Fuzzy 5

Murat Osmanoglu

Uncertainty

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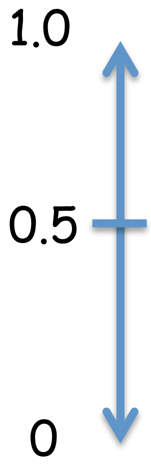

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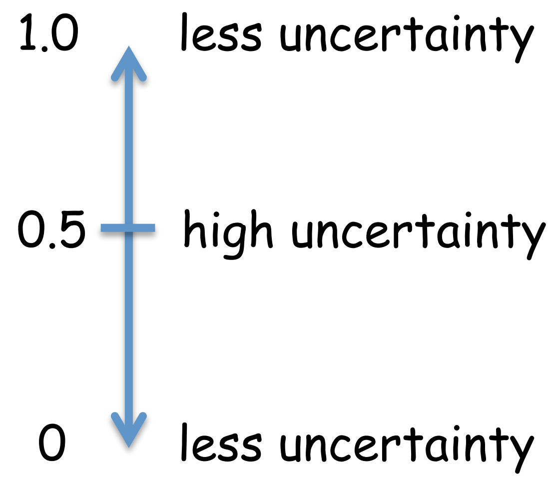

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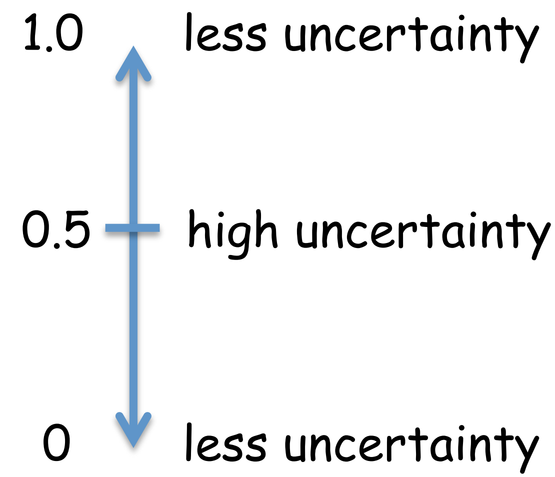

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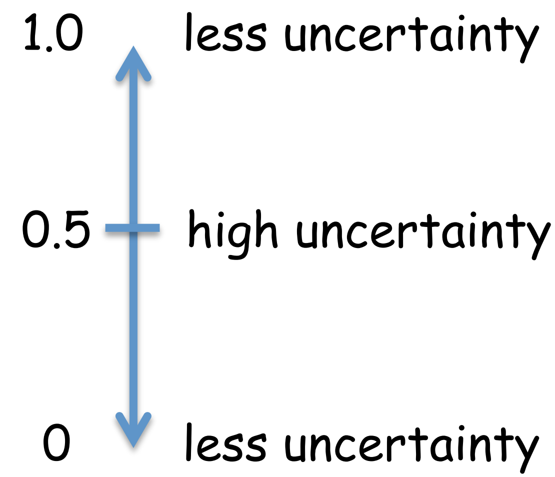

- Given fuzzy set $A = \{(a, 1.0), (b, 0.7), (c, 0.4), (d, 0.1)\}$,

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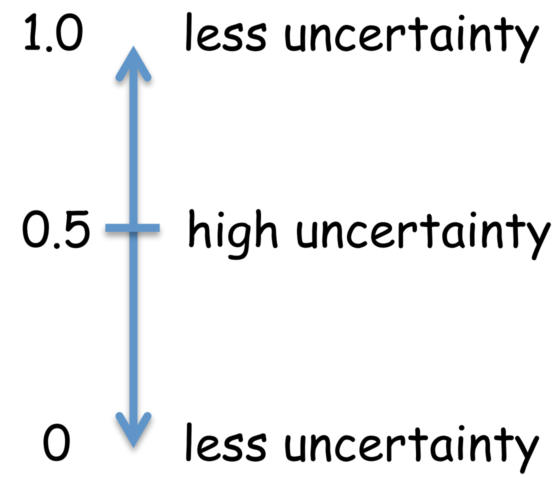

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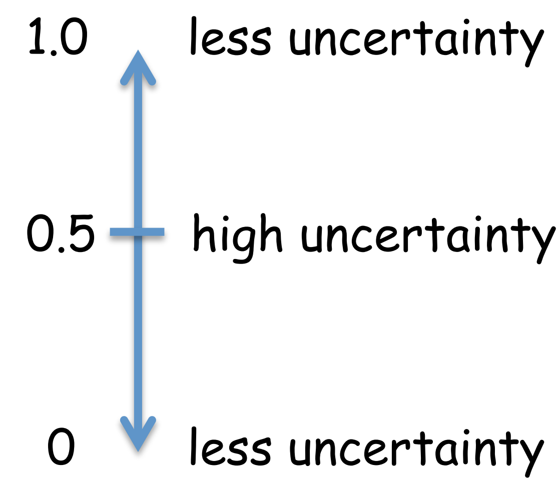

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  How do we measure the fuzziness of the fuzzy set A ?

  How do we compare the fuzziness of two fuzzy sets ?

  How do we decide which one is more uncertain ?

Measure of Fuzziness

$$f : P(X) \rightarrow R$$

all subsets of the universal set

real numbers

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- Given fuzzy sets

$A = \{(a, 1.0), (b, 0.7), (c, 0.4)\}$, $B = \{(a, 0.5), (b, 0.5), (c, 0.5)\}$

$f(A) < f(B)$ and $f(B)$ should be maximum

Measure of Fuzziness

Measure with Entropy

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$p(00) = 3/4$, $p(01) = 1/8$, $p(10) = 1/16$, $p(11) = 1/16$

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$p(00) = 3/4, \; p(01) = 1/8, \; p(10) = 1/16, \; p(11) = 1/16$

$$H(p(x)) = - (3/4) \log (3/4) - (1/8) \log (1/8) - (1/16) \log (1/16) - (1/16) \log (1/16)$$

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$$H(p(x)) = (3/2 + 3/8 + 1/4 + 1/4) - \log 3$$

$$H(p(x)) \approx 0.791$$

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$$H(p(x)) = 1/2 + 1/2 + 1/2 + 1/2$$

$$H(p(x)) = 2$$

Measure of Fuzziness

Measure with Entropy

- $f(A) = -\sum_{x \text{ in } X} [\mu_A(x).\log \mu_A(x) + (1 - \mu_A(x)).\log (1 - \mu_A(x))]$

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- $A = \{(a, 0.6), (b, 0.5), (c, 0.2)\}$

  $f(A) = - (0.6 \log 0.6 + 0.5 \log 0.5 + 0.2 \log 0.2$

  $0.4 \log 0.4 + 0.5 \log 0.5 + 0.8 \log 0.8)$

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$f(A) = 2.686$

Measure of Fuzziness

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  $f(A) = 2.686$, $f'(A) = 2.686/3 = 0.89$

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- $B = \{(a, 0.5), (b, 0.5), (c, 0.5)\}$

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  $f(B) = 3$

Measure of Fuzziness

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  $f(B) = 3$, $f'(B) = 1$

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Measure with Metric Distance

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- $f(A) = \Sigma_{x \text{ in } X} (0.5 - |\mu_A(x) - 0.5|)$

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- $f(A) = \Sigma_{x \text{ in } X} (0.5 - |\mu_A(x) - 0.5|)$
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  $f(A) = (0.5 - |0.6 - 0.5|) + (0.5 - |0.5 - 0.5|) + (0.5 - |0.2 - 0.5|)$

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  $f(A) = 0.4 + 0.5 + 0.2 = 1.1$, $f'(A) = 1.1/1.5 = 0.73$

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- $B = \{(a, 0.5), (b, 0.5), (c, 0.5)\}$

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