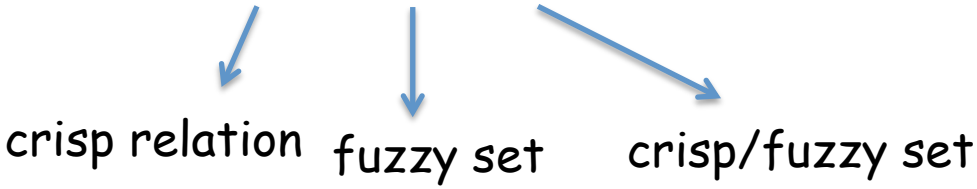
Fuzzy 4

Murat Osmanoglu

Extension of Fuzzy Set

$$R \subseteq A \times B$$



Extension by Crisp Relation

- $B' \subseteq B$ induced by the crisp relation R and the fuzzy set A:

$$B' = \{(y, \mu_{B'}(y)) \mid \mu_{B'}(y) = \max_{x \text{ s.t. } (x,y) \text{ in } R} \mu_A(x)\}$$

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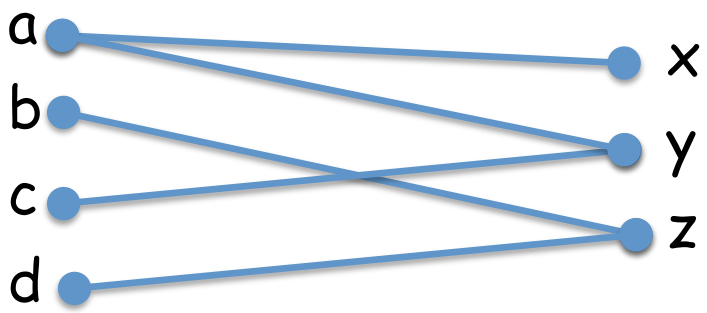

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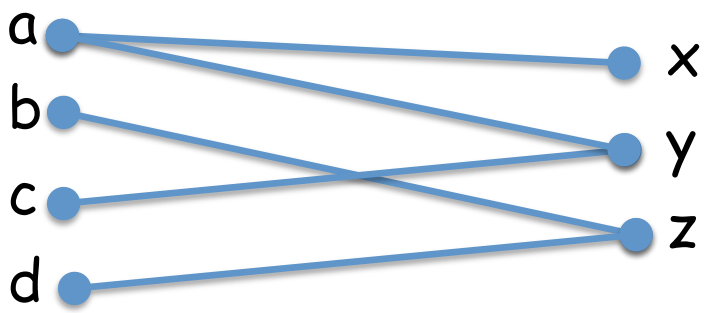

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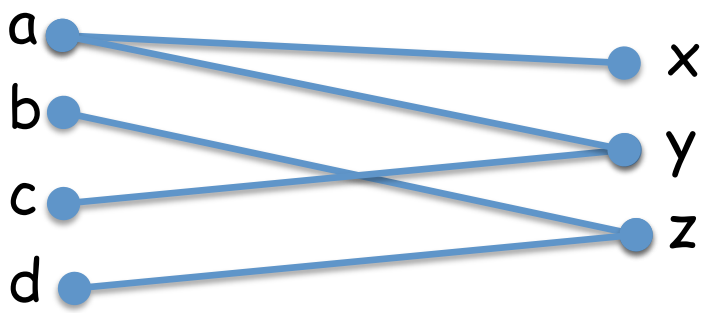

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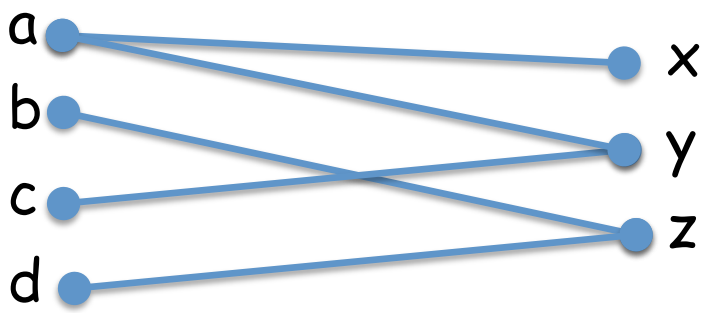

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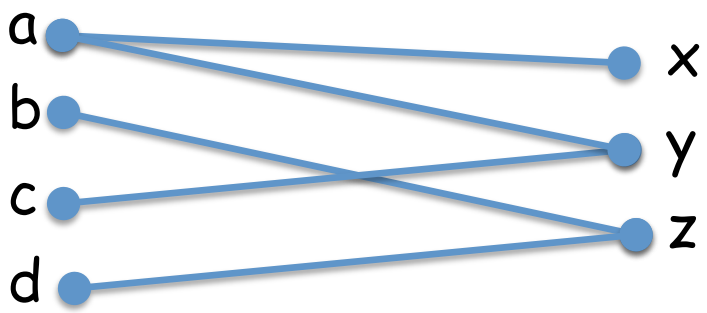

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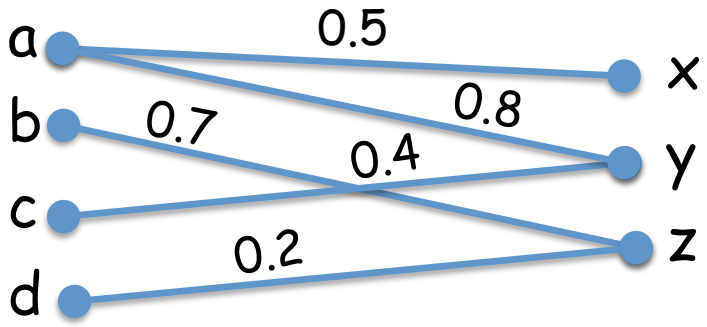

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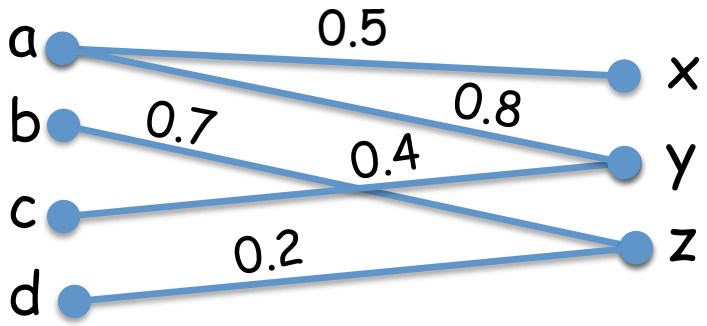

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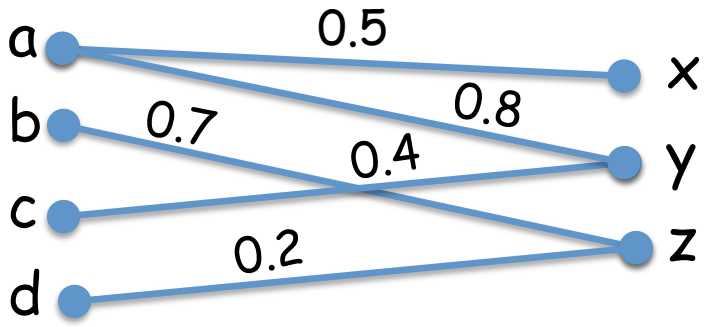

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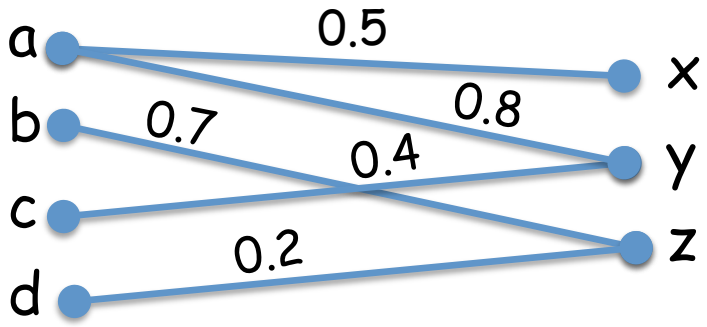

B' = {(x, 0.5), (y, 0.6), (z, 0.7)}

Fuzzy Distance

- calculate the fuzzy distance between the fuzzy sets
  $A = \{(1, 0.5), (2, 1.0), (3, 0.7)\}$ and $B = \{(3, 0.6), (4, 1.0), (5, 0.3)\}$

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Probability vs. Possibility

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- A, B, C, D organize a chess tournament. The following table shows the probabilities and the possibilities of the players on the tournament

	A	B	C	D
P(x)	0.5	0.3	0.2	0
μ(x)	1.0	0.7	0.4	0.1

$p(x) \leq \mu(x)$

Fuzzy Event

Crisp Probability of Fuzzy Event

- consider the sample space S = {a, b, c, d} with the probabilities

$$p(a) = 0.4,\ p(b) = 0.2,\ p(c) = 0.1,\ p(d) = 0.3$$

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- the probability of the fuzzy event A={(a, 0.5), (b, 1.0), (c, 0.3)}

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$$p(A) = 0.4x0.5 + 0.2x1.0 + 0.1x0.3 = 0.43$$

Fuzzy Event

Fuzzy Probability of Fuzzy Event

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$$A_{0.3} = \{a, b, c\},\ A_{0.5} = \{a, b\},\ A_{1.0} = \{b\}$$

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$$p(A_{0.3}) = 0.7,\ \ p(A_{0.5}) = 0.6,\ \ p(A_{1.0}) = 0.2$$

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$$p(A) = \{(0.7, 0.3), (0.6, 0.5), (0.2, 1.0)\}$$