_{3} \mathrm{OH}\right] \\
\mathrm{CO}(\mathrm{~g})+2 \mathrm{H}_{2}(\mathrm{~g}) \stackrel{\substack{k_{1} \\
k_{-1}}}{\mathrm{CH}_{3} \mathrm{OH}(\mathrm{~g})} \\
R_{\mathrm{fwrd}}=R_{\mathrm{rvrs}}
\end{gathered}
$$

The study of physical equilibrium yields useful information, such as the equilibrium vapor pressure (see Section 11.8). However, chemists are particularly interested in chemical equilibrium processes, such as the reversible reaction involving nitrogen dioxide $\left(\mathrm{NO}_{2}\right)$ and dinitrogen tetroxide $\left(\mathrm{N}_{2} \mathrm{O}_{4}\right)$ (Figure 14.1). The progress of the reaction

$$
\mathrm{N}_{2} \mathrm{O}_{4}(g) \rightleftharpoons 2 \mathrm{NO}_{2}(g)
$$

can be monitored easily because $\mathrm{N}_{2} \mathrm{O}_{4}$ is a colorless gas, whereas $\mathrm{NO}_{2}$ has a darkbrown color that makes it sometimes visible in polluted air. Suppose that $\mathrm{N}_{2} \mathrm{O}_{4}$ is injected into an evacuated flask. Some brown color appears immediately, indicating the formation of $\mathrm{NO}_{2}$ molecules. The color intensifies as the dissociation of $\mathrm{N}_{2} \mathrm{O}_{4}$ continues until eventually equilibrium is reached. Beyond that point, no further change in color is evident because the concentrations of both $\mathrm{N}_{2} \mathrm{O}_{4}$ and $\mathrm{NO}_{2}$ remain constant. We can also bring about an equilibrium state by starting with pure $\mathrm{NO}_{2}$. As some of the $\mathrm{NO}_{2}$ molecules combine to form $\mathrm{N}_{2} \mathrm{O}_{4}$, the color fades. Yet another way to create an equilibrium state is to start with a mixture of $\mathrm{NO}_{2}$ and $\mathrm{N}_{2} \mathrm{O}_{4}$ and monitor the system until the color stops changing.


These studies demonstrate that the preceding reaction is indeed reversible, because a pure component $\left(\mathrm{N}_{2} \mathrm{O}_{4}\right.$ or $\left.\mathrm{NO}_{2}\right)$ reacts to give the other gas. The important thing to keep in mind is that at equilibrium, the conversions of $\mathrm{N}_{2} \mathrm{O}_{4}$ to $\mathrm{NO}_{2}$ and of $\mathrm{NO}_{2}$ to $\mathrm{N}_{2} \mathrm{O}_{4}$ are still going on. We do not see a color change because the two rates are equal-the removal of $\mathrm{NO}_{2}$ molecules takes place as fast as the production of $\mathrm{NO}_{2}$ molecules, and $\mathrm{N}_{2} \mathrm{O}_{4}$ molecules are formed as quickly as they dissociate. Figure 14.2 summarizes these three situations.


Figure 14.2 Change in the concentrations of $\mathrm{NO}_{2}$ and $\mathrm{N}_{2} \mathrm{O}_{4}$ with time, in three situations. (a) Initially only $\mathrm{NO}_{2}$ is present. (b) Initially only $\mathrm{N}_{2} \mathrm{O}_{4}$ is present. (c) Initially a mixture of $\mathrm{NO}_{2}$ and $\mathrm{N}_{2} \mathrm{O}_{4}$ is present. In each case, equilibrium is established to the right of the vertical line.

## The Equilibrium Constant

Table 14.1 shows some experimental data for the reaction just described at $25^{\circ} \mathrm{C}$. The gas concentrations are expressed in molarity, which can be calculated from the number of moles of gases present initially and at equilibrium and the volume of the flask in liters. Note that the equilibrium concentrations of $\mathrm{NO}_{2}$ and $\mathrm{N}_{2} \mathrm{O}_{4}$ vary, depending on the starting concentrations. We can look for relationships between $\left[\mathrm{NO}_{2}\right]$ and [ $\mathrm{N}_{2} \mathrm{O}_{4}$ ] present at equilibrium by comparing the ratio of their concentrations. The simplest ratio, that is, $\left[\mathrm{NO}_{2}\right] /\left[\mathrm{N}_{2} \mathrm{O}_{4}\right]$, gives scattered values. But if we examine other possible mathematical relationships, we find that the ratio $\left[\mathrm{NO}_{2}\right]^{2} /\left[\mathrm{N}_{2} \mathrm{O}_{4}\right]$ at equilibrium gives a nearly constant value that averages $4.63 \times 10^{-3}$, regardless of the initial concentrations present:

$$
\begin{equation*}
K=\frac{\left[\mathrm{NO}_{2}\right]^{2}}{\left[\mathrm{~N}_{2} \mathrm{O}_{4}\right]}=4.63 \times 10^{-3} \tag{14.1}
\end{equation*}
$$

where $K$ is a constant. Note that the exponent 2 for $\left[\mathrm{NO}_{2}\right]$ in this expression is the same as the stoichiometric coefficient for $\mathrm{NO}_{2}$ in the reversible reaction.

| TABLE | The NO | , Syst | ${ }^{\text {t }} 2{ }^{\circ} \mathrm{C}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Initial Concentrations (M) |  | Equilibrium Concentrations (M) |  | Ratio of Concentrations at Equilibrium |  |
| $\left[\mathrm{NO}_{2}\right]$ | [ $\mathrm{N}_{2} \mathrm{O}_{4}$ ] | $\left[\mathrm{NO}_{2}\right]$ | [ $\mathrm{N}_{2} \mathrm{O}_{4}$ ] | $\frac{\left[\mathrm{NO}_{2}\right]}{\left[\mathrm{N}_{2} \mathrm{O}_{4}\right]}$ | $\frac{\left[\mathrm{NO}_{2}\right]^{2}}{\left[\mathrm{~N}_{2} \mathrm{O}_{4}\right]}$ |
| 0.000 | 0.670 | 0.0547 | 0.643 | 0.0851 | $4.65 \times 10^{-3}$ |
| 0.0500 | 0.446 | 0.0457 | 0.448 | 0.102 | $4.66 \times 10^{-3}$ |
| 0.0300 | 0.500 | 0.0475 | 0.491 | 0.0967 | $4.60 \times 10^{-3}$ |
| 0.0400 | 0.600 | 0.0523 | 0.594 | 0.0880 | $4.60 \times 10^{-3}$ |
| 0.200 | 0.000 | 0.0204 | 0.0898 | 0.227 | $4.63 \times 10^{-3}$ |

We can generalize this phenomenon with the following reaction at equilibrium:

$$
a \mathrm{~A}+b \mathrm{~B} \rightleftharpoons c \mathrm{C}+d \mathrm{D}
$$

where $a, b, c$, and $d$ are the stoichiometric coefficients for the reacting species $\mathrm{A}, \mathrm{B}$, C, and D. For the reaction at a particular temperature

$$
\begin{equation*}
K=\frac{[\mathrm{C}]^{c}[\mathrm{D}]^{d}}{[\mathrm{~A}]^{a}[\mathrm{~B}]^{b}} \tag{14.2}
\end{equation*}
$$

where $K$ is the equilibrium constant. Equation (14.2) was formulated by two Norwegian chemists, Cato Guldberg ${ }^{\dagger}$ and Peter Waage, ${ }^{\ddagger}$ in 1864. It is the mathematical expression of their law of mass action, which holds that for a reversible reaction at equilibrium and a constant temperature, a certain ratio of reactant and product concentrations has a constant value, $K$ (the equilibrium constant). Note that although the concentrations may vary, as long as a given reaction is at equilibrium and the temperature does not change, according to the law of mass action, the value of $K$ remains constant. The validity of Equation (14.2) and the law of mass action has been established by studying many reversible reactions.

The equilibrium constant, then, is defined by a quotient, the numerator of which is obtained by multiplying together the equilibrium concentrations of the products, each raised to a power equal to its stoichiometric coefficient in the balanced equation. Applying the same procedure to the equilibrium concentrations of reactants gives the denominator. The magnitude of the equilibrium constant tells us whether an equilibrium reaction favors the products or reactants. If $K$ is much greater than 1 (that is, $K \gg 1$ ), the equilibrium will lie to the right and favors the products. Conversely, if the equilibrium constant is much smaller than 1 (that is, $K \ll 1$ ), the equilibrium will lie to the left and favor the reactants (Figure 14.3). In this context, any number greater than 10 is considered to be much greater than 1 , and any number less than 0.1 is much less than 1 .


## Writing Equilibrium Constant Expressions

The concept of equilibrium constants is extremely important in chemistry. As you will soon see, equilibrium constants are the key to solving a wide variety of stoichiometry problems involving equilibrium systems.

To use equilibrium constants, we must express them in terms of the reactant and product concentrations. Our only guide is the law of mass action [Equation (14.2)], which is the general formula for finding equilibrium concentrations. However, because the concentrations of the reactants and products can be expressed in different units and because the reacting species are not always in the same phase, there may be more than one way to express the equilibrium constant for the same reaction.

## Homogeneous Equilibria

The term homogeneous equilibrium applies to reactions in which all reacting species are in the same phase. An example of homogeneous gas-phase equilibrium is the dissociation of $\mathrm{N}_{2} \mathrm{O}_{4}$. The equilibrium constant, as given in Equation (14.1), is

$$
\begin{gathered}
\mathrm{N}_{2} \mathrm{O}_{4}(g) \rightleftharpoons 2 \mathrm{NO}_{2}(g) \\
K_{\mathrm{c}}=\frac{\left[\mathrm{NO}_{2}\right]^{2}}{\left[\mathrm{~N}_{2} \mathrm{O}_{4}\right]}
\end{gathered}
$$

Note that the subscript in Kc indicates that the concentrations of the reacting species are expressed in molarity or moles per liter. The concentrations of reactants and products in gaseous reactions can also be expressed in terms of their partial pressures.
we see that at constant temperature, the pressure $P$ of a gas is directly related to the concentration in $\mathrm{mol} / \mathrm{L}$ of the gas; that is, $P=(n / V) R T$. Thus, for the equilibrium process

$$
\mathrm{N}_{2} \mathrm{O}_{4}(g) \rightleftharpoons 2 \mathrm{NO}_{2}(g)
$$

we can write

$$
\begin{equation*}
K_{P}=\frac{P_{\mathrm{NO}_{2}}^{2}}{P_{\mathrm{N}_{2} \mathrm{O}_{4}}} \tag{14.3}
\end{equation*}
$$

where $P_{\mathrm{NO}_{2}}$ and $P_{\mathrm{N}_{2} \mathrm{O}_{4}}$ are the equilibrium partial pressures (in atm) of $\mathrm{NO}_{2}$ and $\mathrm{N}_{2} \mathrm{O}_{4}$, respectively. The subscript in $K_{P}$ tells us that equilibrium concentrations are expressed in terms of pressure.

In general, $K_{\mathrm{c}}$ is not equal to $K_{P}$, because the partial pressures of reactants and products are not equal to their concentrations expressed in moles per liter. A simple relationship between $K_{P}$ and $K_{\mathrm{c}}$ can be derived as follows. Let us consider the following equilibrium in the gas phase:

$$
a \mathrm{~A}(g) \rightleftharpoons b \mathrm{~B}(g)
$$

where $a$ and $b$ are stoichiometric coefficients. The equilibrium constant $K_{\mathrm{c}}$ is given by

$$
K_{\mathrm{c}}=\frac{[\mathrm{B}]^{b}}{[\mathrm{~A}]^{a}}
$$

and the expression for $K_{P}$ is $\quad K_{P}=\frac{P_{\mathrm{B}}^{b}}{P_{\mathrm{A}}^{a}}$
where $P_{\mathrm{A}}$ and $P_{\mathrm{B}}$ are the partial pressures of A and B. Assuming ideal gas behavior,

$$
\begin{aligned}
P_{\mathrm{A}} V & =n_{\mathrm{A}} R T \\
P_{\mathrm{A}} & =\frac{n_{\mathrm{A}} R T}{V}
\end{aligned}
$$

where $V$ is the volume of the container in liters. Also

$$
\begin{aligned}
P_{\mathrm{B}} V & =n_{\mathrm{B}} R T \\
P_{\mathrm{B}} & =\frac{n_{\mathrm{B}} R I}{V}
\end{aligned}
$$

Substituting these relations into the expression for $K_{P}$, we obtain

$$
K_{P}=\frac{\left(\frac{n_{\mathrm{B}} R T}{V}\right)^{b}}{\left(\frac{n_{\mathrm{A}} R T}{V}\right)^{a}}=\frac{\left(\frac{n_{\mathrm{B}}}{V}\right)^{b}}{\left(\frac{n_{\mathrm{A}}}{V}\right)^{a}}(R T)^{b-a}
$$

Now both $n_{\mathrm{A}} / V$ and $n_{\mathrm{B}} / V$ have units of $\mathrm{mol} / \mathrm{L}$ and can be replaced by $[\mathrm{A}]$ and $[\mathrm{B}]$, so that

$$
\begin{align*}
K_{P} & =\frac{[\mathrm{B}]^{b}}{[\mathrm{~A}]^{a}}(R T)^{\Delta n} \\
& =K_{\mathrm{c}}(R T)^{\Delta n} \tag{14.4}
\end{align*}
$$

where

$$
\begin{aligned}
\Delta n & =b-a \\
& =\text { moles of gaseous products }- \text { moles of gaseous reactants }
\end{aligned}
$$

Because pressures are usually expressed in atm, the gas constant $R$ is given by 0.0821
$\mathrm{L} \cdot \mathrm{atm} / \mathrm{K} \cdot \mathrm{mol}$, and we can write the relationship between $K_{P}$ and $K_{\mathrm{c}}$ as

$$
\begin{equation*}
K_{P}=K_{\mathrm{c}}(0.0821 T)^{\Delta n} \tag{14.5}
\end{equation*}
$$

## EXAMPLE 14.1

Write expressions for $K_{\mathrm{c}}$, and $K_{P}$ if applicable, for the following reversible reactions at equilibrium:
(a) $\mathrm{HF}(a q)+\mathrm{H}_{2} \mathrm{O}(l) \rightleftharpoons \mathrm{H}_{3} \mathrm{O}^{+}(a q)+\mathrm{F}^{-}(a q)$
(b) $2 \mathrm{NO}(g)+\mathrm{O}_{2}(g) \rightleftharpoons 2 \mathrm{NO}_{2}(g)$
(c) $\mathrm{CH}_{3} \mathrm{COOH}(a q)+\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{OH}(a q) \rightleftharpoons \mathrm{CH}_{3} \mathrm{COOC}_{2} \mathrm{H}_{5}(a q)+\mathrm{H}_{2} \mathrm{O}(l)$

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(c) $\mathrm{CH}_{3} \mathrm{COOH}(a q)+\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{OH}(a q) \rightleftharpoons \mathrm{CH}_{3} \mathrm{COOC}_{2} \mathrm{H}_{5}(a q)+\mathrm{H}_{2} \mathrm{O}(l)$

Strategy Keep in mind the following facts: (1) the $K_{P}$ expression applies only to gaseous reactions and (2) the concentration of solvent (usually water) does not appear in the equilibrium constant expression.

Solution (a) Because there are no gases present, $K_{P}$ does not apply and we have only $K_{\mathrm{c}}$.

$$
K_{\mathrm{c}}^{\prime}=\frac{\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]\left[\mathrm{F}^{-}\right]}{[\mathrm{HF}]\left[\mathrm{H}_{2} \mathrm{O}\right]}
$$

HF is a weak acid, so that the amount of water consumed in acid ionizations is negligible compared with the total amount of water present as solvent. Thus, we can rewrite the equilibrium constant as
(b)

$$
\begin{gathered}
K_{\mathrm{c}}=\frac{\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]\left[\mathrm{F}^{-}\right]}{[\mathrm{HF}]} \\
K_{\mathrm{c}}=\frac{\left[\mathrm{NO}_{2}\right]^{2}}{\left[\mathrm{NO}^{2}\left[\mathrm{O}_{2}\right]\right.} \quad K_{P}=\frac{P_{\mathrm{NO}_{2}}^{2}}{P_{\mathrm{NO}}^{2} P_{\mathrm{O}_{2}}}
\end{gathered}
$$

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$$

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(b)

$$
\begin{gathered}
K_{\mathrm{c}}=\frac{\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]\left[\mathrm{F}^{-}\right]}{[\mathrm{HF}]} \\
K_{\mathrm{c}}=\frac{\left[\mathrm{NO}_{2}\right]^{2}}{[\mathrm{NO}]^{2}\left[\mathrm{O}_{2}\right]} \quad K_{P}=\frac{P_{\mathrm{NO}_{2}}^{2}}{P_{\mathrm{NO}}^{2} P_{\mathrm{O}_{2}}}
\end{gathered}
$$

(c) The equilibrium constant $K_{\mathrm{c}}^{\prime}$ is given by

$$
K_{\mathrm{c}}^{\prime}=\frac{\left[\mathrm{CH}_{3} \mathrm{COOC}_{2} \mathrm{H}_{5}\right]\left[\mathrm{H}_{2} \mathrm{O}\right]}{\left[\mathrm{CH}_{3} \mathrm{COOH}\right]\left[\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{OH}\right]}
$$

Because the water produced in the reaction is negligible compared with the water solvent, the concentration of water does not change. Thus, we can write the new equilibrium constant as

$$
K_{\mathrm{c}}=\frac{\left[\mathrm{CH}_{3} \mathrm{COOC}_{2} \mathrm{H}_{5}\right]}{\left[\mathrm{CH}_{3} \mathrm{COOH}\right]\left[\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{OH}\right]}
$$

## EXAMPLE 14.2

The following equilibrium process has been studied at $230^{\circ} \mathrm{C}$ :

$$
2 \mathrm{NO}(g)+\mathrm{O}_{2}(g) \rightleftharpoons 2 \mathrm{NO}_{2}(g)
$$

In one experiment, the concentrations of the reacting species at equilibrium are found to be $[\mathrm{NO}]=0.0542 \mathrm{M},\left[\mathrm{O}_{2}\right]=0.127 \mathrm{M}$, and $\left[\mathrm{NO}_{2}\right]=15.5 \mathrm{M}$. Calculate the equilibrium constant $\left(K_{\mathrm{c}}\right)$ of the reaction at this temperature.

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Strategy The concentrations given are equilibrium concentrations. They have units of $\mathrm{mol} / \mathrm{L}$, so we can calculate the equilibrium constant $\left(K_{\mathrm{c}}\right)$ using the law of mass action [Equation (14.2)].

Solution The equilibrium constant is given by

$$
K_{\mathrm{c}}=\frac{\left[\mathrm{NO}_{2}\right]^{2}}{[\mathrm{NO}]^{2}\left[\mathrm{O}_{2}\right]}
$$

Substituting the concentrations, we find that

$$
K_{\mathrm{c}}=\frac{(15.5)^{2}}{(0.0542)^{2}(0.127)}=6.44 \times 10^{5}
$$

Dinitrogen tetroxide, $\mathrm{N}_{2} \mathrm{O}_{4}(\mathrm{l})$, is an important component of rocket fuels-for example, as an oxidizer of liquid hydrazine in the Titan rocket. At $25^{\circ} \mathrm{C}, \mathrm{N}_{2} \mathrm{O}_{4}$ is a colorless gas that partially dissociates into $\mathrm{NO}_{2}$, a red-brown gas. The color of an equilibrium mixture of these two gases depends on their relative proportions, which in turn depends on the temperature (Fig. 15-7).

(a)

A FIGURE 15-7
The equilibrium $\mathrm{N}_{2} \mathrm{O}_{4}(\mathrm{~g}) \rightleftharpoons 2 \mathrm{NO}_{2}(\mathrm{~g})$
(a) At dry ice temperatures, $\mathrm{N}_{2} \mathrm{O}_{4}$ exists as a solid.

The gas in equilibrium with the solid is mostly colorless $\mathrm{N}_{2} \mathrm{O}_{4}$, with only a trace of brown $\mathrm{NO}_{2}$. (b) When warmed to room temperature and above, the $\mathrm{N}_{2} \mathrm{O}_{4}$ melts and vaporizes. The proportion of $\mathrm{NO}_{2}(\mathrm{~g})$ at equilibrium increases over that at low temperatures, and the equilibrium mixture of $\mathrm{N}_{2} \mathrm{O}_{4}(\mathrm{~g})$ and $\mathrm{NO}_{2}(\mathrm{~g})$ has a red-brown color.

Equilibrium is established in the reaction $\mathrm{N}_{2} \mathrm{O}_{4}(\mathrm{~g}) \rightleftharpoons 2 \mathrm{NO}_{2}(\mathrm{~g})$ at $25^{\circ} \mathrm{C}$. The quantities of the two gases present in a 3.00 L vessel are $7.64 \mathrm{~g} \mathrm{~N}_{2} \mathrm{O}_{4}$ and $1.56 \mathrm{~g} \mathrm{NO}_{2}$. What is the value of $K_{c}$ for this reaction?

$$
\left(\mathrm{N}_{2} \mathrm{O}_{4}=92 \mathrm{~g} / \mathrm{mol} \quad \mathrm{NO}_{2}=46 \mathrm{~g} / \mathrm{mol}\right)
$$

## Analyze

We are given the equilibrium amounts (in terms of mass) of the reactants and products, along with the volume of the reaction vessel. We use these values to determine the equilibrium concentrations and plug them into the equilibrium constant expression.

## Solve

Convert the mass of $\mathrm{N}_{2} \mathrm{O}_{4}$ to moles.

Convert moles of $\mathrm{N}_{2} \mathrm{O}_{4}$ to $\mathrm{mol} / \mathrm{L}$.

Convert the mass of $\mathrm{NO}_{2}$ to moles.

Convert moles of $\mathrm{NO}_{2}$ to $\mathrm{mol} / \mathrm{L}$.
Write the equilibrium constant expression, substitute the equilibrium concentrations, and solve for $K_{c}$.

$$
\begin{aligned}
& \mathrm{mol} \mathrm{~N}_{2} \mathrm{O}_{4}=7.64 \mathrm{~g} \mathrm{~N}_{2} \mathrm{O}_{4} \times \frac{1 \mathrm{~mol} \mathrm{~N}_{2} \mathrm{O}_{4}}{92.01 \mathrm{~g} \mathrm{~N}_{2} \mathrm{O}_{4}}=8.303 \times 10^{-2} \mathrm{~mol} \\
& {\left[\mathrm{~N}_{2} \mathrm{O}_{4}\right]=\frac{8.303 \times 10^{2} \mathrm{~mol}}{3.00 \mathrm{~L}}=0.0277 \mathrm{M}} \\
& \mathrm{~mol} \mathrm{NO}_{2}=1.56 \mathrm{~g} \mathrm{NO}_{2} \times \frac{1 \mathrm{~mol} \mathrm{NO}_{2}}{46.01 \mathrm{~g} \mathrm{NO}_{2}}=3.391 \times 10^{-2} \mathrm{~mol} \\
& {\left[\mathrm{NO}_{2}\right]=\frac{3.391 \times 10^{-2}}{3.00 \mathrm{~L}}=0.0113 \mathrm{M}} \\
& K_{c}=\frac{\left[\mathrm{NO}_{2}\right]^{2}}{\left[\mathrm{~N}_{2} \mathrm{O}_{4}\right]}=\frac{(0.0113)^{2}}{(0.0277)}=4.61 \times 10^{-3}
\end{aligned}
$$

Practice Exercise Carbonyl chloride $\left(\mathrm{COCl}_{2}\right)$, also called phosgene, was used in World War I as a poisonous gas. The equilibrium concentrations for the reaction between carbon monoxide and molecular chlorine to form carbonyl chloride

$$
\mathrm{CO}(g)+\mathrm{Cl}_{2}(g) \rightleftharpoons \mathrm{COCl}_{2}(g)
$$

at $74^{\circ} \mathrm{C}$ are $[\mathrm{CO}]=1.2 \times 10^{-2} \mathrm{M},\left[\mathrm{Cl}_{2}\right]=0.054 \mathrm{M}$, and $\left[\mathrm{COCl}_{2}\right]=0.14 \mathrm{M}$. Calculate the equilibrium constant ( $K_{\mathrm{c}}$ ).

## EXAMPLE 14.3

The equilibrium constant $K_{P}$ for the decomposition of phosphorus pentachloride to phosphorus trichloride and molecular chlorine

$$
\mathrm{PCl}_{5}(g) \rightleftharpoons \mathrm{PCl}_{3}(g)+\mathrm{Cl}_{2}(g)
$$

is found to be 1.05 at $250^{\circ} \mathrm{C}$. If the equilibrium partial pressures of $\mathrm{PCl}_{5}$ and $\mathrm{PCl}_{3}$ are 0.875 atm and 0.463 atm , respectively, what is the equilibrium partial pressure of $\mathrm{Cl}_{2}$ at $250^{\circ} \mathrm{C}$ ?


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Strategy The concentrations of the reacting gases are given in atm, so we can express the equilibrium constant in $K_{P}$. From the known $K_{P}$ value and the equilibrium pressures of $\mathrm{PCl}_{3}$ and $\mathrm{PCl}_{5}$, we can solve for $P_{\mathrm{Cl}_{2}}$.

Solution First, we write $K_{P}$ in terms of the partial pressures of the reacting species

$$
K_{P}=\frac{P_{\mathrm{PCl}_{3}} P_{\mathrm{Cl}_{2}}}{P_{\mathrm{PCl}_{5}}}
$$

Knowing the partial pressures, we write
or

$$
\begin{gathered}
1.05=\frac{(0.463)\left(P_{\mathrm{Cl}_{2}}\right)}{(0.875)} \\
P_{\mathrm{Cl}_{2}}=\frac{(1.05)(0.875)}{(0.463)}=1.98 \mathrm{~atm}
\end{gathered}
$$

## EXAMPLE 14.4

Methanol $\left(\mathrm{CH}_{3} \mathrm{OH}\right)$ is manufactured industrially by the reaction

$$
\mathrm{CO}(\mathrm{~g})+2 \mathrm{H}_{2}(\mathrm{~g}) \rightleftharpoons \mathrm{CH}_{3} \mathrm{OH}(\mathrm{~g})
$$

The equilibrium constant ( $K_{\mathrm{c}}$ ) for the reaction is 10.5 at $220^{\circ} \mathrm{C}$. What is the value of $K_{P}$ at this temperature?

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$$

The equilibrium constant ( $K_{\mathrm{c}}$ ) for the reaction is 10.5 at $220^{\circ} \mathrm{C}$. What is the value of $K_{P}$ at this temperature?
Strategy The relationship between $K_{\mathrm{c}}$ and $K_{P}$ is given by Equation (14.5). What is the change in the number of moles of gases from reactants to product? Recall that

$$
\Delta n=\text { moles of gaseous products }- \text { moles of gaseous reactants }
$$

What unit of temperature should we use?
Solution The relationship between $K_{\mathrm{c}}$ and $K_{P}$ is

$$
K_{P}=K_{\mathrm{c}}(0.0821 T)^{\Delta n}
$$

Because $T=273+220=493 \mathrm{~K}$ and $\Delta n=1-3=-2$, we have

$$
\begin{aligned}
K_{P} & =(10.5)(0.0821 \times 493)^{-2} \\
& =6.41 \times 10^{-3}
\end{aligned}
$$

## Heterogeneous Equilibria

As you might expect, a heterogeneous equilibrium results from a reversible reaction involving reactants and products that are in different phases. For example, when calcium carbonate is heated in a closed vessel, the following equilibrium is attained:

$$
\mathrm{CaCO}_{3}(s) \rightleftharpoons \mathrm{CaO}(s)+\mathrm{CO}_{2}(g)
$$

The two solids and one gas constitute three separate phases. At equilibrium, we might write the equilibrium constant as

$$
K_{\mathrm{c}}^{\prime}=\frac{[\mathrm{CaO}]\left[\mathrm{CO}_{2}\right]}{\left[\mathrm{CaCO}_{3}\right]}
$$

The "concentration" of a solid, like its density, is an intensive property and does not depend on how much of the substance is present.


For this reason, the terms $\left[\mathrm{CaCO}_{3}\right]$ and $[\mathrm{CaO}]$ are themselves constants and can be combined with the equilibrium constant. We can simplify Equation (14.6) by writing

$$
\begin{equation*}
\frac{\left[\mathrm{CaCO}_{3}\right]}{[\mathrm{CaO}]} K_{\mathrm{c}}^{\prime}=K_{\mathrm{c}}=\left[\mathrm{CO}_{2}\right] \tag{14.7}
\end{equation*}
$$

where $K_{\mathrm{c}}$, the "new" equilibrium constant, is conveniently expressed in terms of a single concentration, that of $\mathrm{CO}_{2}$. Note that the value of $K_{\mathrm{c}}$ does not depend on how much $\mathrm{CaCO}_{3}$ and CaO are present, as long as some of each is present at equilibrium

Alternatively, we can express the equilibrium constant as

$$
\begin{equation*}
K_{P}=P_{\mathrm{CO}_{2}} \tag{14.8}
\end{equation*}
$$

The equilibrium constant in this case is numerically equal to the pressure of $\mathrm{CO}_{2}$ gas, an easily measurable quantity.

## EXAMPLE 14.5

Write the equilibrium constant expression $K_{\mathrm{c}}$, and $K_{P}$ if applicable, for each of the following heterogeneous systems:
(a) $\left(\mathrm{NH}_{4}\right)_{2} \mathrm{Se}(\mathrm{s}) \rightleftharpoons 2 \mathrm{NH}_{3}(g)+\mathrm{H}_{2} \mathrm{Se}(\mathrm{g})$
(b) $\mathrm{AgCl}(s) \rightleftharpoons \mathrm{Ag}^{+}(a q)+\mathrm{Cl}^{-}(a q)$
(c) $\mathrm{P}_{4}(s)+6 \mathrm{Cl}_{2}(g) \rightleftharpoons 4 \mathrm{PCl}_{3}(l)$

## EXAMPLE 14.5

Write the equilibrium constant expression $K_{\mathrm{c}}$, and $K_{P}$ if applicable, for each of the following heterogeneous systems:
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(b) $\mathrm{AgCl}(s) \rightleftharpoons \mathrm{Ag}^{+}(a q)+\mathrm{Cl}^{-}(a q)$
(c) $\mathrm{P}_{4}(s)+6 \mathrm{Cl}_{2}(g) \rightleftharpoons 4 \mathrm{PCl}_{3}(l)$

Strategy We omit any pure solids or pure liquids in the equilibrium constant expression because their activities are unity.
Solution (a) Because $\left(\mathrm{NH}_{4}\right)_{2} \mathrm{Se}$ is a solid, the equilibrium constant $K_{\mathrm{c}}$ is given by

$$
K_{\mathrm{c}}=\left[\mathrm{NH}_{3}\right]^{2}\left[\mathrm{H}_{2} \mathrm{Se}\right]
$$

Alternatively, we can express the equilibrium constant $K_{P}$ in terms of the partial pressures of $\mathrm{NH}_{3}$ and $\mathrm{H}_{2} \mathrm{Se}$ :

$$
K_{P}=P_{\mathrm{NH}_{3}}^{2} P_{\mathrm{H}_{2} \mathrm{Se}}
$$

(b) Here AgCl is a solid so the equilibrium constant is given by

$$
K_{\mathrm{c}}=\left[\mathrm{Ag}^{+}\right]\left[\mathrm{Cl}^{-}\right]
$$

Because no gases are present, there is no $K_{P}$ expression.
(c) We note that $\mathrm{P}_{4}$ is a solid and $\mathrm{PCl}_{3}$ is a liquid, so they do not appear in the equilibrium constant expression. Thus, $K_{\mathrm{c}}$ is given by

$$
K_{\mathrm{c}}=\frac{1}{\left[\mathrm{Cl}_{2}\right]^{6}} \quad K_{P}=\frac{1}{P_{\mathrm{Cl}_{2}}^{6}}
$$

## EXAMPLE 14.6

Consider the following heterogeneous equilibrium:

$$
\mathrm{CaCO}_{3}(s) \rightleftharpoons \mathrm{CaO}(s)+\mathrm{CO}_{2}(g)
$$

At $800^{\circ} \mathrm{C}$, the pressure of $\mathrm{CO}_{2}$ is 0.236 atm . Calculate (a) $K_{P}$ and (b) $K_{\mathrm{c}}$ for the reaction at this temperature.

## EXAMPLE 14.6

Consider the following heterogeneous equilibrium:

$$
\mathrm{CaCO}_{3}(s) \rightleftharpoons \mathrm{CaO}(s)+\mathrm{CO}_{2}(g)
$$

At $800^{\circ} \mathrm{C}$, the pressure of $\mathrm{CO}_{2}$ is 0.236 atm . Calculate (a) $K_{P}$ and (b) $K_{\mathrm{c}}$ for the reaction at this temperature.

Strategy Remember that pure solids do not appear in the equilibrium constant expression. The relationship between $K_{P}$ and $K_{\mathrm{c}}$ is given by Equation (14.5).

Solution (a) Using Equation (14.8) we write

$$
\begin{aligned}
K_{P} & =P_{\mathrm{CO}_{2}} \\
& =0.236
\end{aligned}
$$

(b) From Equation (14.5), we know

$$
K_{P}=K_{\mathrm{c}}(0.0821 T)^{\Delta n}
$$

In this case, $T=800+273=1073 \mathrm{~K}$ and $\Delta n=1$, so we substitute these values in the equation and obtain

$$
\begin{aligned}
0.236 & =K_{\mathrm{c}}(0.0821 \times 1073) \\
K_{\mathrm{c}} & =2.68 \times 10^{-3}
\end{aligned}
$$

Practice Exercise Consider the following equilibrium at 395 K :

$$
\mathrm{NH}_{4} \mathrm{HS}(s) \rightleftharpoons \mathrm{NH}_{3}(g)+\mathrm{H}_{2} \mathrm{~S}(g)
$$

The partial pressure of each gas is 0.265 atm . Calculate $K_{P}$ and $K_{\mathrm{c}}$ for the reaction.

## Multiple Equilibria

The reactions we have considered so far are all relatively simple. A more complicated situation is one in which the product molecules in one equilibrium system are involved in a second equilibrium process:

The products formed in the first reaction, C and D , react further to form products E and F . At equilibrium we can write two separate equilibrium constants:

$$
K_{\mathrm{c}}^{\prime}=\frac{[\mathrm{C}][\mathrm{D}]}{[\mathrm{A}][\mathrm{B}]}
$$

and

$$
K_{\mathrm{c}}^{\prime \prime}=\frac{[\mathrm{E}][\mathrm{F}]}{[\mathrm{C}][\mathrm{D}]}
$$

The overall reaction is given by the sum of the two reactions

$$
\text { Overall reaction: } \quad \begin{array}{ll}
\mathrm{A}+\mathrm{B} \rightleftharpoons \mathrm{C}+\mathrm{D} & K_{\mathrm{c}}^{\prime} \\
\mathrm{C}+\mathrm{D} \rightleftharpoons \mathrm{E}+\mathrm{F} & K_{\mathrm{c}}^{\prime \prime} \\
\mathrm{A}+\mathrm{B} \rightleftharpoons \mathrm{E}+\mathrm{F} & K_{\mathrm{c}}
\end{array}
$$

and the equilibrium constant $K_{\mathrm{c}}$ for the overall reaction is

$$
K_{\mathrm{c}}=\frac{[\mathrm{E}][\mathrm{F}]}{[\mathrm{A}][\mathrm{B}]}
$$

We obtain the same expression if we take the product of the expressions for $K_{\mathrm{c}}^{\prime}$ and $K_{\mathrm{c}}^{\prime \prime}$ :

$$
K_{\mathrm{c}}^{\prime} K_{\mathrm{c}}^{\prime \prime}=\frac{[\mathrm{C}][\mathrm{D}]}{[\mathrm{A}][\mathrm{B}]} \times \frac{[\mathrm{E}][\mathrm{F}]}{[\mathrm{C}][\mathrm{D}]}=\frac{[\mathrm{E}][\mathrm{F}]}{[\mathrm{A}][\mathrm{B}]}
$$

Therefore,

$$
\begin{equation*}
K_{\mathrm{c}}=K_{\mathrm{c}}^{\prime} K_{\mathrm{c}}^{\prime \prime} \tag{14.9}
\end{equation*}
$$

## Relationship of $K$ to the Balanced Chemical Equation

The equilibrium constant expression and the value of $K$ both depend on how we write the equation for a reaction. Here are some general rules to keep in mind.

- When we reverse an equation, we invert the value of $K$.
- When we multiply the coefficients in a balanced equation by a common factor $(2,3, \ldots)$, we raise the equilibrium constant to the corresponding power $(2,3, \ldots)$.
- When we divide the coefficients in a balanced equation by a common factor $(2,3, \ldots)$, we take the corresponding root of the equilibrium constant (square root, cube root, ...).

To illustrate these points, let us consider the synthesis of methanol (methyl alcohol) from a carbon monoxide-hydrogen mixture called synthesis gas. This reaction is likely to become increasingly important as methanol and its mixtures with gasoline find greater use as motor fuels. The balanced reaction is

$$
\mathrm{CO}(\mathrm{~g})+2 \mathrm{H}_{2}(\mathrm{~g}) \rightleftharpoons \mathrm{CH}_{3} \mathrm{OH}(\mathrm{~g}) \quad K=9.23 \times 10^{-3}
$$

Suppose that in discussing the synthesis of $\mathrm{CH}_{3} \mathrm{OH}$ from CO and $\mathrm{H}_{2}$, we had written the reverse reaction, that is,

$$
\mathrm{CH}_{3} \mathrm{OH}(\mathrm{~g}) \rightleftharpoons \mathrm{CO}(\mathrm{~g})+2 \mathrm{H}_{2}(\mathrm{~g}) \quad K^{\prime}=?
$$

Now, according to the generalized equilibrium constant expression (15.7), we should write

$$
K^{\prime}=\frac{a_{\mathrm{CO}(\mathrm{~g})} a_{\mathrm{H}_{2}(\mathrm{~g})}}{a_{\mathrm{CH}}^{3 \mathrm{OH}(\mathrm{~g})}}=\frac{1}{\frac{a_{\mathrm{CH}}^{3} \mathrm{OH}(\mathrm{~g})}{a_{\mathrm{CO}(\mathrm{~g})} a_{\mathrm{H}_{2}(\mathrm{~g})}}}=\frac{1}{K}=\frac{1}{9.23 \times 10^{-3}}=1.08 \times 10^{2}
$$

In the preceding expression, the equilibrium constant expression and $K$ value for the reaction, as originally written, are printed in blue. We see that $K^{\prime}=1 / K$.

Suppose that for a certain application we want an equation based on synthesizing two moles of $\mathrm{CH}_{3} \mathrm{OH}(\mathrm{g})$.

$$
2 \mathrm{CO}(\mathrm{~g})+4 \mathrm{H}_{2}(\mathrm{~g}) \rightleftharpoons 2 \mathrm{CH}_{3} \mathrm{OH}(\mathrm{~g}) \quad K^{m}=\text { ? }
$$

Here, $K^{\prime \prime}=K^{2}$. That is,

$$
K^{\prime \prime}=\frac{a_{\mathrm{CH}_{3} \mathrm{OH}(\mathrm{~g})}^{2}}{a_{\mathrm{CO}(\mathrm{~g})}^{2} a_{\mathrm{H}_{2}(\mathrm{~g})}^{4}}=\left(\frac{a_{\mathrm{CH} 3} \mathrm{OH}(\mathrm{~g})}{a_{\mathrm{CO}(\mathrm{~g})} a_{\mathrm{H}_{2}(\mathrm{~g})}}\right)^{2}=(K)^{2}=\left(9.23 \times 10^{-3}\right)^{2}=8.52 \times 10^{-5}
$$

## EXAMPLE 15-3 Relating $K$ to the Balanced Chemical Equation

The following $K$ value is given at 298 K for the synthesis of $\mathrm{NH}_{3}(\mathrm{~g})$ from its elements.

$$
\mathrm{N}_{2}(\mathrm{~g})+3 \mathrm{H}_{2}(\mathrm{~g}) \rightleftharpoons 2 \mathrm{NH}_{3}(\mathrm{~g}) \quad K=5.8 \times 10^{5}
$$

What is the value of $K$ at 298 K for the following reaction?

$$
\mathrm{NH}_{3}(\mathrm{~g}) \rightleftharpoons \frac{1}{2} \mathrm{~N}_{2}(\mathrm{~g})+\frac{3}{2} \mathrm{H}_{2}(\mathrm{~g}) \quad K=?
$$

## EXAMPLE 15-3 Relating $K$ to the Balanced Chemical Equation

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$$
\mathrm{N}_{2}(\mathrm{~g})+3 \mathrm{H}_{2}(\mathrm{~g}) \rightleftharpoons 2 \mathrm{NH}_{3}(\mathrm{~g}) \quad K=5.8 \times 10^{5}
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What is the value of $K$ at 298 K for the following reaction?

$$
\mathrm{NH}_{3}(\mathrm{~g}) \rightleftharpoons \frac{1}{2} \mathrm{~N}_{2}(\mathrm{~g})+\frac{3}{2} \mathrm{H}_{2}(\mathrm{~g}) \quad K=?
$$

Analyze
The solution to this problem lies in recognizing that the reaction is the reverse and one-half of the given reaction. In this example we apply two of the rules given above that relate $K$ to balanced chemical reactions.

## Solve

First, reverse the given equation. This puts $\mathrm{NH}_{3}(\mathrm{~g})$ on $\quad 2 \mathrm{NH}_{3}(\mathrm{~g}) \rightleftharpoons \mathrm{N}_{2}(\mathrm{~g})+3 \mathrm{H}_{2}(\mathrm{~g})$ the left side of the equation, where we need it.

The equilibrium constant $K^{\prime}$ becomes

$$
K^{\prime}=1 /\left(5.8 \times 10^{5}\right)=1.7 \times 10^{-6}
$$

Then, to base the equation on $1 \mathrm{~mol}_{\mathrm{NH}_{3}(\mathrm{~g})}$, divide all coefficients by 2.
$\mathrm{NH}_{3}(\mathrm{~g}) \rightleftharpoons \frac{1}{2} \mathrm{~N}_{2}(\mathrm{~g})+\frac{3}{2} \mathrm{H}_{2}(\mathrm{~g})$
This requires the square root of $K^{\prime}$.

$$
K=\sqrt{1.7 \times 10^{-6}}=1.3 \times 10^{-3}
$$

PRACTICE EXAMPLE B: For the reaction $\mathrm{NO}(\mathrm{g})+\frac{1}{2} \mathrm{O}_{2}(\mathrm{~g}) \rightleftharpoons \mathrm{NO}_{2}(\mathrm{~g})$ at $184^{\circ} \mathrm{C}, \mathrm{K}=1.2 \times 10^{2}$. What is the value of $K$ at $184^{\circ} \mathrm{C}$ for the reaction $2 \mathrm{NO}_{2}(\mathrm{~g}) \rightleftharpoons 2 \mathrm{NO}(\mathrm{g})+\mathrm{O}_{2}(\mathrm{~g})$ ?

## EXAMPLE 14.7

The reaction for the production of ammonia can be written in a number of ways:
(a) $\mathrm{N}_{2}(g)+3 \mathrm{H}_{2}(g) \rightleftharpoons 2 \mathrm{NH}_{3}(g)$
(b) $\frac{1}{2} \mathrm{~N}_{2}(g)+\frac{3}{2} \mathrm{H}_{2}(g) \rightleftharpoons \mathrm{NH}_{3}(g)$
(c) $\frac{1}{3} \mathrm{~N}_{2}(g)+\mathrm{H}_{2}(g) \rightleftharpoons \frac{2}{3} \mathrm{NH}_{3}(g)$

Write the equilibrium constant expression for each formulation. (Express the concentrations of the reacting species in $\mathrm{mol} / \mathrm{L}$.)
(d) How are the equilibrium constants related to one another?

## EXAMPLE 14.7

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(b) $\frac{1}{2} \mathrm{~N}_{2}(g)+\frac{3}{2} \mathrm{H}_{2}(g) \rightleftharpoons \mathrm{NH}_{3}(g)$
(c) $\frac{1}{3} \mathrm{~N}_{2}(g)+\mathrm{H}_{2}(g) \rightleftharpoons \frac{2}{3} \mathrm{NH}_{3}(g)$

Write the equilibrium constant expression for each formulation. (Express the concentrations of the reacting species in $\mathrm{mol} / \mathrm{L}$.)
(d) How are the equilibrium constants related to one another?

Strategy We are given three different expressions for the same reacting system. Remember that the equilibrium constant expression depends on how the equation is balanced, that is, on the stoichiometric coefficients used in the equation.

## Solution

(a)

$$
\begin{align*}
K_{a} & =\frac{\left[\mathrm{NH}_{3}\right]^{2}}{\left[\mathrm{~N}_{2}\right]\left[\mathrm{H}_{2}\right]^{3}} \\
K_{b} & =\frac{\left[\mathrm{NH}_{3}\right]}{\left[\mathrm{N}_{2}\right]^{\frac{1}{2}}\left[\mathrm{H}_{2}\right]^{\frac{3}{2}}}  \tag{b}\\
K_{c} & =\frac{\left[\mathrm{NH}_{3}\right]^{\frac{2}{3}}}{\left[\mathrm{~N}_{2}\right]^{\frac{1}{3}}\left[\mathrm{H}_{2}\right]} \\
K_{a} & =K_{b}^{2} \\
K_{a} & =K_{c}^{3} \\
K_{b}^{2} & =K_{c}^{3} \quad \text { or } \quad K_{b}=K_{c}^{\frac{3}{2}}
\end{align*}
$$

(c)
(d)

## Predicting the Direction of a Reaction

The equilibrium constant $K_{\mathrm{c}}$ for the formation of hydrogen iodide from molecular hydrogen and molecular iodine in the gas phase

$$
\mathrm{H}_{2}(g)+\mathrm{I}_{2}(g) \rightleftharpoons 2 \mathrm{HI}(g)
$$

is 54.3 at $430^{\circ} \mathrm{C}$. Suppose that in a certain experiment we place 0.243 mole of $\mathrm{H}_{2}$, 0.146 mole of $\mathrm{I}_{2}$, and 1.98 moles of HI all in a $1.00-\mathrm{L}$ container at $430^{\circ} \mathrm{C}$. Will there be a net reaction to form more $\mathrm{H}_{2}$ and $\mathrm{I}_{2}$ or more HI ? Inserting the starting concentrations in the equilibrium constant expression, we write

$$
\frac{[\mathrm{HI}]_{0}^{2}}{\left[\mathrm{H}_{2}\right]_{0}\left[\mathrm{I}_{2}\right]_{0}}=\frac{(1.98)^{2}}{(0.243)(0.146)}=111
$$

where the subscript 0 indicates initial concentrations (before equilibrium is reached). Because the quotient $[\mathrm{HI}]_{0}^{2} /\left[\mathrm{H}_{2}\right]_{0}\left[\mathrm{I}_{2}\right]_{0}$ is greater than $K_{\mathrm{c}}$, this system is not at equilibrium.

For reactions that have not reached equilibrium, such as the formation of HI considered above, we obtain the reaction quotient $\left(\boldsymbol{Q}_{c}\right)$, instead of the equilibrium constant by substituting the initial concentrations into the equilibrium constant expression. To determine the direction in which the net reaction will proceed to achieve equilibrium, we compare the values of $Q_{\mathrm{c}}$ and $K_{\mathrm{c}}$. The three possible cases are as follows:

- $Q_{\mathrm{c}}<K_{\mathrm{c}} \quad$ The ratio of initial concentrations of products to reactants is too small. To reach equilibrium, reactants must be converted to products. The system proceeds from left to right (consuming reactants, forming products) to reach equilibrium.
- $Q_{\mathrm{c}}=K_{\mathrm{c}} \quad$ The initial concentrations are equilibrium concentrations. The system is at equilibrium.
- $Q_{\mathrm{c}}>K_{\mathrm{c}} \quad$ The ratio of initial concentrations of products to reactants is too large. To reach equilibrium, products must be converted to reactants. The system proceeds from right to left (consuming products, forming reactants) to reach equilibrium.



## EXAMPLE 14.8

At the start of a reaction, there are $0.249 \mathrm{~mol} \mathrm{~N}_{2}, 3.21 \times 10^{-2} \mathrm{~mol} \mathrm{H}_{2}$, and $6.42 \times$ $10^{-4} \mathrm{~mol} \mathrm{NH}_{3}$ in a $3.50-\mathrm{L}$ reaction vessel at $375^{\circ} \mathrm{C}$. If the equilibrium constant $\left(K_{\mathrm{c}}\right)$ for the reaction

$$
\mathrm{N}_{2}(g)+3 \mathrm{H}_{2}(g) \rightleftharpoons 2 \mathrm{NH}_{3}(g)
$$

is 1.2 at this temperature, decide whether the system is at equilibrium. If it is not, predict which way the net reaction will proceed.

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\mathrm{N}_{2}(g)+3 \mathrm{H}_{2}(g) \rightleftharpoons 2 \mathrm{NH}_{3}(g)
$$

is 1.2 at this temperature, decide whether the system is at equilibrium. If it is not, predict which way the net reaction will proceed.
Strategy We are given the initial amounts of the gases (in moles) in a vessel of known volume (in liters), so we can calculate their molar concentrations and hence the reaction quotient $\left(Q_{\mathrm{c}}\right)$. How does a comparison of $Q_{\mathrm{c}}$ with $K_{\mathrm{c}}$ enable us to determine if the system is at equilibrium or, if not, in which direction will the net reaction proceed to reach equilibrium?
Solution The initial concentrations of the reacting species are

$$
\begin{aligned}
{\left[\mathrm{N}_{2}\right]_{0} } & =\frac{0.249 \mathrm{~mol}}{3.50 \mathrm{~L}}=0.0711 \mathrm{M} \\
{\left[\mathrm{H}_{2}\right]_{0} } & =\frac{3.21 \times 10^{-2} \mathrm{~mol}}{3.50 \mathrm{~L}}=9.17 \times 10^{-3} \mathrm{M} \\
{\left[\mathrm{NH}_{3}\right]_{0} } & =\frac{6.42 \times 10^{-4} \mathrm{~mol}}{3.50 \mathrm{~L}}=1.83 \times 10^{-4} \mathrm{M}
\end{aligned}
$$

Next we write

$$
Q_{\mathrm{c}}=\frac{\left[\mathrm{NH}_{3}\right]_{0}^{2}}{\left[\mathrm{~N}_{2}\right]_{0}\left[\mathrm{H}_{2}\right]_{0}^{3}}=\frac{\left(1.83 \times 10^{-4}\right)^{2}}{(0.0711)\left(9.17 \times 10^{-3}\right)^{3}}=0.611
$$

Because $Q_{\mathrm{c}}$ is smaller than $K_{\mathrm{c}}(1.2)$, the system is not at equilibrium. The net result will be an increase in the concentration of $\mathrm{NH}_{3}$ and a decrease in the concentrations of $\mathrm{N}_{2}$ and $\mathrm{H}_{2}$. That is, the net reaction will proceed from left to right until equilibrium is reached.

Practice Exercise The equilibrium constant $\left(K_{\mathrm{c}}\right)$ for the formation of nitrosyl chloride, an orange-yellow compound, from nitric oxide and molecular chlorine

$$
2 \mathrm{NO}(g)+\mathrm{Cl}_{2}(g) \rightleftharpoons 2 \mathrm{NOCl}(g)
$$

is $6.5 \times 10^{4}$ at $35^{\circ} \mathrm{C}$. In a certain experiment, $2.0 \times 10^{-2}$ mole of $\mathrm{NO}, 8.3 \times 10^{-3}$ mole of $\mathrm{Cl}_{2}$, and 6.8 moles of NOCl are mixed in a $2.0-\mathrm{L}$ flask. In which direction will the system proceed to reach equilibrium?

## Calculating Equilibrium Concentrations

If we know the equilibrium constant for a particular reaction, we can calculate the concentrations in the equilibrium mixture from the initial concentrations. Commonly, only the initial reactant concentrations are given. Let us consider the following system involving two organic compounds, cis-stilbene and trans-stilbene, in a nonpolar hydrocarbon solvent (Figure 14.6):

$$
\text { cis-stilbene } \rightleftharpoons \text { trans-stilbene }
$$

The equilibrium constant $\left(K_{\mathrm{c}}\right)$ for this system is 24.0 at $200^{\circ} \mathrm{C}$. Suppose that initially only cis-stilbene is present at a concentration of $0.850 \mathrm{~mol} / \mathrm{L}$. How do we calculate the concentrations of cis- and trans-stilbene at equilibrium? From the

|  | cis-stilbene | $\rightleftharpoons$ |
| :--- | :---: | :---: |
| Initial $(M):$ | 0.850 | 0 |
| Change $(M):$ | $-x$ | $+x$ |
| Equilibrium $(M):$ | $(0.850-x)$ | $x$ |

A positive $(+)$ change represents an increase and a negative $(-)$ change a decrease in concentration at equilibrium. Next, we set up the equilibrium constant expression

$$
\begin{aligned}
K_{\mathrm{c}} & =\frac{[\text { trans-stilbene }]}{[\text { cis-stilbene }]} \\
24.0 & =\frac{x}{0.850-x} \\
x & =0.816 \mathrm{M}
\end{aligned}
$$

Having solved for $x$, we calculate the equilibrium concentrations of cis-stilbene and trans-stilbene as follows:

$$
\begin{aligned}
{[\text { cis-stilbene }] } & =(0.850-0.816) M=0.034 \mathrm{M} \\
{[\text { trans-stilbene }] } & =0.816 \mathrm{M}
\end{aligned}
$$

To check the results we could use the equilibrium concentrations to calculate $K_{\mathrm{c}}$. We summarize our approach to solving equilibrium constant problems as follows:

1. Express the equilibrium concentrations of all species in terms of the initial concentrations and a single unknown $x$, which represents the change in concentration.
2. Write the equilibrium constant expression in terms of the equilibrium concentrations. Knowing the value of the equilibrium constant, solve for $x$.
3. Having solved for $x$, calculate the equilibrium concentrations of all species.

## EXAMPLE 14.9

A mixture of $0.500 \mathrm{~mol} \mathrm{H}_{2}$ and $0.500 \mathrm{~mol}_{2}$ was placed in a 1.00-L stainless-steel flask at $430^{\circ} \mathrm{C}$. The equilibrium constant $K_{\mathrm{c}}$ for the reaction $\mathrm{H}_{2}(\mathrm{~g})+\mathrm{I}_{2}(\mathrm{~g}) \rightleftharpoons 2 \mathrm{HI}(\mathrm{g})$ is 54.3 at this temperature. Calculate the concentrations of $\mathrm{H}_{2}, \mathrm{I}_{2}$, and HI at equilibrium.

## EXAMPLE 14.9

A mixture of $0.500 \mathrm{~mol} \mathrm{H}_{2}$ and $0.500 \mathrm{~mol}_{2}$ was placed in a 1.00-L stainless-steel flask at $430^{\circ} \mathrm{C}$. The equilibrium constant $K_{\mathrm{c}}$ for the reaction $\mathrm{H}_{2}(g)+\mathrm{I}_{2}(\mathrm{~g}) \rightleftharpoons 2 \mathrm{HI}(\mathrm{g})$ is 54.3 at this temperature. Calculate the concentrations of $\mathrm{H}_{2}, \mathrm{I}_{2}$, and HI at equilibrium.

Strategy We are given the initial amounts of the gases (in moles) in a vessel of known volume (in liters), so we can calculate their molar concentrations. Because initially no HI was present, the system could not be at equilibrium. Therefore, some $\mathrm{H}_{2}$ would react with the same amount of $\mathrm{I}_{2}$ (why?) to form HI until equilibrium was established.

Solution We follow the preceding procedure to calculate the equilibrium concentrations.
Step 1: The stoichiometry of the reaction is $1 \mathrm{~mol}_{2}$ reacting with $1 \mathrm{~mol}_{2}$ to yield 2 mol HI . Let $x$ be the depletion in concentration ( $\mathrm{mol} / \mathrm{L}$ ) of $\mathrm{H}_{2}$ and $\mathrm{I}_{2}$ at equilibrium. It follows that the equilibrium concentration of HI must be $2 x$. We summarize the changes in concentrations as follows:


Step 2: The equilibrium constant is given by

$$
K_{\mathrm{c}}=\frac{[\mathrm{HI}]^{2}}{\left[\mathrm{H}_{2}\right]\left[\mathrm{I}_{2}\right]}
$$

Substituting, we get

$$
54.3=\frac{(2 x)^{2}}{(0.500-x)(0.500-x)}
$$

Taking the square root of both sides, we get

$$
\begin{aligned}
7.37 & =\frac{2 x}{0.500-x} \\
x & =0.393 \mathrm{M}
\end{aligned}
$$

Step 3: At equilibrium, the concentrations are

$$
\begin{aligned}
{\left[\mathrm{H}_{2}\right] } & =(0.500-0.393) M=0.107 M \\
{\left[\mathrm{I}_{2}\right] } & =(0.500-0.393) M=0.107 \mathrm{M} \\
{[\mathrm{HI}] } & =2 \times 0.393 M=0.786 \mathrm{M}
\end{aligned}
$$

## Factors That Affect Chemical Equilibrium

Chemical equilibrium represents a balance between forward and reverse reactions. In most cases, this balance is quite delicate. Changes in experimental conditions may disturb the balance and shift the equilibrium position so that more or less of the desired product is formed.

When we say that an equilibrium position shifts to the right, for example, we mean that the net reaction is now from left to right. Variables that can be controlled experimentally are concentration, pressure, volume, and temperature.

## Le Châtelier's Principle

There is a general rule that helps us to predict the direction in which an equilibrium reaction will move when a change in concentration, pressure, volume, or temperature occurs.

The rule, known as Le Châtelier'st principle, states that if an external stress is applied to a system at equilibrium, the system adjusts in such a way that the stress is partially offset as the system reaches a new equilibrium position.

The word "stress" here means a change in concentration, pressure, volume, or temperature that removes the system from the equilibrium state. We will use Le Châtelier's principle to assess the effects of such changes.

## Changes in Concentration

Iron(III) thiocyanate $\left[\mathrm{Fe}(\mathrm{SCN})_{3}\right]$ dissolves readily in water to give a red solution. The red color is due to the presence of hydrated $\mathrm{FeSCN}^{2+}$ ion. The equilibrium between undissociated $\mathrm{FeSCN}^{2+}$ and the $\mathrm{Fe}^{3+}$ and $\mathrm{SCN}^{-}$ions is given by

$$
\underset{\quad \mathrm{FeSCN}^{2+}(a q)}{\quad \text { red }} \rightleftharpoons \underset{\text { pale yellow }}{\mathrm{Fe}^{3+}(a q)}+\underset{\mathrm{SCN}^{-}(a q)}{\text { colorless }}
$$

What happens if we add some sodium thiocyanate ( NaSCN ) to this solution? In this case, the stress applied to the equilibrium system is an increase in the concentration of $\mathrm{SCN}^{-}$(from the dissociation of NaSCN). To offset this stress, some $\mathrm{Fe}^{3+}$ ions react with the added $\mathrm{SCN}^{-}$ions, and the equilibrium shifts from right to left:

$$
\mathrm{FeSCN}^{2+}(a q) \longleftarrow \mathrm{Fe}^{3+}(a q)+\mathrm{SCN}^{-}(a q)
$$

Consequently, the red color of the solution deepens (Figure 14.7). Similarly, if we added iron(III) nitrate $\left[\mathrm{Fe}\left(\mathrm{NO}_{3}\right)_{3}\right]$ to the original solution, the red color would also deepen because the additional $\mathrm{Fe}^{3+}$ ions $\left[\right.$ from $\mathrm{Fe}\left(\mathrm{NO}_{3}\right)_{3}$ ] would shift the equilibrium from right to left.

Now suppose we add some oxalic acid $\left(\mathrm{H}_{2} \mathrm{C}_{2} \mathrm{O}_{4}\right)$ to the original solution. Oxalic acid ionizes in water to form the oxalate ion, $\mathrm{C}_{2} \mathrm{O}_{4}^{2-}$, which binds strongly to the $\mathrm{Fe}^{3+}$ ions. The formation of the stable yellow ion $\mathrm{Fe}\left(\mathrm{C}_{2} \mathrm{O}_{4}\right)_{3}^{3-}$ removes free $\mathrm{Fe}^{3+}$ ions in solution. Consequently, more $\mathrm{FeSCN}^{2+}$ units dissociate and the equilibrium shifts from left to right:

$$
\mathrm{FeSCN}^{2+}(a q) \longrightarrow \mathrm{Fe}^{3+}(a q)+\mathrm{SCN}^{-}(a q)
$$

The red solution will turn yellow due to the formation of $\mathrm{Fe}\left(\mathrm{C}_{2} \mathrm{O}_{4}\right)_{3}^{3-}$ ions.


Figure 14.7 Effect of concentration change on the position of equilibrium. (a) An aqueous $\mathrm{Fe}(\mathrm{SCN})_{3}$ solution. The color of the solution is due to both the red $\mathrm{FeSCN}{ }^{2+}$ and the yellow $\mathrm{Fe}^{3+}$ ions. (b) After the addition of some NaSCN to the solution in (a), the equilibrium shifts to the left. (c) After the addition of some $\mathrm{Fe}\left(\mathrm{NO}_{3}\right)_{3}$ to the solution in (a), the equilibrium shifts to the left. (d) After the addition of some $\mathrm{H}_{2} \mathrm{C}_{2} \mathrm{O}_{4}$ to the solution in (a), the equilibrium shifts to the right. The yellow color is due to the $\mathrm{Fe}\left(\mathrm{C}_{2} \mathrm{O}_{4}\right)_{3}^{3-}$ ions.

This experiment demonstrates that all reactants and products are present in the reacting system at equilibrium. Second, increasing the concentrations of the products $\left(\mathrm{Fe}^{3+}\right.$ or $\left.\mathrm{SCN}^{-}\right)$shifts the equilibrium to the left, and decreasing the concentration of the product $\mathrm{Fe}^{3+}$ shifts the equilibrium to the right. These results are just as predicted by Le Châtelier's principle.

## EXAMPLE 14.11

At $720^{\circ} \mathrm{C}$, the equilibrium constant $K_{\mathrm{c}}$ for the reaction

$$
\mathrm{N}_{2}(g)+3 \mathrm{H}_{2}(g) \rightleftharpoons 2 \mathrm{NH}_{3}(g)
$$

is $2.37 \times 10^{-3}$. In a certain experiment, the equilibrium concentrations are $\left[\mathrm{N}_{2}\right]=0.683 \mathrm{M}$, $\left[\mathrm{H}_{2}\right]=8.80 \mathrm{M}$, and $\left[\mathrm{NH}_{3}\right]=1.05 \mathrm{M}$. Suppose some $\mathrm{NH}_{3}$ is added to the mixture so that its concentration is increased to 3.65 M . (a) Use Le Châtelier's principle to predict the shift in direction of the net reaction to reach a new equilibrium. (b) Confirm your prediction by calculating the reaction quotient $Q_{\mathrm{c}}$ and comparing its value with $K_{\mathrm{c}}$.

Strategy (a) What is the stress applied to the system? How does the system adjust to offset the stress? (b) At the instant when some $\mathrm{NH}_{3}$ is added, the system is no longer at equilibrium. How do we calculate the $Q_{\mathrm{c}}$ for the reaction at this point? How does a comparison of $Q_{\mathrm{c}}$ with $K_{\mathrm{c}}$ tell us the direction of the net reaction to reach equilibrium.

Solution (a) The stress applied to the system is the addition of $\mathrm{NH}_{3}$. To offset this stress, some $\mathrm{NH}_{3}$ reacts to produce $\mathrm{N}_{2}$ and $\mathrm{H}_{2}$ until a new equilibrium is established. The net reaction therefore shifts from right to left; that is,

$$
\mathrm{N}_{2}(g)+3 \mathrm{H}_{2}(g) \longleftarrow 2 \mathrm{NH}_{3}(g)
$$

(b) At the instant when some of the $\mathrm{NH}_{3}$ is added, the system is no longer at equilibrium. The reaction quotient is given by

$$
\begin{aligned}
Q_{\mathrm{c}} & =\frac{\left[\mathrm{NH}_{3}\right]_{0}^{2}}{\left[\mathrm{~N}_{2}\right]_{0}\left[\mathrm{H}_{2}\right]_{0}^{3}} \\
& =\frac{(3.65)^{2}}{(0.683)(8.80)^{3}} \\
& =2.86 \times 10^{-2}
\end{aligned}
$$

Because this value is greater than $2.37 \times 10^{-3}$, the net reaction shifts from right to left until $Q_{\mathrm{c}}$ equals $K_{\mathrm{c}}$.

## Effect of Temperature on Equilibrium

We can think of changing the temperature of an equilibrium mixture in terms of adding heat (raising the temperature) or removing heat (lowering the temperature). According to Le Châtelier's principle, adding heat favors the reaction in which heat is absorbed (endothermic reaction). Removing heat favors the reaction in which heat is evolved (exothermic reaction). Stated in terms of changing temperature,

Raising the temperature of an equilibrium mixture shifts the equilibrium condition in the direction of the endothermic reaction. Lowering the temperature causes a shift in the direction of the exothermic reaction.

For endothermic reactions, $K$ increases as temperature increases. For exothermic reactions, $K$ decreases as temperature increases.

## EXAMPLE 15-8 Applying Le Châtelier's Principle: Effect of Temperature on Equilibrium

Consider the reaction

$$
2 \mathrm{SO}_{2}(\mathrm{~g})+\mathrm{O}_{2}(\mathrm{~g}) \rightleftharpoons 2 \mathrm{SO}_{3}(\mathrm{~g}) \quad \Delta_{\mathrm{r}} H^{\circ}=-197.8 \mathrm{~kJ} \mathrm{~mol}^{-1}
$$

Will the amount of $\mathrm{SO}_{3}(\mathrm{~g})$ formed from given amounts of $\mathrm{SO}_{2}(\mathrm{~g})$ and $\mathrm{O}_{2}(\mathrm{~g})$ be greater at high or low temperatures?

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Will the amount of $\mathrm{SO}_{3}(\mathrm{~g})$ formed from given amounts of $\mathrm{SO}_{2}(\mathrm{~g})$ and $\mathrm{O}_{2}(\mathrm{~g})$ be greater at high or low temperatures?

## Analyze

We must think of the impact made by changing the temperature. In general, an increase in temperature causes a shift in the direction of the endothermic reaction.

## Solve

The sign of $\Delta_{\mathrm{r}} H^{\circ}$ tells us that the forward reaction is exothermic. Thus, the reverse reaction is endothermic. In this case, increasing the temperature will favor the reverse reaction and lowering the temperature will favor the forward reaction. The conversion of $\mathrm{SO}_{2}$ to $\mathrm{SO}_{3}$ is favored at low temperatures.

## Changes in Volume and Pressure

Changes in pressure ordinarily do not affect the concentrations of reacting species in condensed phases (say, in an aqueous solution) because liquids and solids are virtually incompressible. On the other hand, concentrations of gases are greatly affected by changes in pressure.

## Effect of Changes in Pressure or Volume on Equilibrium

There are three ways to change the pressure of a constant-temperature equilibrium mixture.

1. Add or remove a gaseous reactant or product. The effect of these actions on the equilibrium condition is simply that caused by adding or removing a reaction component, as described previously.
2. Add an inert gas to the constant-volume reaction mixture. This has the effect of increasing the total pressure, but the partial pressures of the reacting species are all unchanged. An inert gas added to a constant-volume equilibrium mixture has no effect on the equilibrium condition.
3. Change the pressure by changing the volume of the system. Decreasing the volume of the system increases the pressure, and increasing the system volume decreases the pressure. Thus, the effect of this type of pressure change is simply that of a volume change.

In general, an increase in pressure (decrease in volume) favors the net reaction that decreases the total number of moles of gases (the reverse reaction, in this case), and a decrease in pressure (increase in volume) favors the net reaction that increases the total number of moles of gases (here, the forward reaction).

For reactions in which there is no change in the number of moles of gases, a pressure (or volume) change has no effect on the position of equilibrium.

When the volume of an equilibrium mixture of gases is reduced, a net change occurs in the direction that produces fewer moles of gas. When the volume is increased, a net change occurs in the direction that produces more moles of gas.

## EXAMPLE 14.12

Consider the following equilibrium systems:
(a) $2 \mathrm{PbS}(s)+3 \mathrm{O}_{2}(g) \rightleftharpoons 2 \mathrm{PbO}(s)+2 \mathrm{SO}_{2}(g)$
(b) $\mathrm{PCl}_{5}(g) \rightleftharpoons \mathrm{PCl}_{3}(g)+\mathrm{Cl}_{2}(g)$
(c) $\mathrm{H}_{2}(g)+\mathrm{CO}_{2}(g) \rightleftharpoons \mathrm{H}_{2} \mathrm{O}(g)+\mathrm{CO}(g)$

Predict the direction of the net reaction in each case as a result of increasing the pressure (decreasing the volume) on the system at constant temperature.

```
(a) \(2 \mathrm{PbS}(s)+3 \mathrm{O}_{2}(g) \rightleftharpoons 2 \mathrm{PbO}(s)+2 \mathrm{SO}_{2}(g)\)
(b) \(\mathrm{PCl}_{5}(g) \rightleftharpoons \mathrm{PCl}_{3}(\mathrm{~g})+\mathrm{Cl}_{2}(\mathrm{~g})\)
(c) \(\mathrm{H}_{2}(\mathrm{~g})+\mathrm{CO}_{2}(\mathrm{~g}) \rightleftharpoons \mathrm{H}_{2} \mathrm{O}(\mathrm{g})+\mathrm{CO}(\mathrm{g})\)
```

Strategy A change in pressure can affect only the volume of a gas, but not that of a solid because solids (and liquids) are much less compressible. The stress applied is an increase in pressure. According to Le Châtelier's principle, the system will adjust to partially offset this stress. In other words, the system will adjust to decrease the pressure. This can be achieved by shifting to the side of the equation that has fewer moles of gas. Recall that pressure is directly proportional to moles of gas: $P V=n R T$ so $P \propto n$.

Solution (a) Consider only the gaseous molecules. In the balanced equation, there are 3 moles of gaseous reactants and 2 moles of gaseous products. Therefore, the net reaction will shift toward the products (to the right) when the pressure is increased.
(b) The number of moles of products is 2 and that of reactants is 1 ; therefore, the net reaction will shift to the left, toward the reactant.
(c) The number of moles of products is equal to the number of moles of reactants, so a change in pressure has no effect on the equilibrium.

## Effect of a Catalyst on Equilibrium

A catalyst is a substance that, when added to a reaction mixture, speeds up both the forward and reverse reactions. Equilibrium is achieved more rapidly, but the equilibrium amounts are unchanged by the catalyst.

## EXAMPLE 14.13

Consider the following equilibrium process between dinitrogen tetrafluoride $\left(\mathrm{N}_{2} \mathrm{~F}_{4}\right)$ and nitrogen difluoride $\left(\mathrm{NF}_{2}\right)$ :

$$
\mathrm{N}_{2} \mathrm{~F}_{4}(g) \rightleftharpoons 2 \mathrm{NF}_{2}(g) \quad \Delta H^{\circ}=38.5 \mathrm{~kJ} / \mathrm{mol}
$$

Predict the changes in the equilibrium if (a) the reacting mixture is heated at constant volume; (b) some $\mathrm{N}_{2} \mathrm{~F}_{4}$ gas is removed from the reacting mixture at constant temperature and volume; (c) the pressure on the reacting mixture is decreased at constant temperature;

Strategy (a) What does the sign of $\Delta H^{\circ}$ indicate about the heat change (endothermic or exothermic) for the forward reaction? (b) Would the removal of some $\mathrm{N}_{2} \mathrm{~F}_{4}$ increase or decrease the $Q_{\mathrm{c}}$ of the reaction? (c) How would the decrease in pressure change the volume of the system? (d) What is the function of a catalyst? How does it affect a reacting system not at equilibrium? at equilibrium?

Solution (a) The stress applied is the heat added to the system. Note that the $\mathrm{N}_{2} \mathrm{~F}_{4} \longrightarrow 2 \mathrm{NF}_{2}$ reaction is an endothermic process ( $\Delta H^{\circ}>0$ ), which absorbs heat from the surroundings. Therefore, we can think of heat as a reactant

$$
\text { heat }+\mathrm{N}_{2} \mathrm{~F}_{4}(g) \rightleftharpoons 2 \mathrm{NF}_{2}(g)
$$

The system will adjust to remove some of the added heat by undergoing a decomposition reaction (from left to right). The equilibrium constant

$$
K_{\mathrm{c}}=\frac{\left[\mathrm{NF}_{2}\right]^{2}}{\left[\mathrm{~N}_{2} \mathrm{~F}_{4}\right]}
$$

will therefore increase with increasing temperature because the concentration of $\mathrm{NF}_{2}$ has increased and that of $\mathrm{N}_{2} \mathrm{~F}_{4}$ has decreased. Recall that the equilibrium constant is a constant only at a particular temperature. If the temperature is changed, then the equilibrium constant will also change.
(b) The stress here is the removal of $\mathrm{N}_{2} \mathrm{~F}_{4}$ gas. The system will shift to replace some of the $\mathrm{N}_{2} \mathrm{~F}_{4}$ removed. Therefore, the system shifts from right to left until equilibrium is reestablished. As a result, some $\mathrm{NF}_{2}$ combines to form $\mathrm{N}_{2} \mathrm{~F}_{4}$.
(c) The stress applied is a decrease in pressure (which is accompanied by an increase in gas volume). The system will adjust to remove the stress by increasing the pressure. Recall that pressure is directly proportional to the number of moles of a gas. In the balanced equation we see that the formation of $\mathrm{NF}_{2}$ from $\mathrm{N}_{2} \mathrm{~F}_{4}$ will increase the total number of moles of gases and hence the pressure. Therefore, the system will shift from left to right to reestablish equilibrium. The equilibrium constant will remain unchanged because temperature is held constant.
(d) The function of a catalyst is to increase the rate of a reaction. If a catalyst is added to a reacting system not at equilibrium, the system will reach equilibrium faster than if left undisturbed. If a system is already at equilibrium, as in this case, the addition of a catalyst will not affect either the concentrations of $\mathrm{NF}_{2}$ and $\mathrm{N}_{2} \mathrm{~F}_{4}$ or the equilibrium constant.

