Fuzzy 2

Murat Osmanoglu

Maximizing and Minimizing Set

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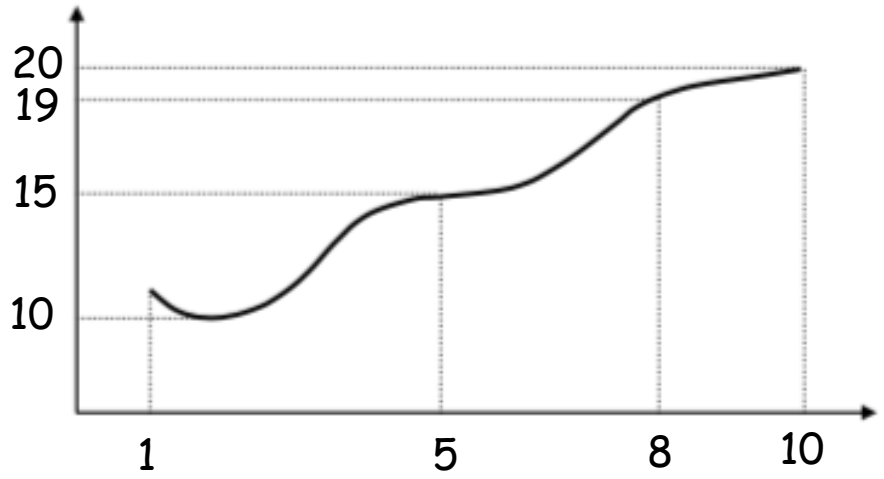
- consider the function f given with the following figure



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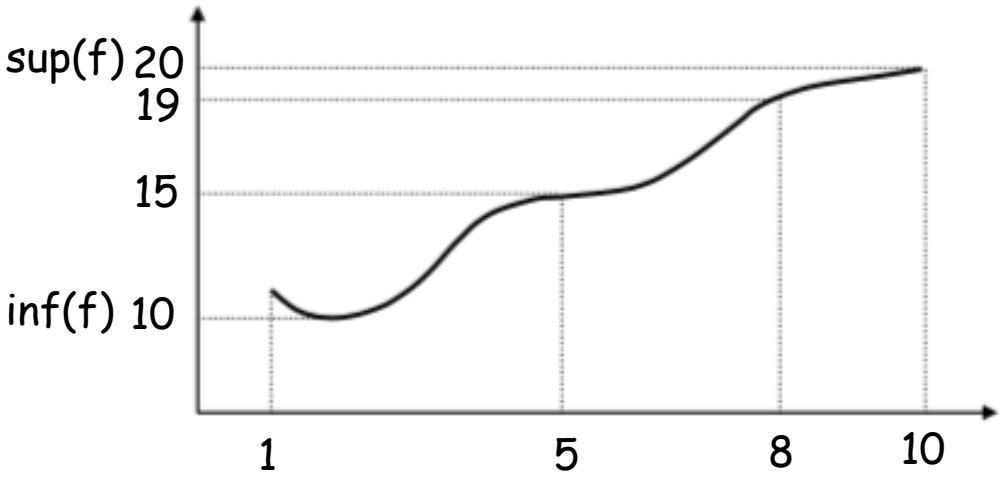

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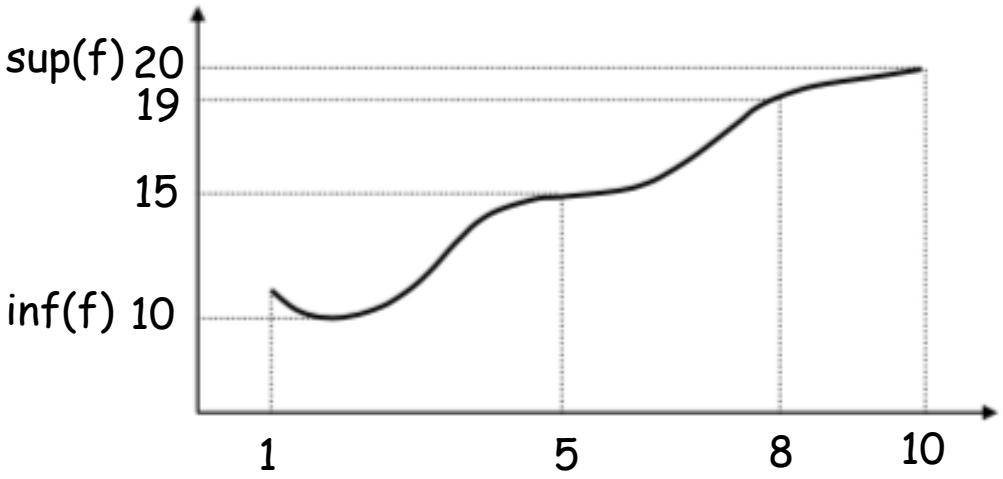

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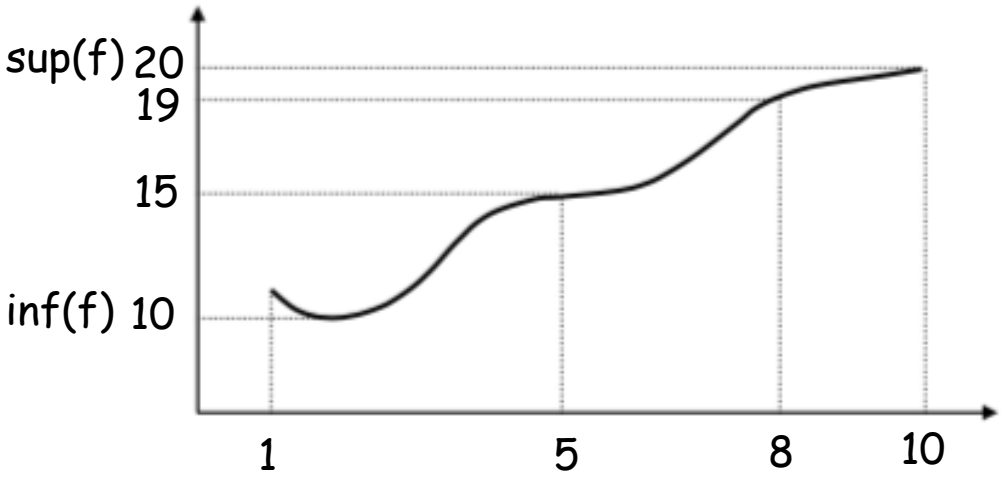

$\mu_M(1) = (11 - 10) / (20 - 10) = 0.1$

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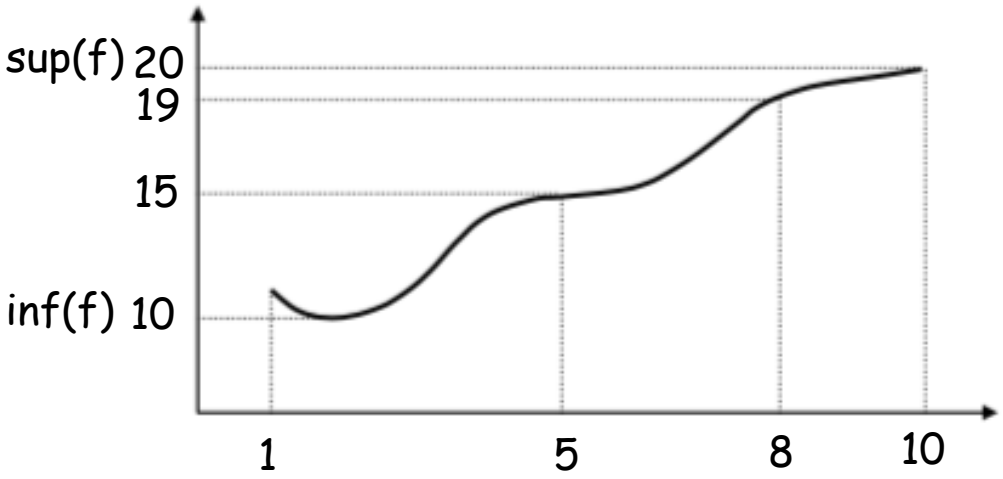

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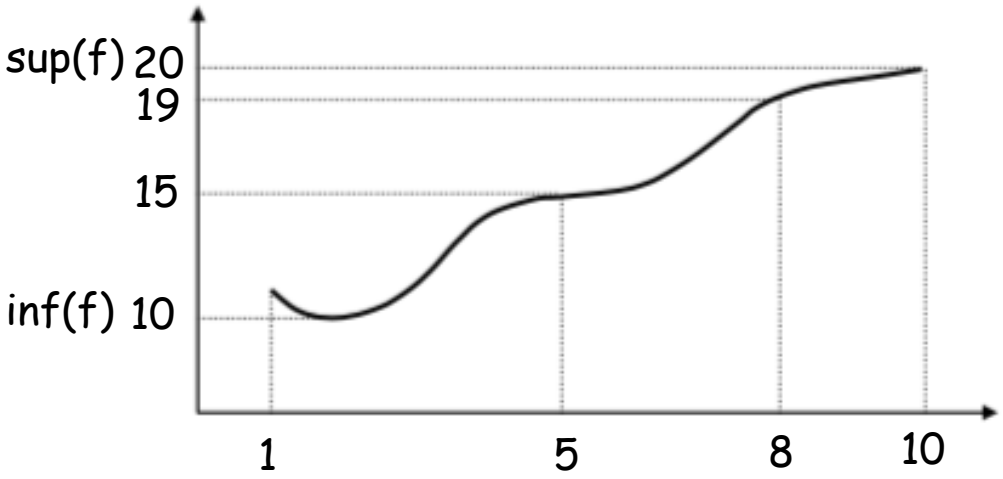

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$$\mu_M(\pi/3) = \frac{\cos(\pi/3) + 1}{2} = 3/4$$

Integration of Fuzzy Function

Integration of fuzzifying function in crisp interval

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Integration of fuzzifying function in crisp interval

- consider the fuzzy bunch of function $F = \{(f_1, 0.6), (f_2, 0.9), (f_3, 0.5)\}$

  where $f_1(x) = 3x$, $f_2(x) = x^2$, $f_3(x) = x - 1$

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- calculate the integration of F in $X = [1, 2]$

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$$I_1(1,2) = \int_1^2 3x dx = \frac{9}{2}$$

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$$\tilde{I}(1,2) = \left\{\left(\frac{9}{2}, 0.6\right), \left(\frac{7}{3}, 0.9\right), \left(\frac{1}{2}, 0.5\right)\right\}$$

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  where A = {(1, 0.5), (2, 1.0), (3, 0.7)} and B = {(3, 0.6), (4, 1.0), (5, 0.3)}

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Differentiation of Fuzzy Function

Differentation of fuzzifying function on crisp point

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- calculate the differentiation of F on $x_0 = 2$

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$$f_1'(x) = 4,\ f_2'(x) = 2x,\ f_3'(x) = 1$$

$$F'(2) = \{(4, 0.6), (4, 0.9), (1, 0.5)\}$$

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- consider the function $f(x) = 3x^3$

Differentiation of Fuzzy Function

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- consider the function $f(x) = 3x^3$
- calculate differentiation of f at A

  where A = {(-2, 0.5), (0, 1.0), (2, 0.7)}

Differentiation of Fuzzy Function

Differentiation of crisp function on fuzzy point

- consider the function $f(x) = 3x^3$
- calculate differentiation of f at A

where $A = \{(-2, 0.5), (0, 1.0), (2, 0.7)\}$

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