























- most popular form of the representation : if-then rule
- the rule is : If x is a, then y is b

the fact is : x is a

the result is : y is b



- most popular form of the representation : if-then rule
- the rule is : If x is a, then y is b

the fact is : x is a

the result is : y is b

modus ponens

fact : x is a rule : if x is a, then y is b result : y is b

- modus tollens
- fact : y is not b
  rule : if x is a, then y is b
  result : a is not b



- most popular form of the representation : if-then rule
- the rule is : If x is a, then y is b

the fact is : x is a

the result is : y is b

modus ponens

fact : x is a
rule : if x is a, then y is b
result : y is b

р	q	p→q	[p∧(p <b>→</b> q)] <b>→</b> q
1	1	1	1
1	0	0	1
0	1	1	1
0	0	1	1



- most popular form of the representation : if-then rule
- the rule is : If x is A, then y is B

the result is : y is B'



- most popular form of the representation : if-then rule
- the rule is : If x is A, then y is B If A(x), then B(y)

the result is : y is B'



- most popular form of the representation : if-then rule
- the rule is : If x is A, then y is B If A(x), then  $B(y) : R(x, y) (A(x) \rightarrow B(y))$ 
  - the fact is  $x ext{ is } A'$
  - the result is : y is B'



- most popular form of the representation : if-then rule
- the rule is : If x is A, then y is B If A(x), then  $B(y) : R(x, y) (A(x) \rightarrow B(y))$

the result is : y is B'

'if temperature is high, then humidity is fairly high' High(x)  $\rightarrow$  Fairly\_High(y)



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the result is : y is B'

modus ponens

fact : x is A'

rule : if x is A, then y is B result : y is B'



- most popular form of the representation : if-then rule
- the rule is : If x is A, then y is B If A(x), then  $B(y) : R(x, y) (A(x) \rightarrow B(y))$

the result is : y is B'

modus ponens

fact: x is A': R(x)rule: if x is A, then y is B : R(x, y)result: y is B':  $R(y) = R(x) \circ R(x, y)$ 



consider the fuzzy rule and the premise given as :

'x and y are approximately equal' and 'x is small'

• consider the fuzzy rule and the premise given as :

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R(x,y)	1	2	3	4
1	1.0	0.5	0	0
2	0.5	1.0	0.5	0
3	0	0.5	1.0	0.5
4	0	0	0.5	1.0

• consider the fuzzy rule and the premise given as :

R(x,y)	1	2	3	4	R(x)	1	2	3	4
1	1.0	0.5	0	0	μ <sub>R</sub> (x)	1.0	0.7	0.4	0.1
2	0.5	1.0	0.5	0					
3	0	0.5	1.0	0.5					
4	0	0	0.5	1.0					

• consider the fuzzy rule and the premise given as :

R(x,y)	1	2	3	4	R(x)	1	2	3	4
1	1.0	0.5	0	0	$\mu_{R}(x)$	1.0	0.7	0.4	0.1
2	0.5	1.0	0.5	0					
3	0	0.5	1.0	0.5	R(y)	1	2	3	4
4	0	0	0.5	1.0	μ <sub>R</sub> (γ)				

• consider the fuzzy rule and the premise given as :

'x and y are approximately equal' and 'x is small' R(x, y) = ApproximatelyEqual(x, y) R(x) = Small(x)

R(x,y)	1	2	3	4	R	(x)	1	2	3	4
1	1.0	0.5	0	0	μ <sup>ε</sup>	(x)	1.0	0.7	0.4	0.1
2	0.5	1.0	0.5	0						
3	0	0.5	1.0	0.5	R	(y)	1	2	3	4
4	0	0	0.5	1.0	μ <sub>ε</sub>	<sub>2</sub> (y)				

 $R(y) = R(x) \circ R(x, y)$ 

• consider the fuzzy rule and the premise given as :

'x and y are approximately equal' and 'x is small' R(x, y) = ApproximatelyEqual(x, y) R(x) = Small(x)

R(x,y)	1	2	3	4	R(x)	1	2	3	4
1	1.0	0.5	0	0	μ <sub>R</sub> (x)	1.0	0.7	0.4	0.1
2	0.5	1.0	0.5	0					
3	0	0.5	1.0	0.5	R(y)	1	2	3	4
4	0	0	0.5	1.0	μ <sub>R</sub> (γ)				

• consider the fuzzy rule and the premise given as :

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R(x,y)	1	2	3	4	R(x)	1	2	3	4
1	1.0	0.5	0	0	$\mu_{R}(x)$	1.0	0.7	0.4	0.1
2	0.5	1.0	0.5	0					
3	0	0.5	1.0	0.5	R(y)	1	2	3	4
4	0	0	0.5	1.0	μ <sub>R</sub> (γ)	1.0			

• consider the fuzzy rule and the premise given as :

'x and y are approximately equal' and 'x is small' R(x, y) = ApproximatelyEqual(x, y) R(x) = Small(x)

R(x,y)	1	2	3	4	R(x)	1	2	3	4
1	1.0	0.5	0	0	μ <sub>R</sub> (x)	1.0	0.7	0.4	0.1
2	0.5	1.0	0.5	0					
3	0	0.5	1.0	0.5	R(y)	1	2	3	4
4	0	0	0.5	1.0	μ <sub>R</sub> (γ)	1.0	0.7	0.5	0.4

• consider the fuzzy rule and the premise given as :

'x and y are approximately equal' and 'x is 2' R(x, y) = ApproximatelyEqual(x, y) R(x) = Small(x)

R(x,y)	1	2	3	4
1	1.0	0.5	0	0
2	0.5	1.0	0.5	0
3	0	0.5	1.0	0.5
4	0	0	0.5	1.0

• consider the fuzzy rule and the premise given as :

'x and y are approximately equal' and 'x is 2' R(x, y) = ApproximatelyEqual(x, y) R(x) = Small(x)

R(x,y)	1	2	3	4	R(x)	1	2	3	4
1	1.0	0.5	0	0	μ <sub>R</sub> (x)	0	1.0	0	0
2	0.5	1.0	0.5	0					
3	0	0.5	1.0	0.5					
4	0	0	0.5	1.0					

• consider the fuzzy rule and the premise given as :

'x and y are approximately equal' and 'x is 2' R(x, y) = ApproximatelyEqual(x, y) R(x) = Small(x)

R(x,y)	1	2	3	4	R(x)	1	2	3	4
1	1.0	0.5	0	0	μ <sub>R</sub> (x)	0	1.0	0	0
2	0.5	1.0	0.5	0					
3	0	0.5	1.0	0.5	R(y)	1	2	3	4
4	0	0	0.5	1.0	μ <sub>R</sub> (γ)	0.5	1.0	0.5	0



- most popular form of the representation : if-then rule
- The rule is :  $R(x, y) (A(x) \rightarrow B(y))$



- most popular form of the representation : if-then rule
- The rule is :  $R(x, y) (A(x) \rightarrow B(y))$

 $\mu_{\mathsf{R}}(\mathsf{x},\mathsf{y}) = \mathsf{f}(\mu_{\mathsf{A}}(\mathsf{x}),\mu_{\mathsf{B}}(\mathsf{y}))$ 



- most popular form of the representation : if-then rule
- The rule is :  $R(x, y) (A(x) \rightarrow B(y))$

 $\mu_{\mathsf{R}}(x, \gamma) = \mathsf{f}(\mu_{\mathsf{A}}(x), \mu_{\mathsf{B}}(\gamma))$ 



 $f(\mu_A(x), \mu_B(y)) = \mu_A(x) \wedge \mu_B(y)$ 



- most popular form of the representation : if-then rule
- The rule is :  $R(x, y) (A(x) \rightarrow B(y))$



Mamdani

 $f(\mu_A(x), \mu_B(y)) = \mu_A(x) \wedge \mu_B(y)$ 

Larsen

 $\mathsf{f}(\mu_{A}(\mathsf{x}),\,\mu_{B}(\mathsf{y}))=\mu_{A}(\mathsf{x})\,.\,\mu_{B}(\mathsf{y})$ 



'if temperature is high, then humidity is fairly high'  $R(t, h) = A(t) \rightarrow B(h)$  where A in T and B in H



'if temperature is high, then humidity is fairly high'

 $R(t, h) = A(t) \rightarrow B(h)$  where A in T and B in H





'if temperature is high, then humidity is fairly high'

 $R(t, h) = A(t) \rightarrow B(h)$  where A in T and B in H

0.6



90

1.0

	Marr	ndani		
R(†,h)	40	60	80	90
10				
20				
30				
40				



'if temperature is high, then humidity is fairly high'

 $R(t, h) = A(t) \rightarrow B(h)$  where A in T and B in H



B = 'fairly high'

Α	10	20	30	40	В		40	60	80	90
μ <sub>Α</sub> (†)	0.1	0.2	0.6	0.9	μ <sub>B</sub> (Ͱ	)	0.3	0.5	0.8	1.0

	Marr	ndani		
R(†,h)	40	60	80	90
10	0.1	0.1	0.1	0.1
20				
30				
40				



'if temperature is high, then humidity is fairly high'

 $R(t, h) = A(t) \rightarrow B(h)$  where A in T and B in H



B = 'fairly high'

Α	10	20	30	40	В	40	60	80	90
μ <sub>A</sub> (†)	0.1	0.2	0.6	0.9	μ <sub>B</sub> (h)	0.3	0.5	0.8	1.0

	Marr			
R(†,h)	40	60	80	90
10	0.1	0.1	0.1	0.1
20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
40	0.3	0.5	0.8	0.9



'if temperature is high, then humidity is fairly high'

 $R(t, h) = A(t) \rightarrow B(h)$  where A in T and B in H



	1	r ·	• .	1 1	•	. 1	1
В	= 1	ra I	Irl	V	n	a	n
-						J	

A	10	20	30	40	
μ <sub>A</sub> (†)	0.1	0.2	0.6	0.9	

В	40	60	80	90
μ <sub>B</sub> (h)	0.3	0.5	0.8	1.0

Mamdani					Lar	sen		
R(†,h)	40	60	80	90	R(†,h)	40	60	80
10	0.1	0.1	0.1	0.1	10			
20	0.2	0.2	0.2	0.2	20			
30	0.3	0.5	0.6	0.6	30			
40	0.3	0.5	0.8	0.9	40			



'if temperature is high, then humidity is fairly high'

 $R(t, h) = A(t) \rightarrow B(h)$  where A in T and B in H



B =	'fairly	high'

A	10	20	30	40	
μ <sub>A</sub> (†)	0.1	0.2	0.6	0.9	-

60

0.1

0.2

0.5

0.5

80

0.1

0.2

0.6

0.8

90

0.1

0.2

0.6

0.9

В	40	60	80	90
μ <sub>B</sub> (h)	0.3	0.5	0.8	1.0

40

0.1

0.2

0.3

0.3

R(t,h)

10

20

30

Larsen								
	R(†,h)	40	60	80	90			
	10	0.03						
	20							
	30							
	40							



'if temperature is high, then humidity is fairly high'

 $R(t, h) = A(t) \rightarrow B(h)$  where A in T and B in H



B =	'fairly	high'

A	10	20	30	40	
μ <sub>A</sub> (†)	0.1	0.2	0.6	0.9	-

В	40	60	80	90
μ <sub>B</sub> (h)	0.3	0.5	0.8	1.0

R(t,h)

10

20

30

Mam	dani			Larsen						
40	60	80	90		R(t,h)	40	60	80	90	_
0.1	0.1	0.1	0.1		10	0.03	0.05			-
0.2	0.2	0.2	0.2		20					
0.3	0.5	0.6	0.6		30					
0.3	0.5	0.8	0.9		40					



'if temperature is high, then humidity is fairly high'

 $R(t, h) = A(t) \rightarrow B(h)$  where A in T and B in H



B	Ξ	'fa	ir		hi	ia	h
	_	10		'7		'9	

1

A	10	20	30	40	_
ı <sub>A</sub> (†)	0.1	0.2	0.6	0.9	-

В	40	60	80	90
μ <sub>B</sub> (h)	0.3	0.5	0.8	1.0

Larsen

Mama	lani
Mana	IUUU

R(†,h)	40	60	80	90	R(†,h)	40	60	80	90
10	0.1	0.1	0.1	0.1	10	0.03	0.05	0.08	
20	0.2	0.2	0.2	0.2	20				
30	0.3	0.5	0.6	0.6	30				
40	0.3	0.5	0.8	0.9	40				



'if temperature is high, then humidity is fairly high'

 $R(t, h) = A(t) \rightarrow B(h)$  where A in T and B in H



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	-	JU		I Y	1110	11
					-	

I

A	10	20	30	40	_
J <sub>A</sub> (†)	0.1	0.2	0.6	0.9	-

60

0.1

0.2

0.5

0.5

80

0.1

0.2

0.6

0.8

90

0.1

0.2

0.6

0.9

40

В	40	60	80	90
μ <sub>B</sub> (h)	0.3	0.5	0.8	1.0

Mamdani	

40

0.1

0.2

0.3

0.3

R(t,h)

10

20

30

R(†,h)	40	60	80	90
10	0.03	0.05	0.08	0.1
20				
30				
	R(t,h) 10 20 30	Lar R(t,h) 40 10 0.03 20 30	LarsenR(t,h)4060100.030.0520	LarsenR(t,h)406080100.030.050.082030



'if temperature is high, then humidity is fairly high'

 $R(t, h) = A(t) \rightarrow B(h)$  where A in T and B in H



Α	10	20	30	40
μ <sub>A</sub> (†)	0.1	0.2	0.6	0.9

B = 'fairly high'

В	40	60	80	90
μ <sub>B</sub> (h)	0.3	0.5	0.8	1.0

	l	
<i>/</i> V\	ama	ani

R(†,h)	40	60	80	90	
10	0.1	0.1	0.1	0.1	
20	0.2	0.2	0.2	0.2	
30	0.3	0.5	0.6	0.6	
40	0.3	0.5	0.8	0.9	

R(†,h)	40	60	80	90
10	0.03	0.05	0.08	0.1
20	0.06	0.1	0.16	0.2
30	0.18	0.3	0.48	0.6
40	0.27	0.45	0.72	0.9



R(t,h) = 'if temperature is high, then humidity is fairly high'

A' = ' temperature is fairly high'



R(t,h) = 'if temperature is high, then humidity is fairly high'

A' = ' temperature is fairly high'

A' = 'fairly high'

A'	10	20	30	40
μ <sub>A</sub> (†)	0.02	0.15	0.5	0.8

R(†,h)	40	60	80	90
10	0.1	0.1	0.1	0.1
20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
40	0.3	0.5	0.8	0.9



R(t,h) = 'if temperature is high, then humidity is fairly high'

T

A' = ' temperature is fairly high'

A 1		11	•		1	•	1 /
A	Ξ	ta	Irl	V	n	<b>  </b>	n
		•				<u> </u>	

0.3

0.5

0.8

 $\mu_{B'}(h)$ 

	<b>^</b> '	10		20	20	40		R(†,h)	40	60	80	90	
	A	10		20	30	40	-	10	0.1	0.1	0.1	0.1	
μ,	<sub>A</sub> (†)	0.0	2	0.15	0.5	0.8			0.1	0.1	0.2	0	
	I							20	0.2	0.2	0.2	0.2	
								30	0.3	0.5	0.6	0.6	
								40	0.3	0.5	0.8	0.9	
Mamdani													
	B		40	)	60	80	90						

0.8

<u>Multiple Input Multiple Output</u>

• R: if  $x_1$  is  $A_1, x_2$  is  $A_2, \dots, x_n$  is  $A_n$ , then  $z_1$  is  $C_1, z_2$  is  $C_2, \dots, z_m$  is  $C_m$ 

Multiple Input Multiple Output

• R: if  $x_1$  is  $A_1, x_2$  is  $A_2, \dots, x_n$  is  $A_n$ , then  $z_1$  is  $C_1, z_2$  is  $C_2, \dots, z_m$  is  $C_m$ 

$$\begin{aligned} & R_{1}: \text{ if } x_{1} \text{ is } A_{1}, x_{2} \text{ is } A_{2}, ..., x_{n} \text{ is } A_{n}, \text{ then } z_{1} \text{ is } C_{1} \\ & R_{2}: \text{ if } x_{1} \text{ is } A_{1}, x_{2} \text{ is } A_{2}, ..., x_{n} \text{ is } A_{n}, \text{ then } z_{2} \text{ is } C_{2} \\ & \dots \\ & R_{m}: \text{ if } x_{1} \text{ is } A_{1}, x_{2} \text{ is } A_{2}, ..., x_{n} \text{ is } A_{n}, \text{ then } z_{m} \text{ is } C_{m} \end{aligned}$$

Multiple Input Multiple Output

• R: if  $x_1$  is  $A_1, x_2$  is  $A_2, \dots, x_n$  is  $A_n$ , then  $z_1$  is  $C_1, z_2$  is  $C_2, \dots, z_m$  is  $C_m$ 

$$R_1$$
: if  $x_1$  is  $A_1$ ,  $x_2$  is  $A_2$ , ...,  $x_n$  is  $A_n$ , then  $z_1$  is  $C_1$   
 $R_2$ : if  $x_1$  is  $A_1$ ,  $x_2$  is  $A_2$ , ...,  $x_n$  is  $A_n$ , then  $z_2$  is  $C_2$   
...

 $R_m$ : if  $x_1$  is  $A_1, x_2$  is  $A_2, ..., x_n$  is  $A_n$ , then  $z_m$  is  $C_m$ 

• a multiple input multiple output fuzzy system can considered as a collection of multiple input single output fuzzy systems

$$R = \{R_1, R_2, ..., R_m\}$$

Two Input Single Output

• input : x is A' and y is B'

. . .

 $R_1$ : if x is  $A_1$  and y is  $B_1$ , then z is  $C_1$  $R_2$ : if x is  $A_2$  and y is  $B_2$ , then z is  $C_2$ 

 $R_m$ : if x is  $A_m$  and y is  $B_2$ , then z is  $C_m$ output : z is C'

Two Input Single Output

• input : x is A' and y is B'

 $R_1$ : if x is  $A_1$  and y is  $B_1$ , then z is  $C_1$ 

 $R_2$ : if x is  $A_2$  and y is  $B_2$ , then z is  $C_2$ 

 $R_m$ : if x is  $A_m$  and y is  $B_2$ , then z is  $C_m$ 

output : z is C'

. . .

•  $R_i$ : if x is  $A_i$  and y is  $B_i$ , then z is  $C_i$  $R_i$ : ( $A_i$  and  $B_i$ )  $\rightarrow C_i$ 

Two Input Single Output

• input : x is A' and y is B'

. . .

 $R_1$ : if x is  $A_1$  and y is  $B_1$ , then z is  $C_1$ 

 $R_2$ : if x is  $A_2$  and y is  $B_2$ , then z is  $C_2$ 

 $R_m$ : if x is  $A_m$  and y is  $B_2$ , then z is  $C_m$ output : z is C'

•  $R_i$ : if x is  $A_i$  and y is  $B_i$ , then z is  $C_i$   $R_i$ :  $(A_i$  and  $B_i$ )  $\rightarrow C_i$  $R_i$ :  $(A_i \rightarrow C_i)$  and  $(B_i \rightarrow C_i)$ 

Two Input Single Output

. . .

• input : x is A' and y is B'

 $R_1$ : if x is  $A_1$  and y is  $B_1$ , then z is  $C_1$ 

 $R_2$ : if x is  $A_2$  and y is  $B_2$ , then z is  $C_2$ 

 $R_m$ : if x is  $A_m$  and y is  $B_2$ , then z is  $C_m$ output : z is C'

•  $R_i$ : if x is  $A_i$  and y is  $B_i$ , then z is  $C_i$   $R_i$ :  $(A_i$  and  $B_i$ )  $\rightarrow C_i$   $R_i$ :  $(A_i \rightarrow C_i)$  and  $(B_i \rightarrow C_i)$  $R_i = R_{i1} \wedge R_{i2}$ 



- most popular form of the representation : if-then rule
- the rule is : If x is A, then z is C If A(x), then  $C(z) : R(x, z) (A(x) \rightarrow C(z))$ 

  - the result is  $z \in C'$

 $\frac{Mamdani}{R(y) = R(x) \circ R(x, z)}$ 

min for the implication

Larsen  $R(y) = R(x) \circ R(x, z)$ 

product for the implication